AdS/CFT
and
condensed matter

Reviews:

arXiv:0907.0008

arXiv:0901.4103

arXiv:0810.3005 (with Markus Mueller)

Talk online: sachdev.physics.harvard.edu

Lars Fritz, Harvard
Victor Galitski, Maryland
Max Metlitski, Harvard
Eun Gook Moon, Harvard
Markus Mueller, Trieste
Yang Qi, Harvard
Joerg Schmalian, Iowa
Cenke Xu, Harvard

Frederik Denef, Harvard
Sean Hartnoll, Harvard
Christopher Herzog, Princeton
Pavel Kovtun, Victoria
Dam Son, Washington

PHYSICS
HARVARD

Outline

A. "Relativistic" field theories of quantum phase transitions

1. Coupled dimer antiferromagnets

2. Triangular lattice antiferromagnets

3. Graphene

4. AdS/CFT and quantum critical transport

Outline

B. Finite density quantum matter

1. Graphene

   Fermi surfaces and Fermi liquids

2. Quantum phase transitions of Fermi liquids

   Pomeranchuk instability and spin density waves;
   Fermi surfaces and "non-Fermi liquids"

3. AdS_2 theory

4. Cuprate superconductivity

Outline

A. "Relativistic" field theories of quantum phase transitions

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A. "Relativistic" field theories of quantum phase transitions

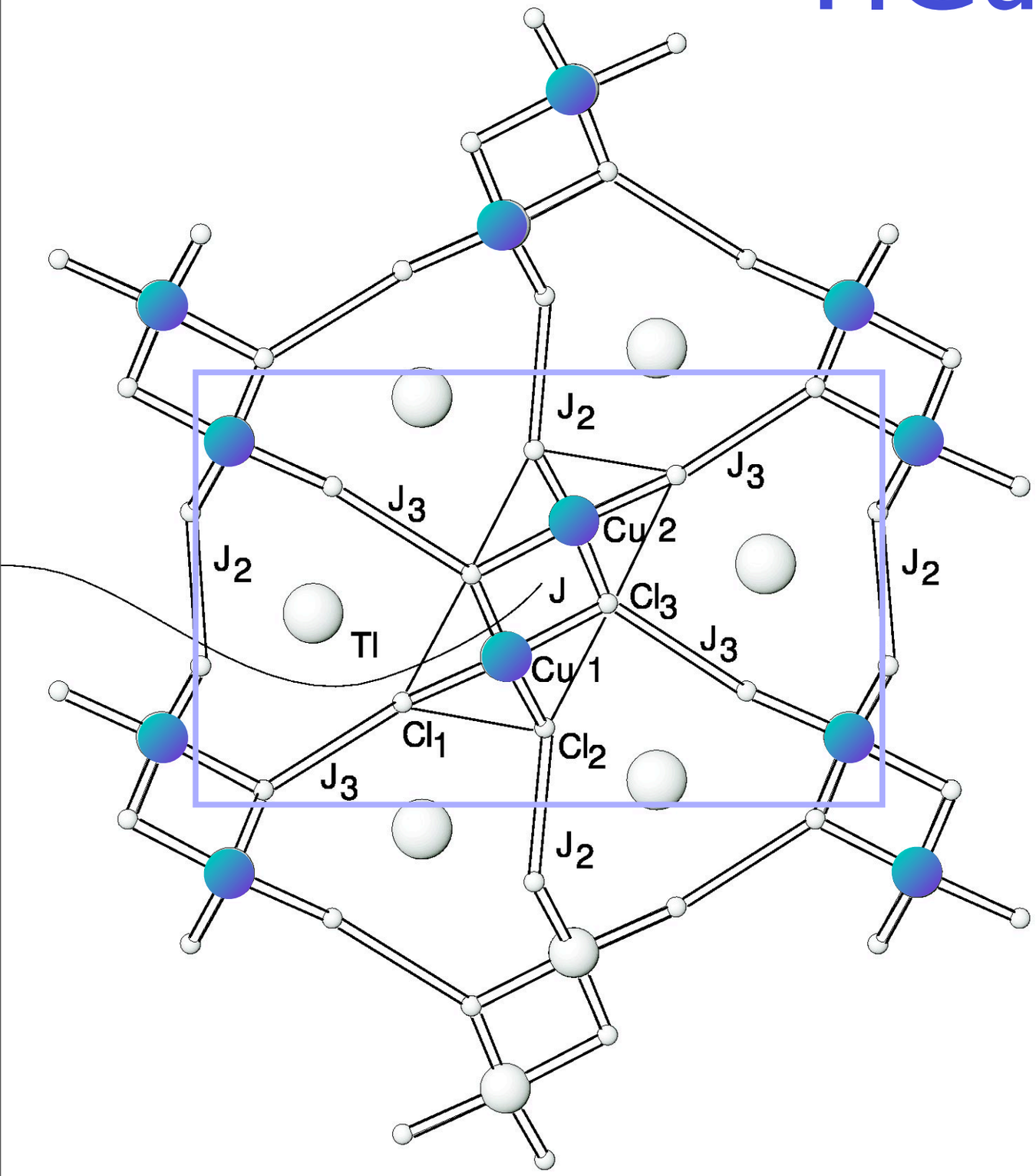
1. Coupled dimer antiferromagnets

2. Triangular lattice antiferromagnets

3. Graphene

4. AdS/CFT and quantum critical transport

$TlCuCl_3$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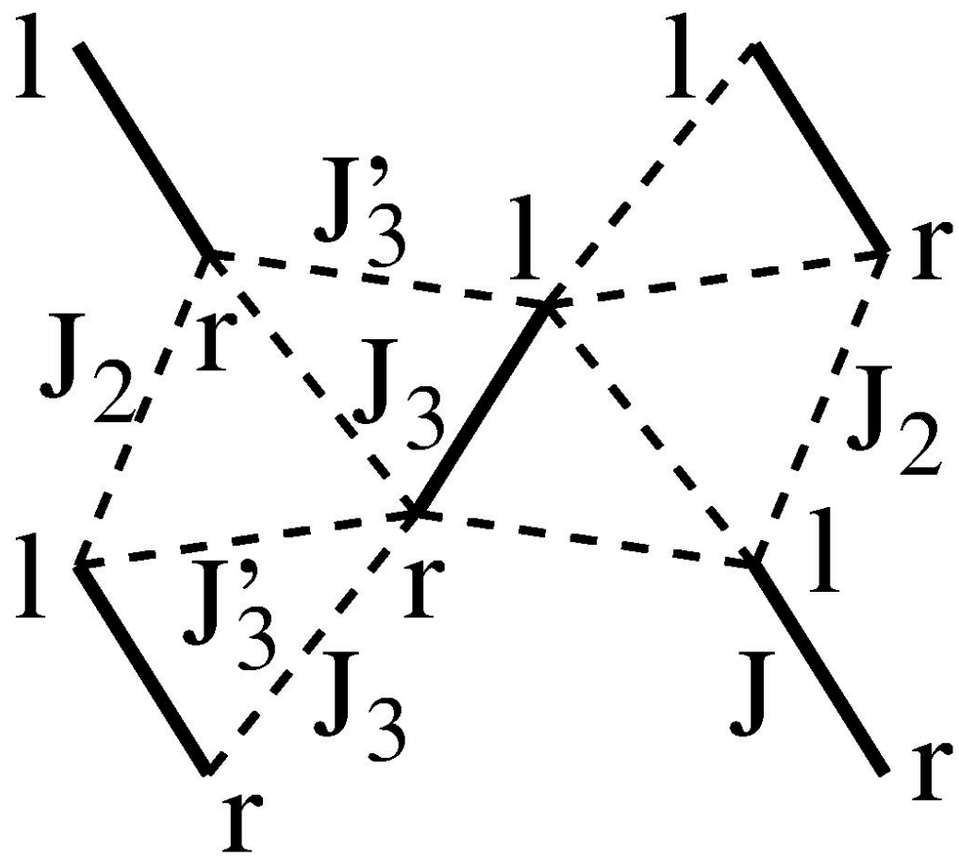
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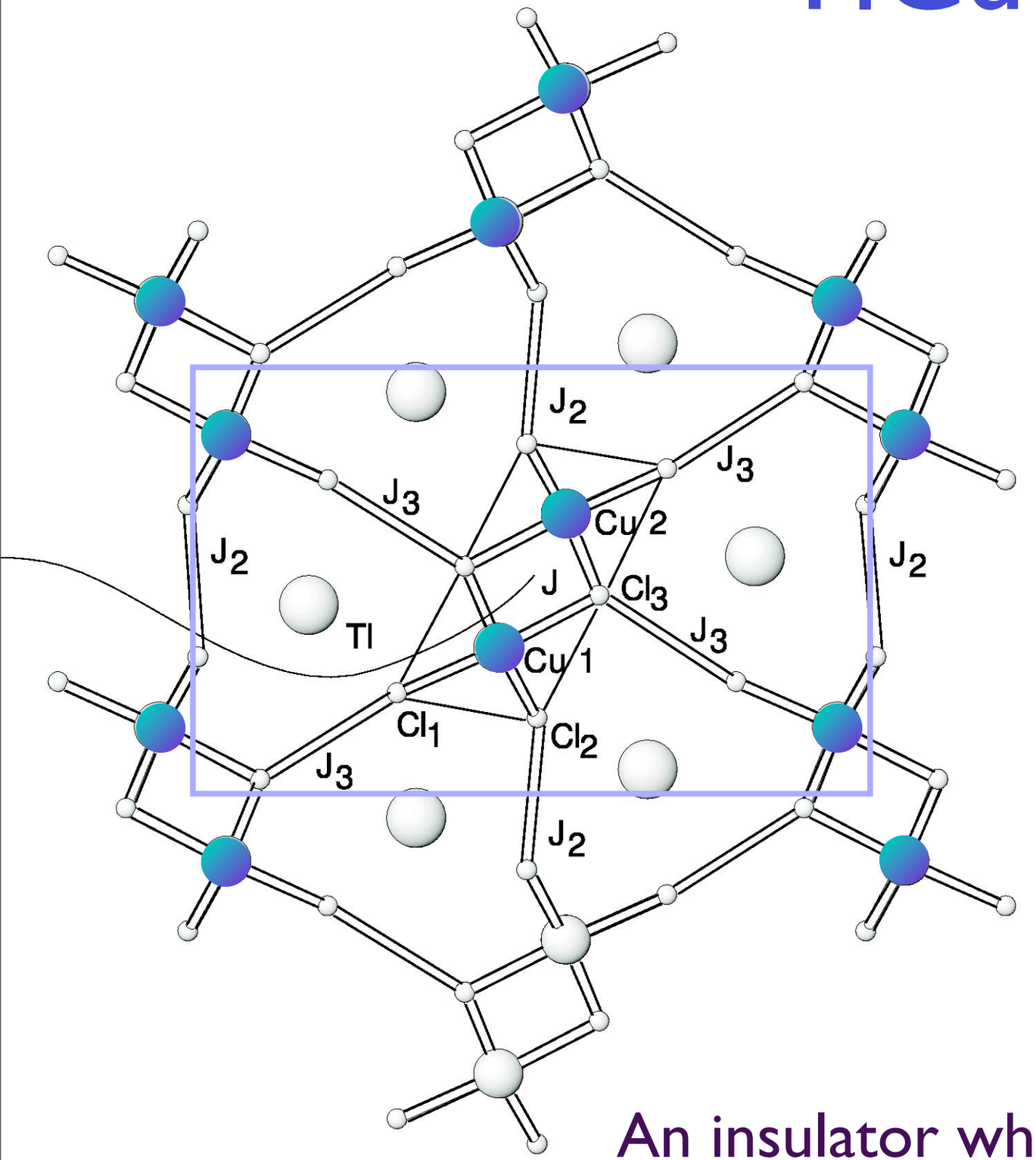
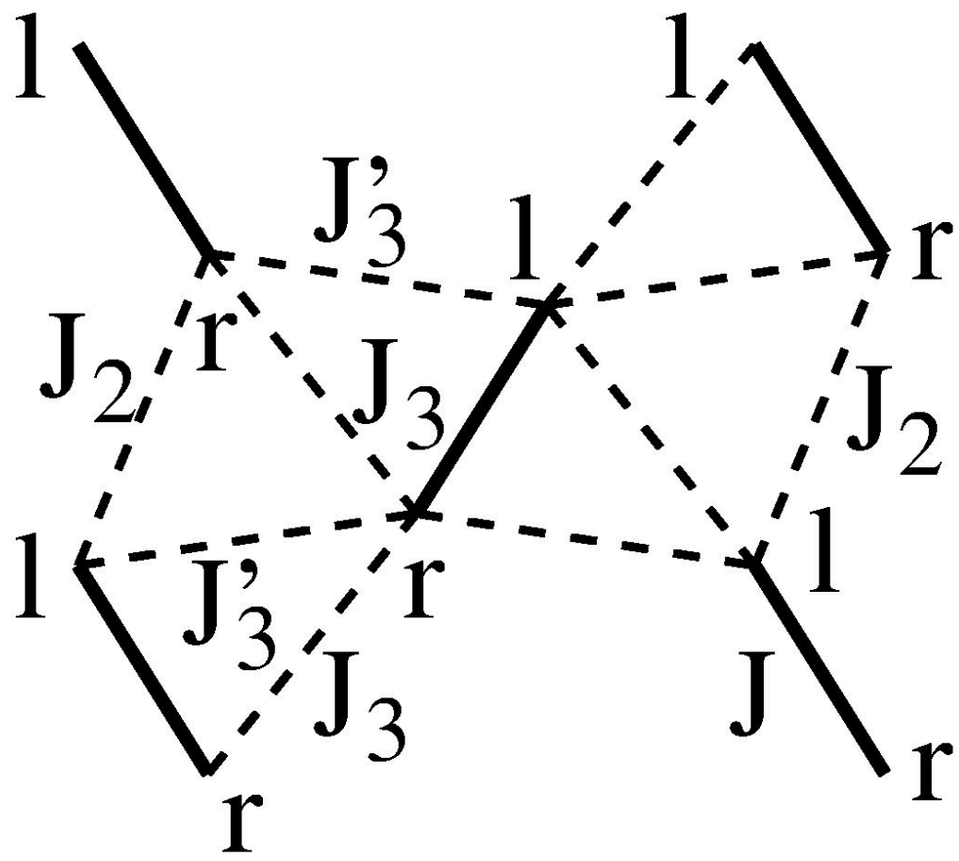
$TlCuCl_3$

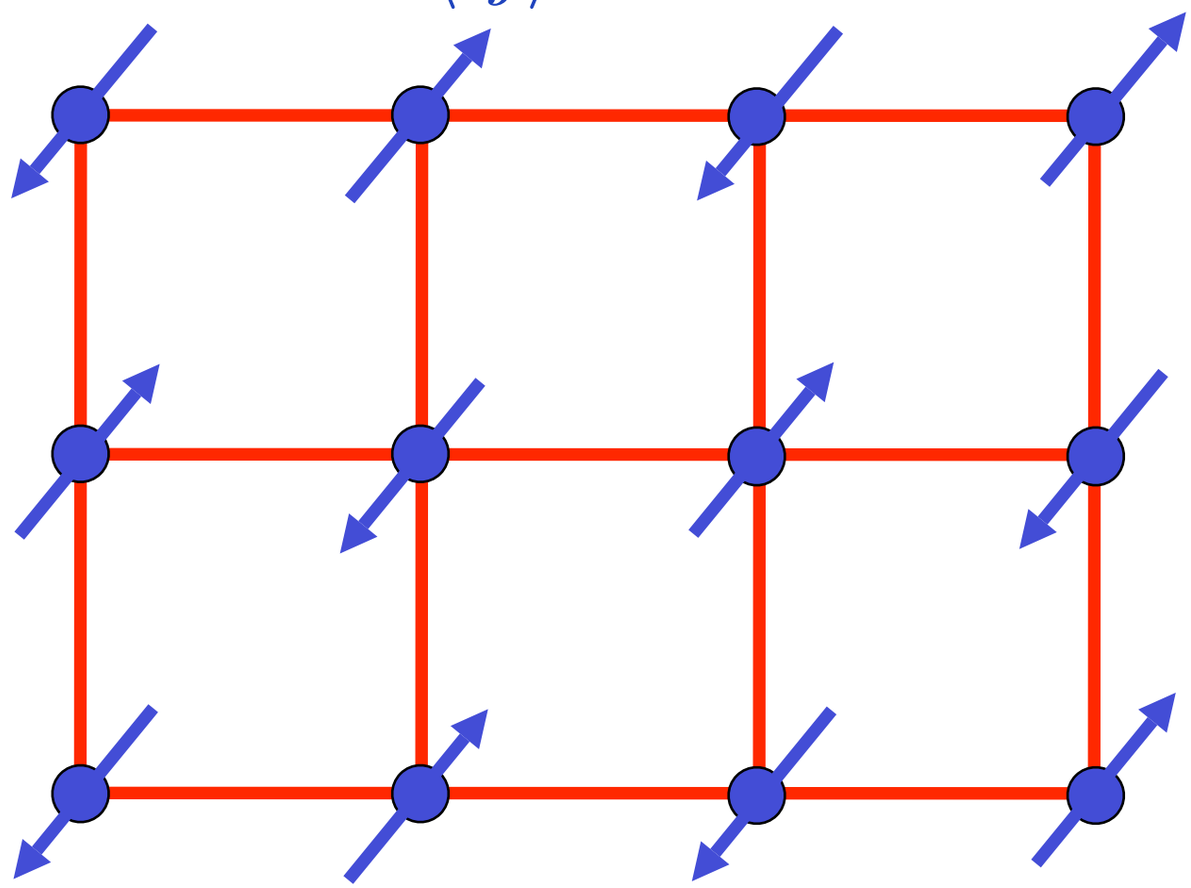




An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

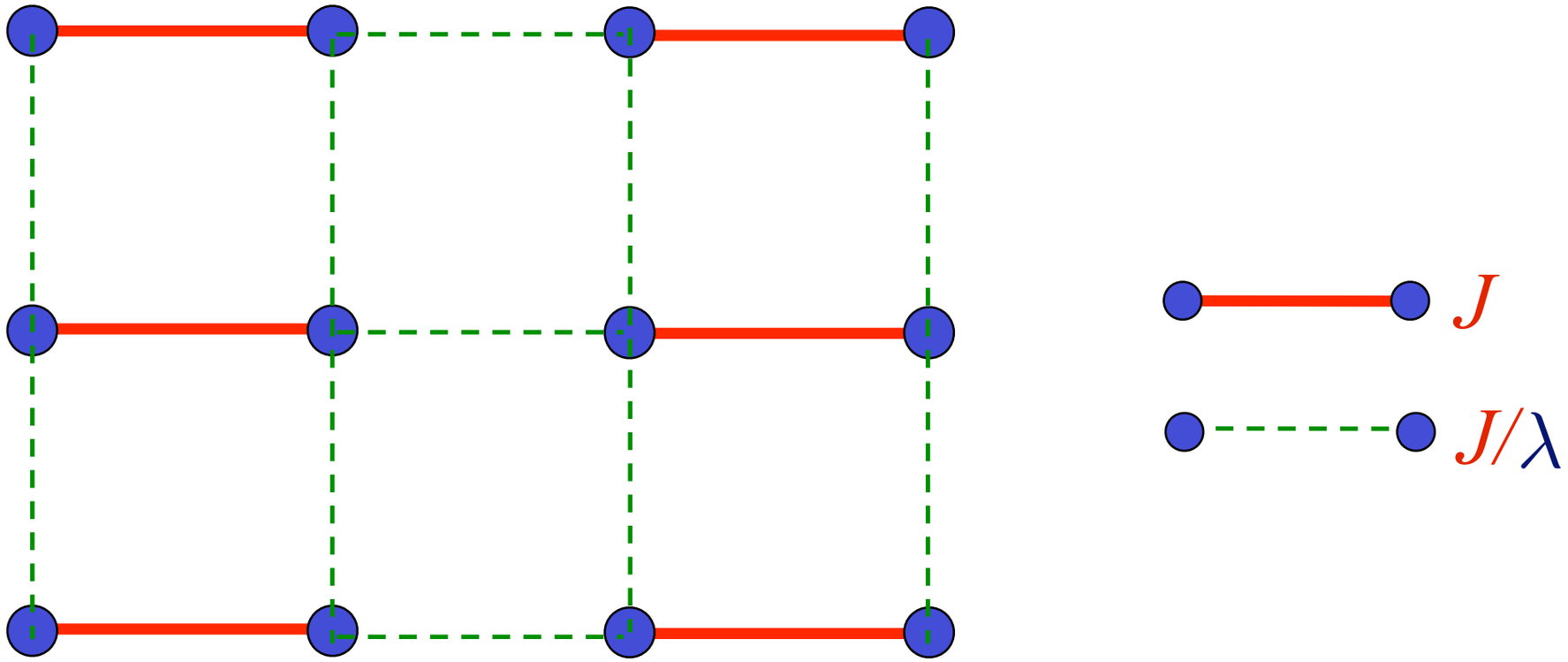
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

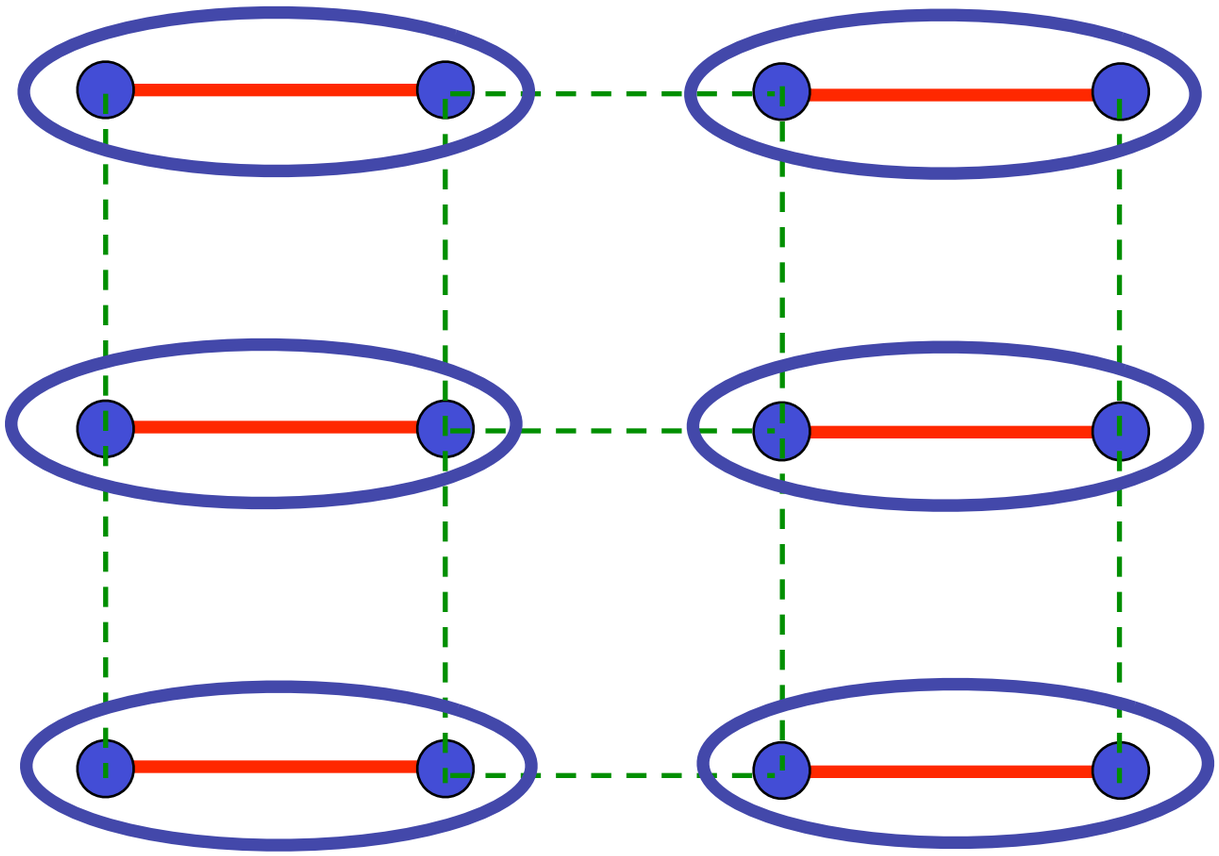
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

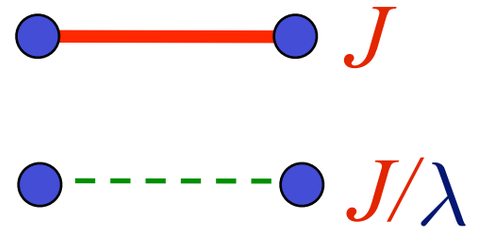


Weaken some bonds to induce spin entanglement in a new quantum phase

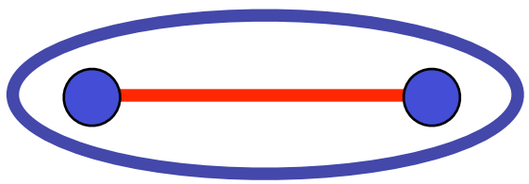
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

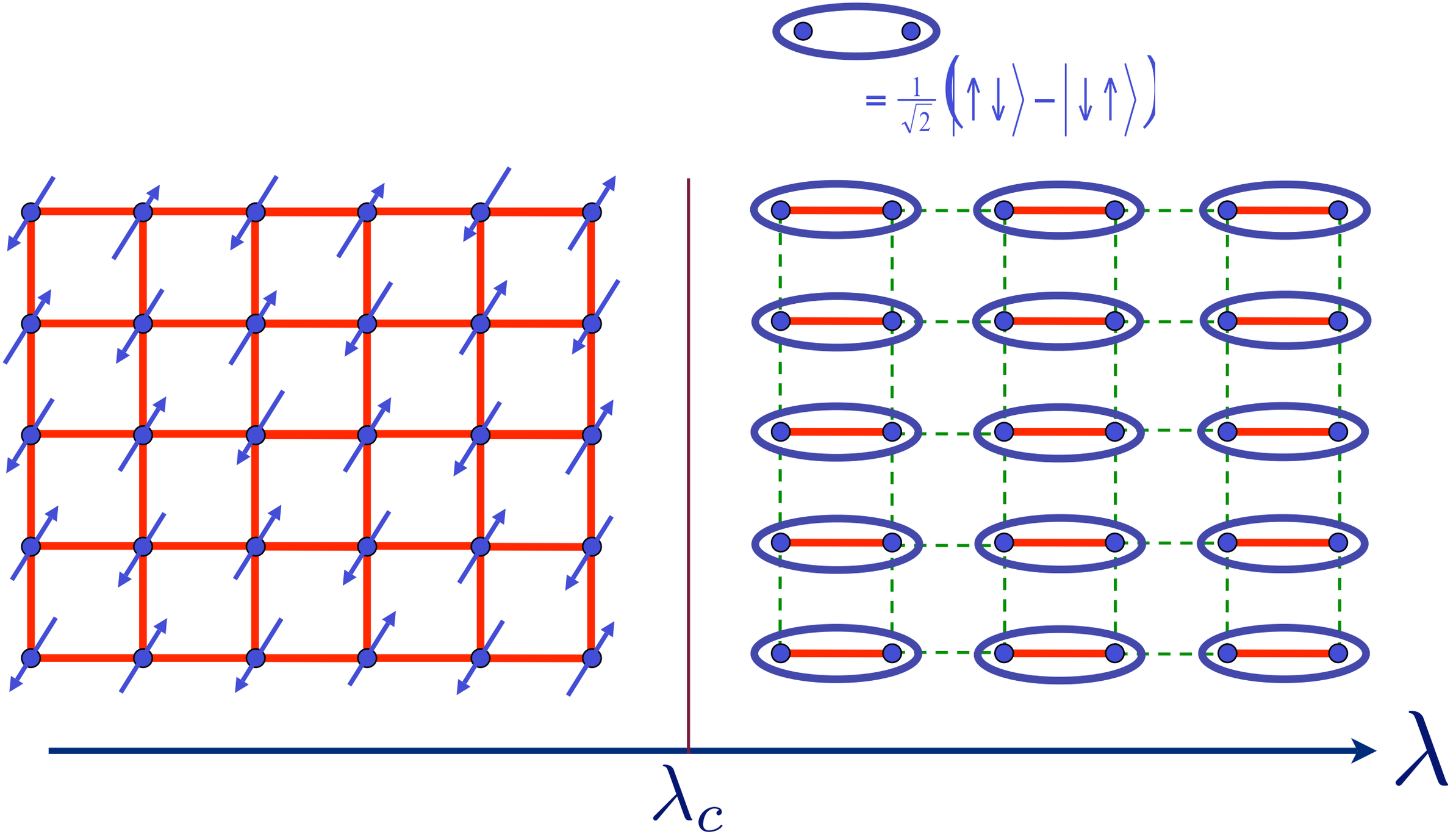




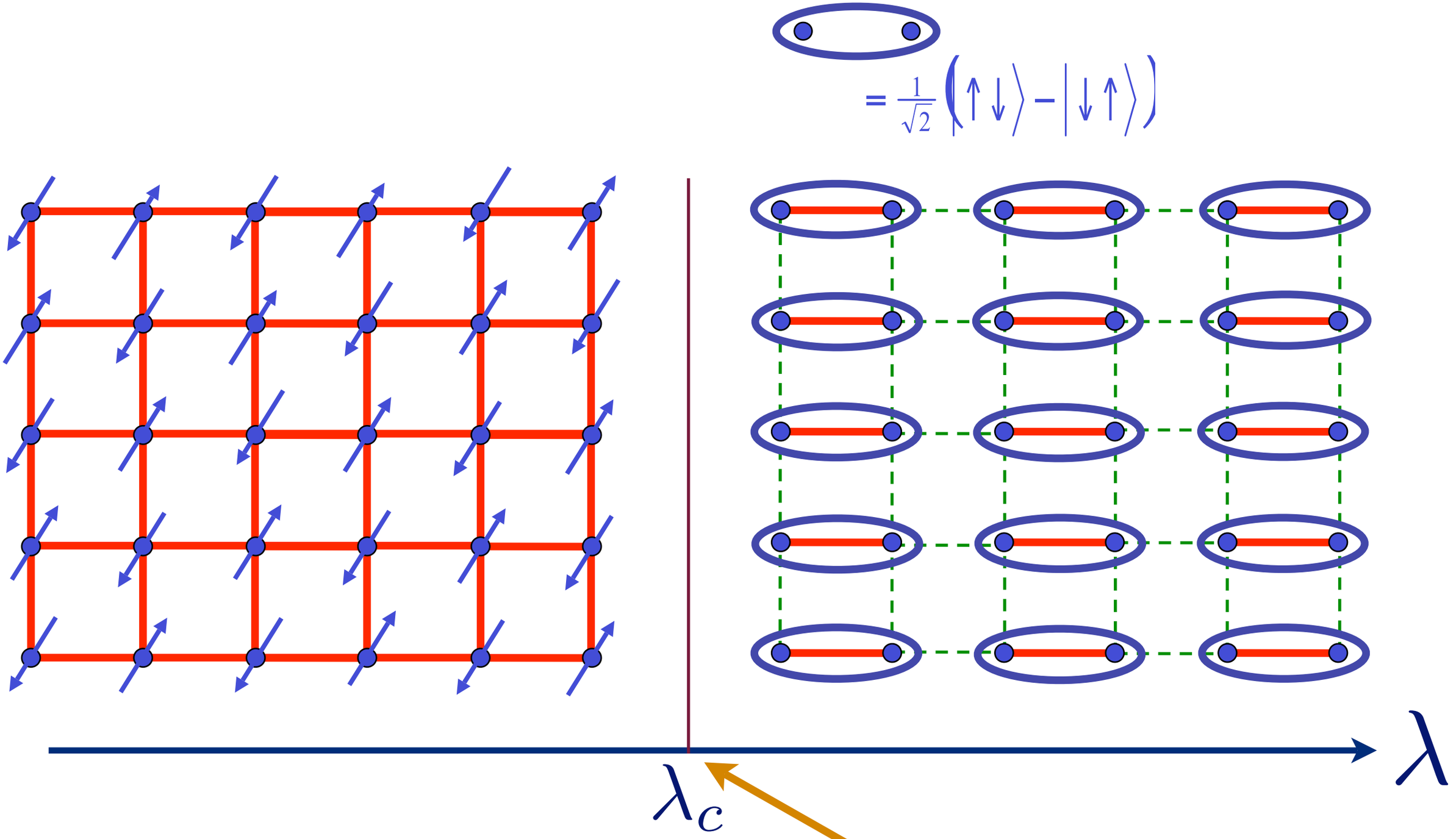
Ground state is a "quantum paramagnet"
with spins locked in valence bond singlets



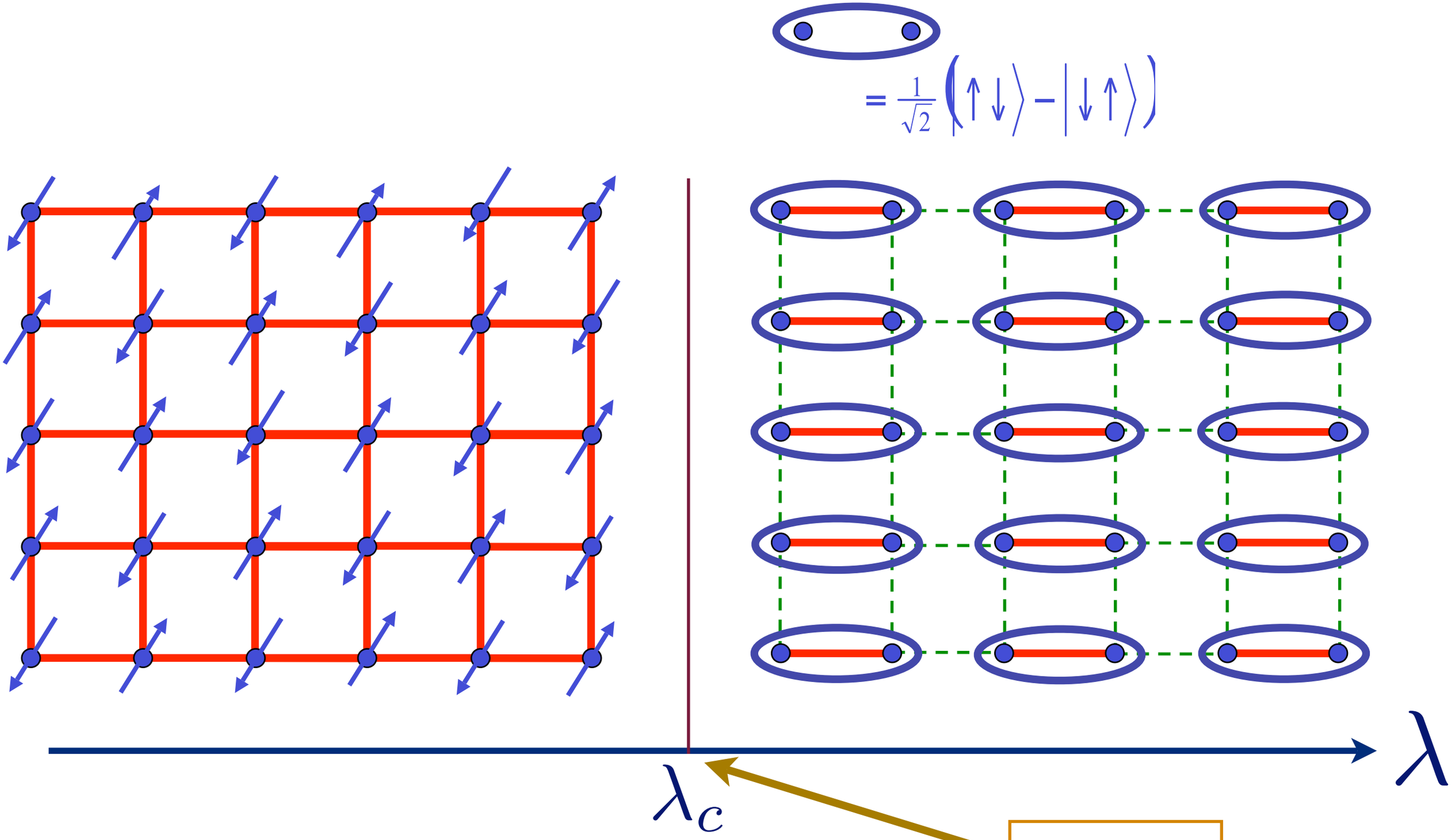
$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$



Pressure in $TlCuCl_3$



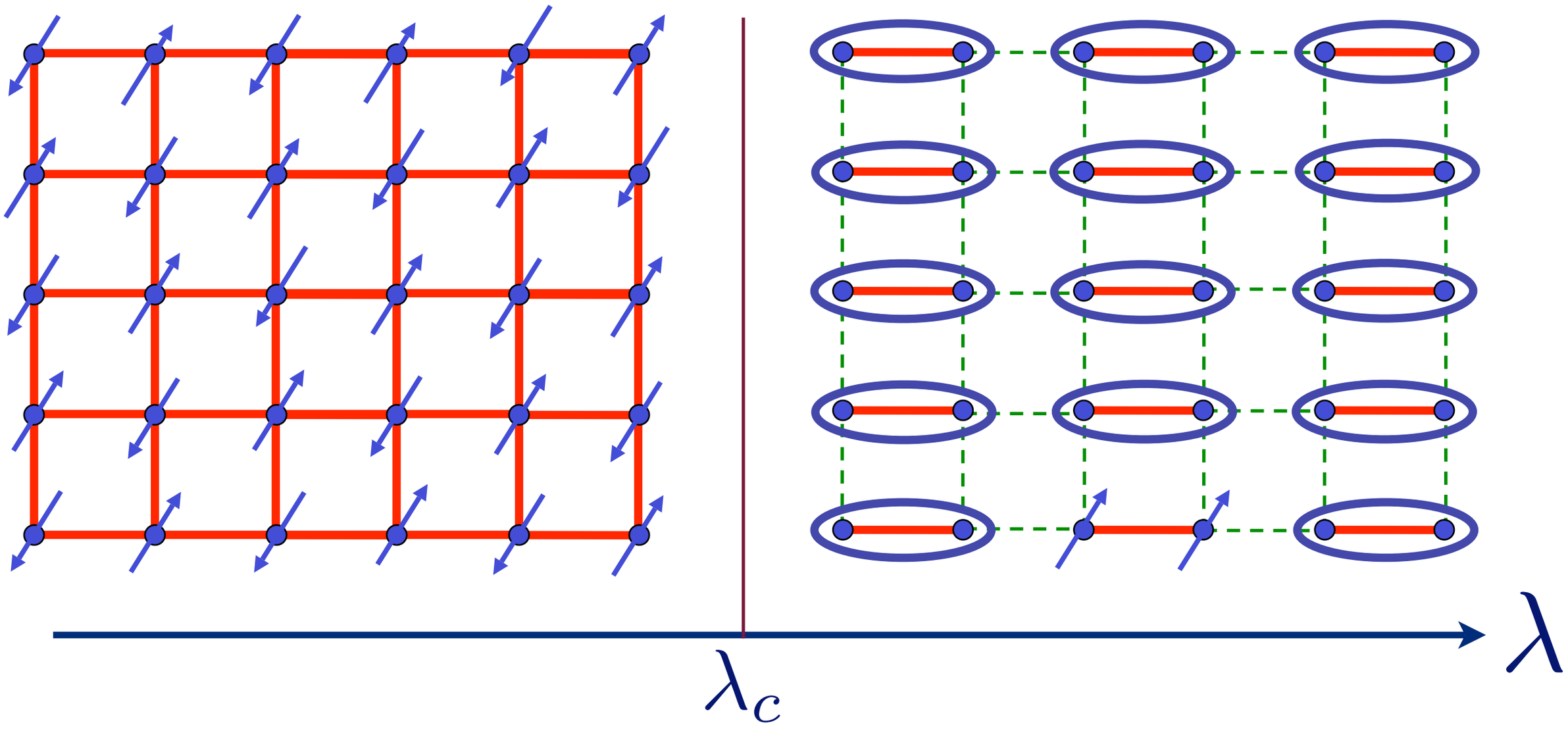
Quantum critical point with non-local entanglement in spin wavefunction



O(3) order parameter $\vec{\varphi}$

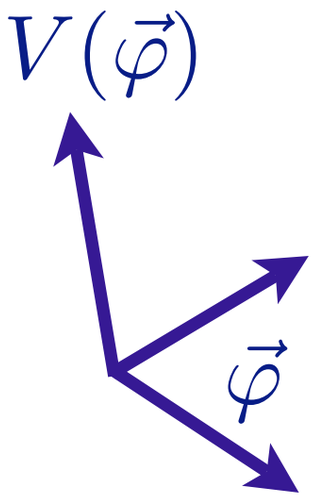
$$\mathcal{S} = \int d^2r d\tau \left[(\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2 \right]$$

Excitation spectrum in the paramagnetic phase



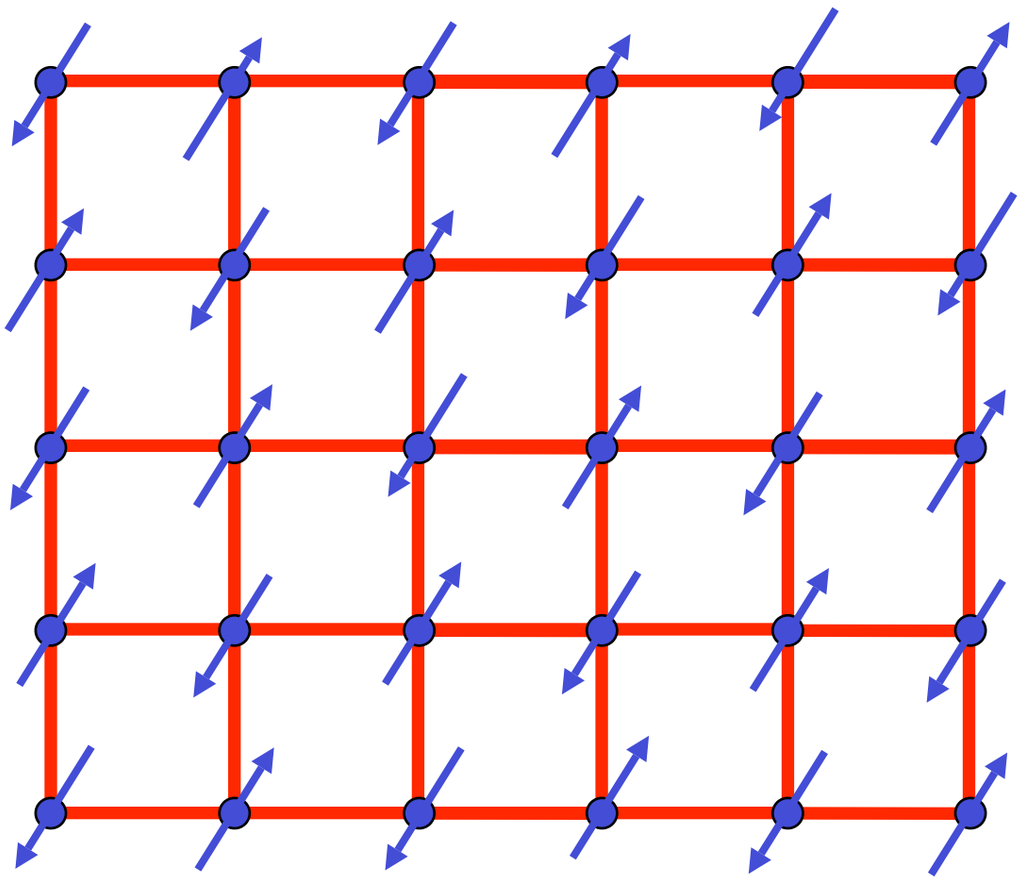
$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

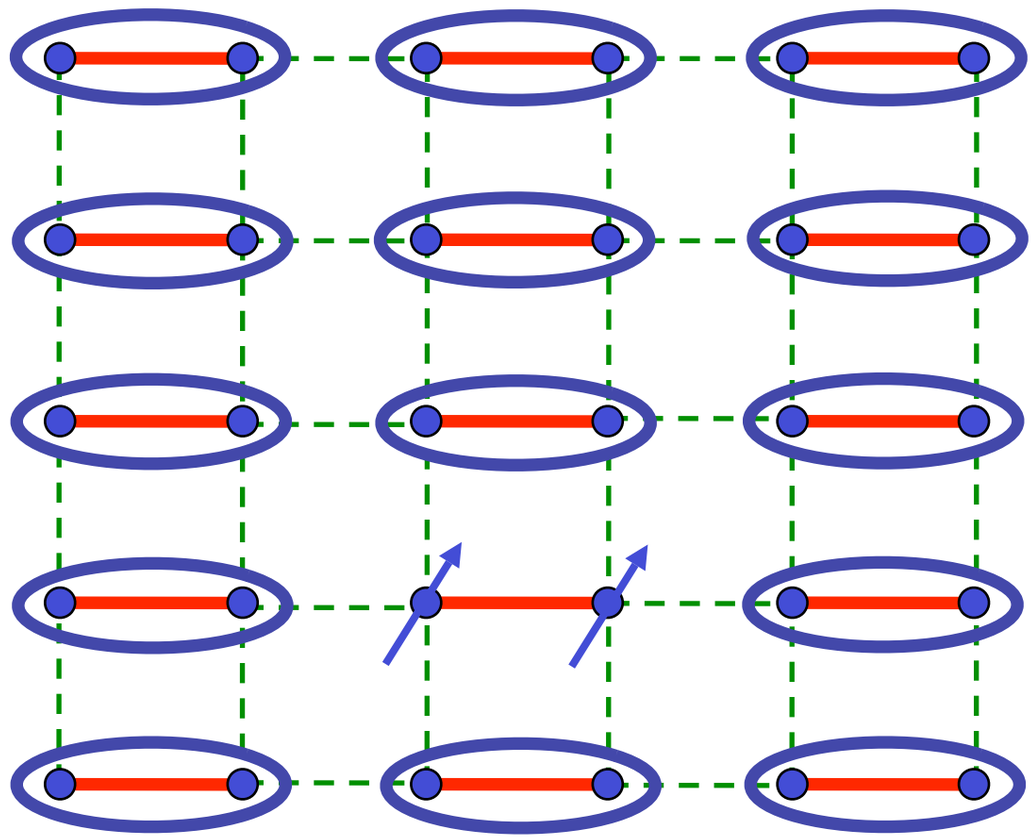
$$\lambda > \lambda_c$$



Spin $S = 1$
"triplon"

Excitation spectrum in the paramagnetic phase





λ

λ_c

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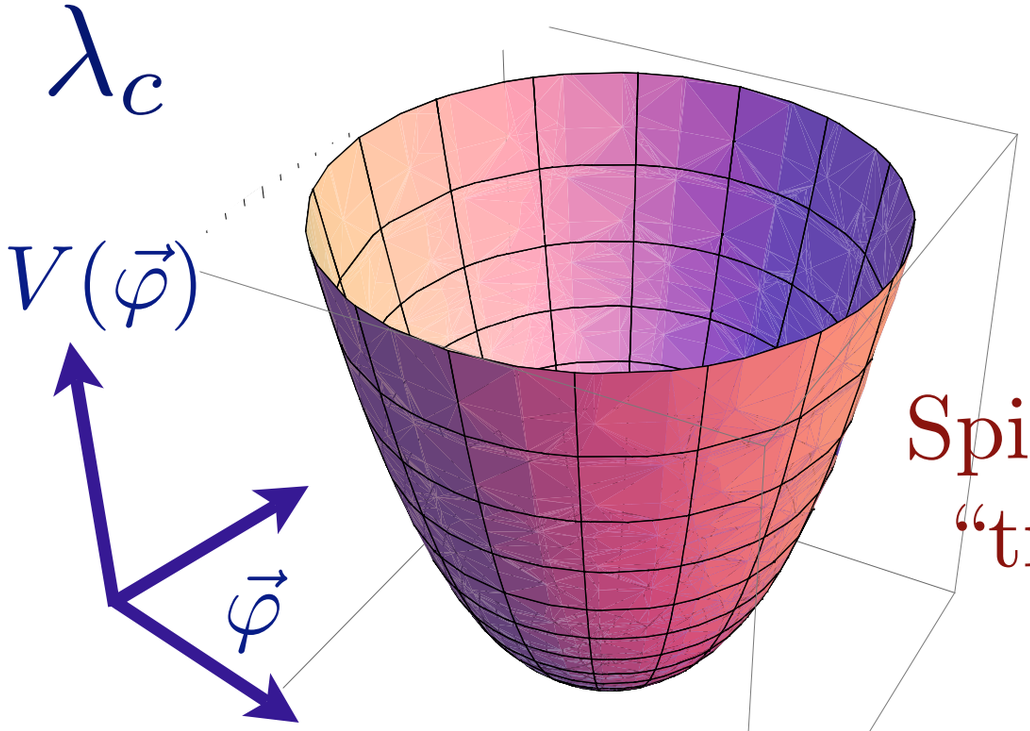

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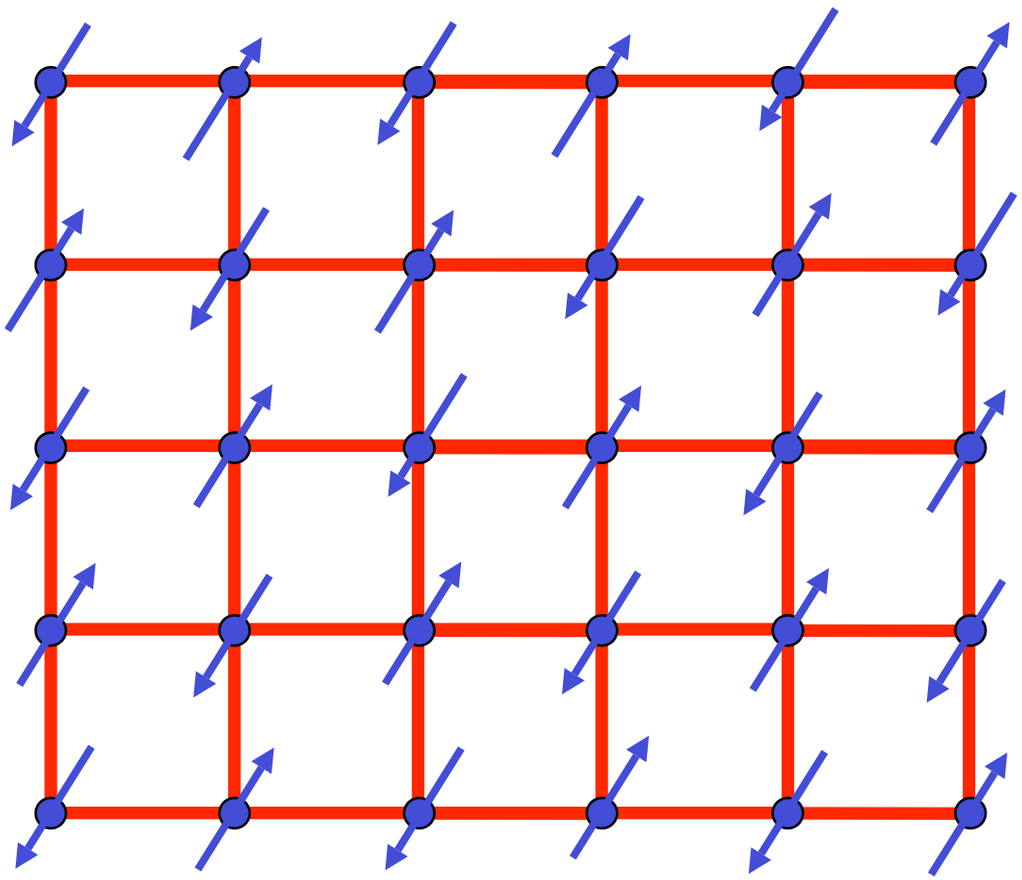

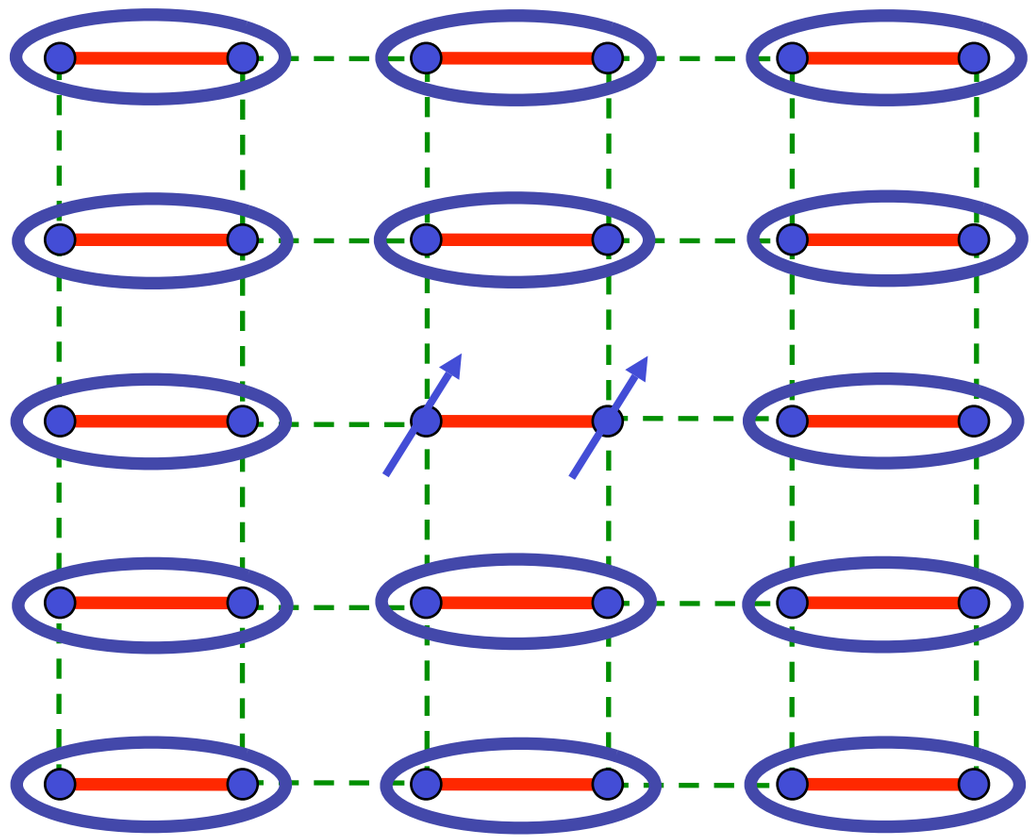

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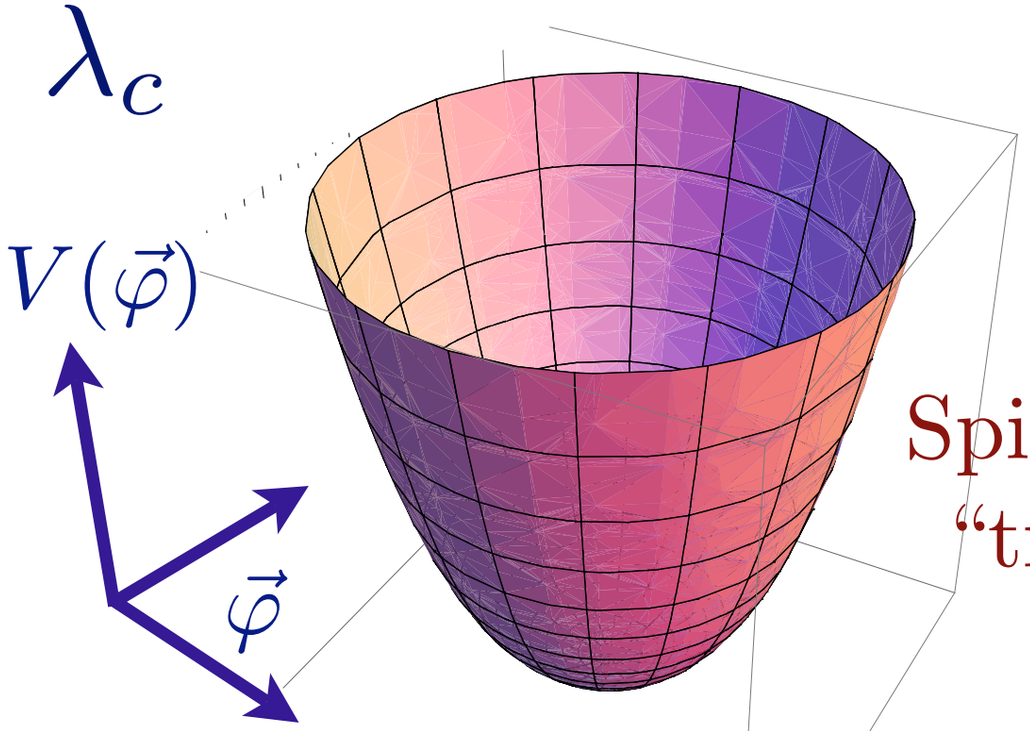

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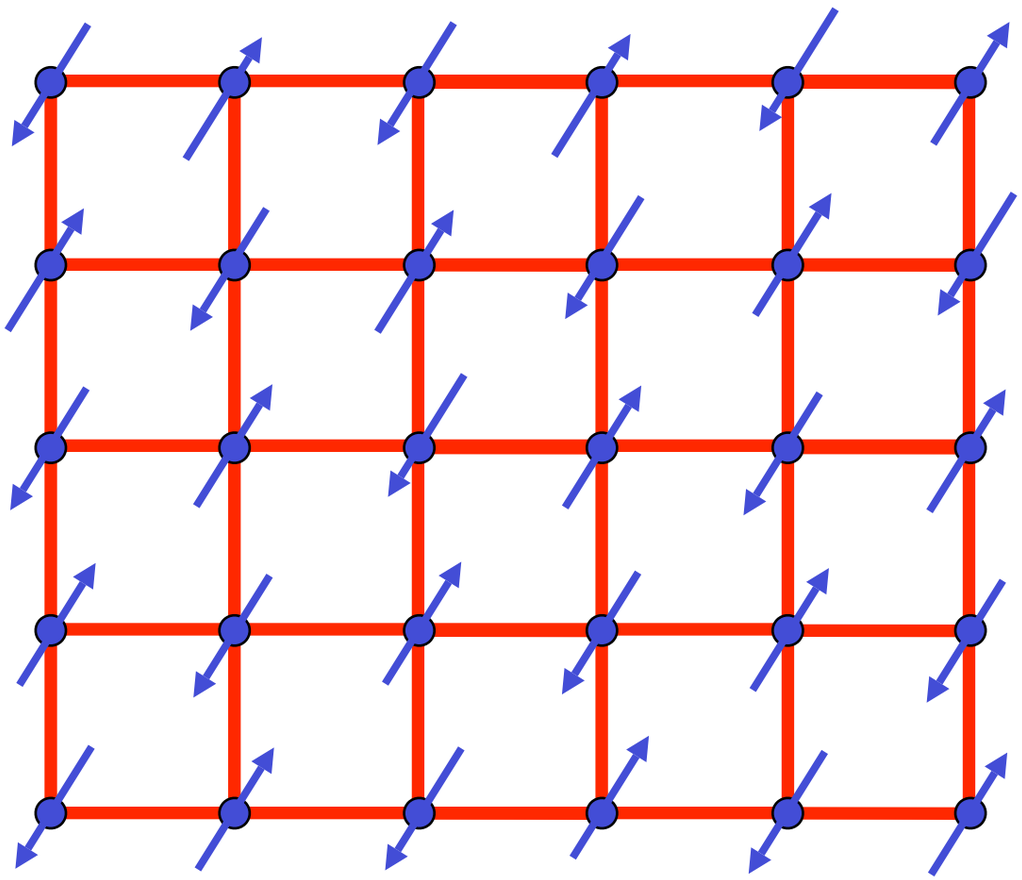

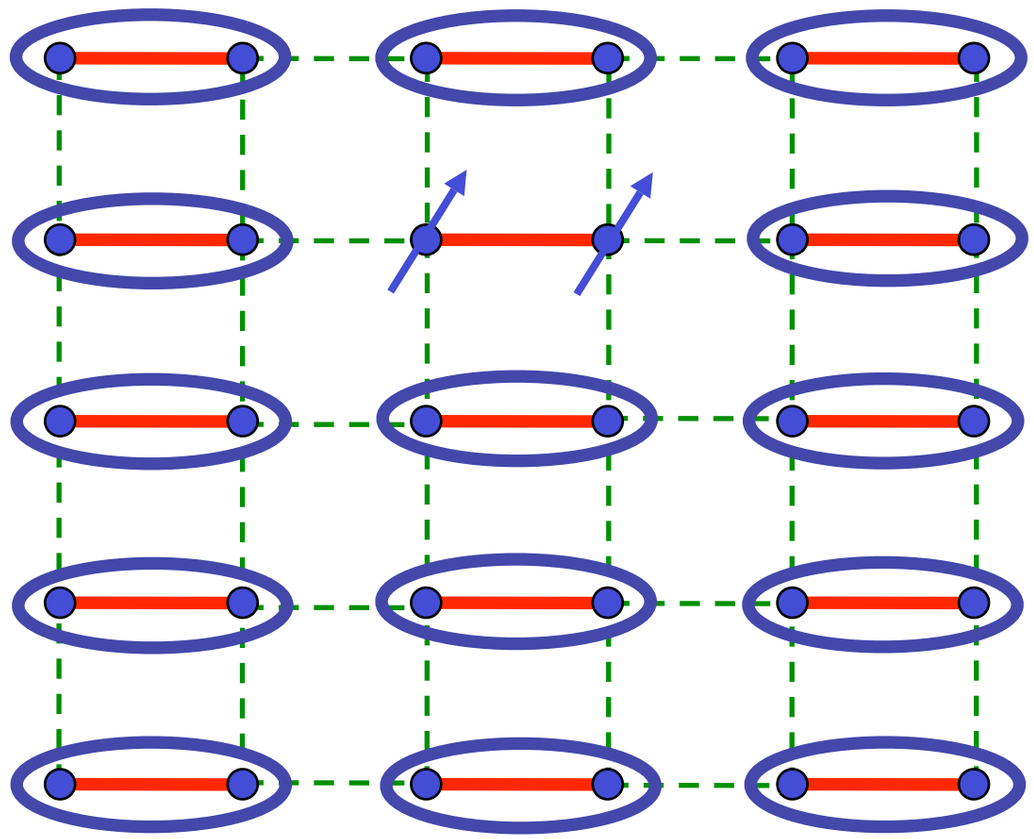

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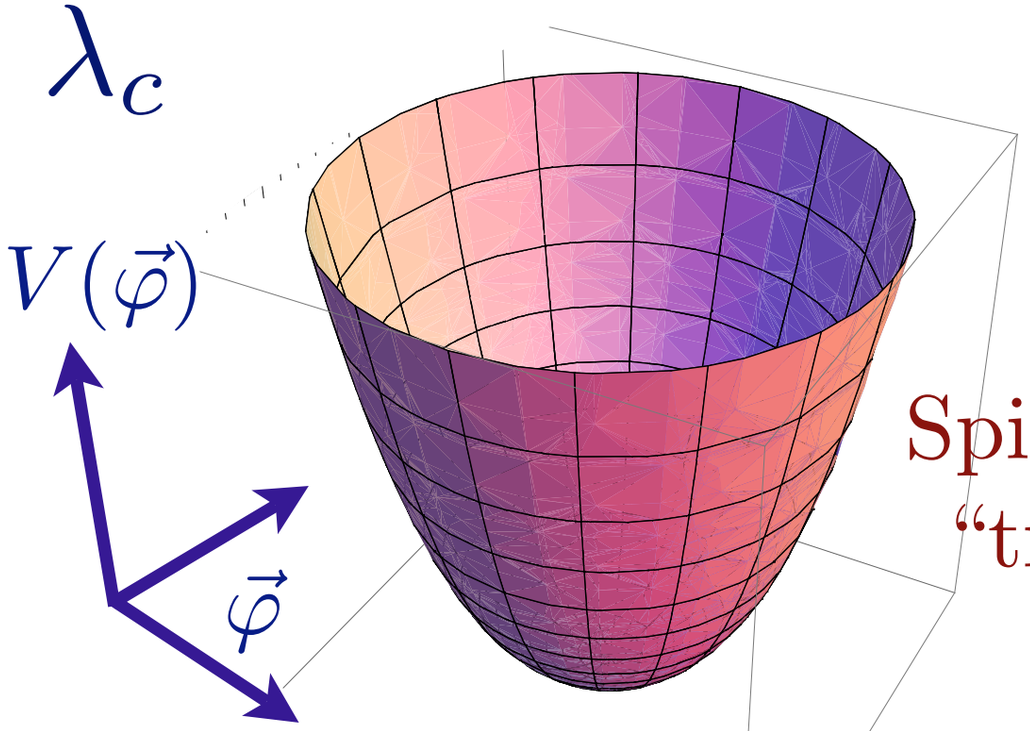

Spin $S = 1$
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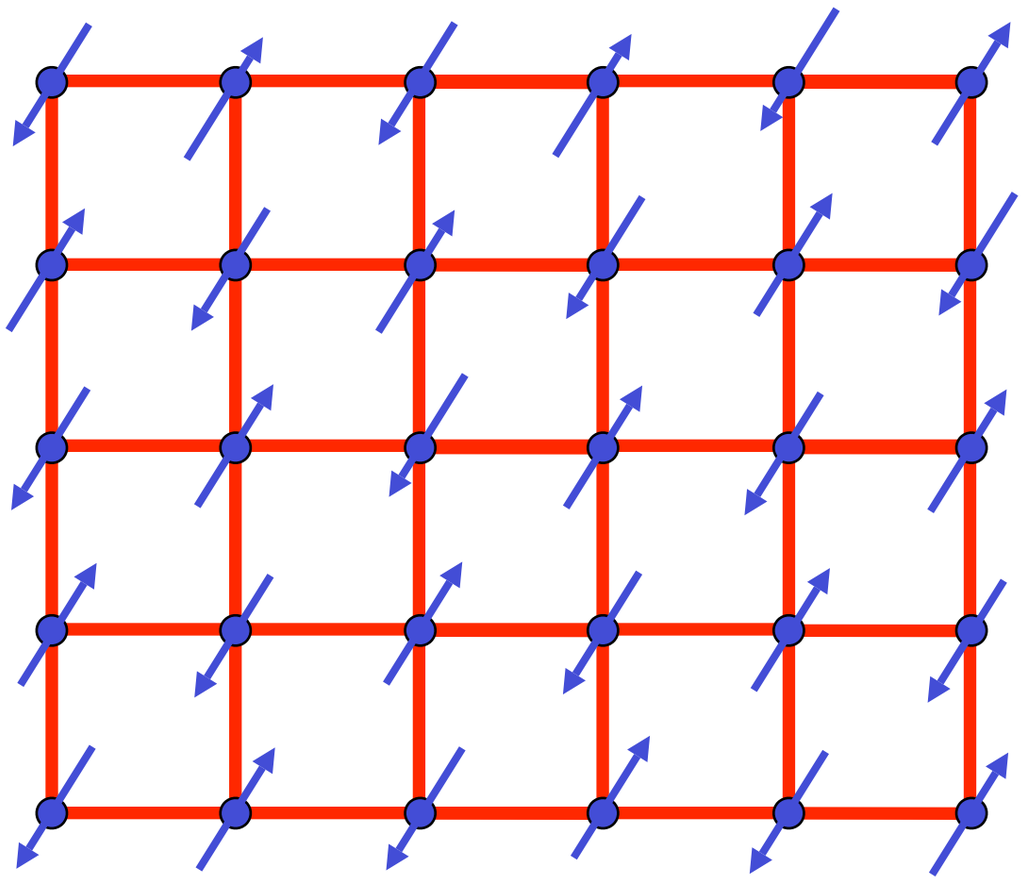
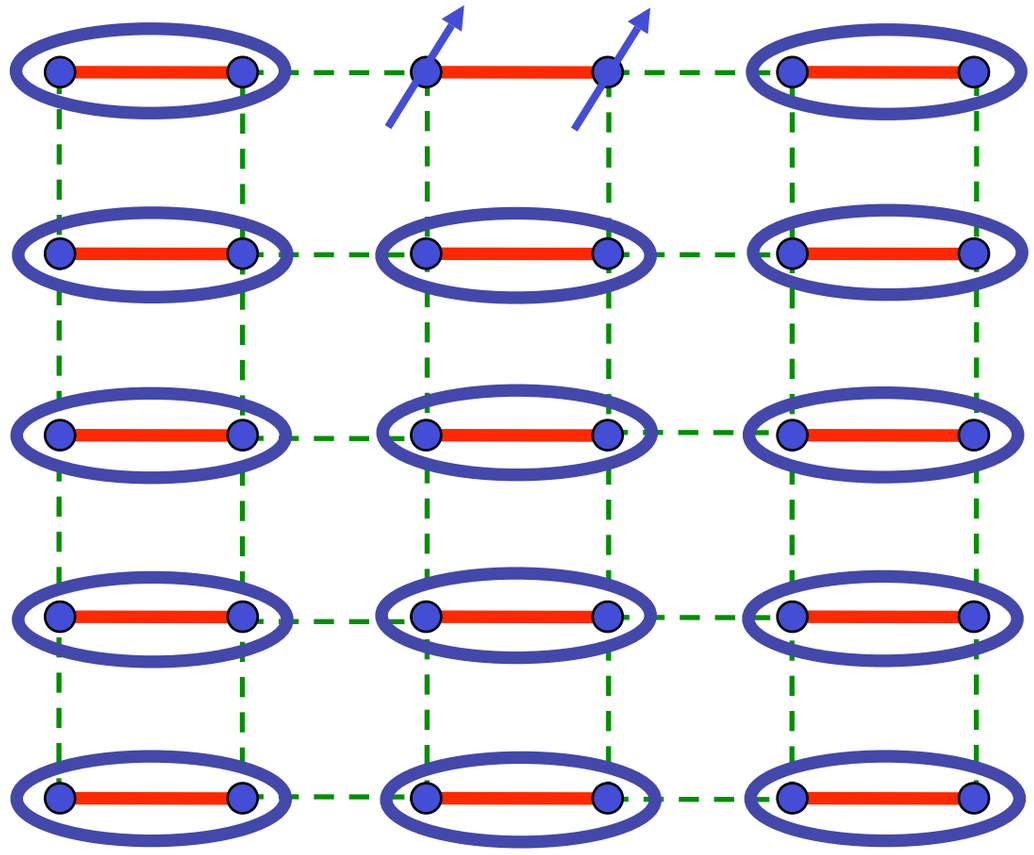
Excitation spectrum in the paramagnetic phase





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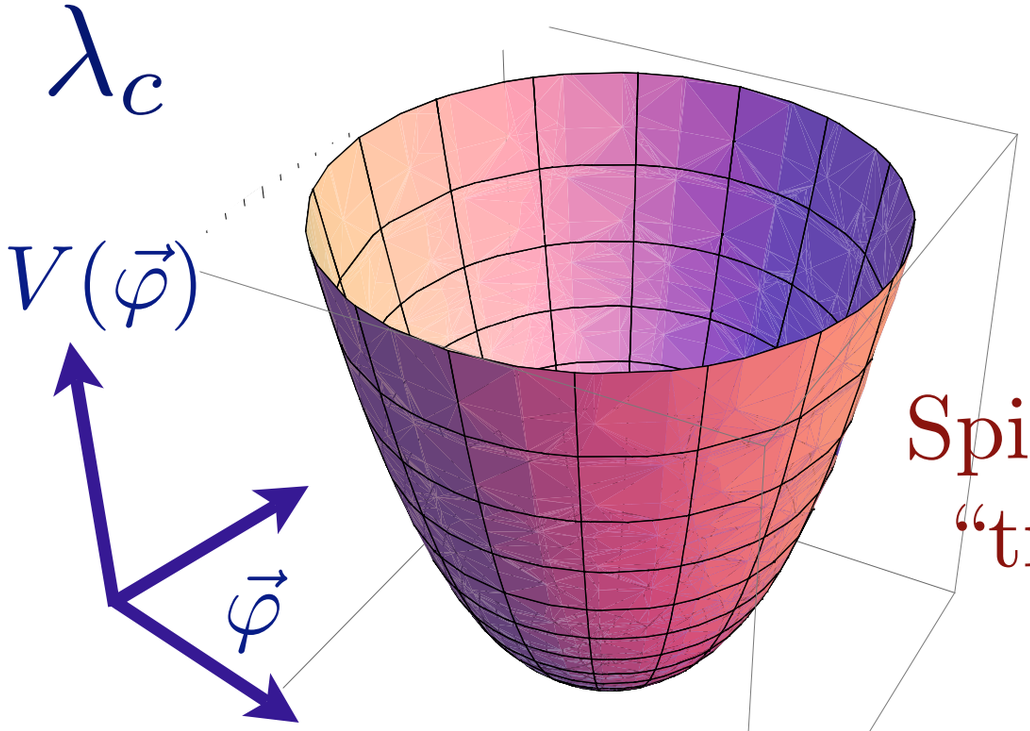
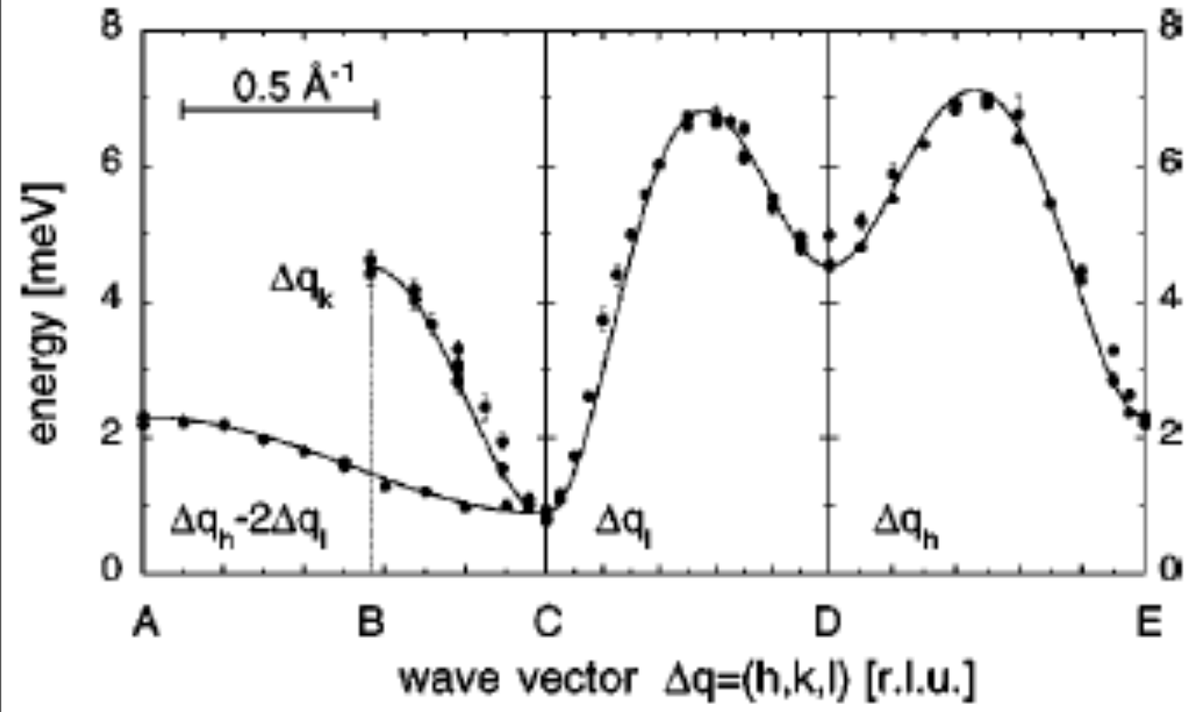


Spin $S = 1$
"triplon"

$TlCuCl_3$ at ambient pressure



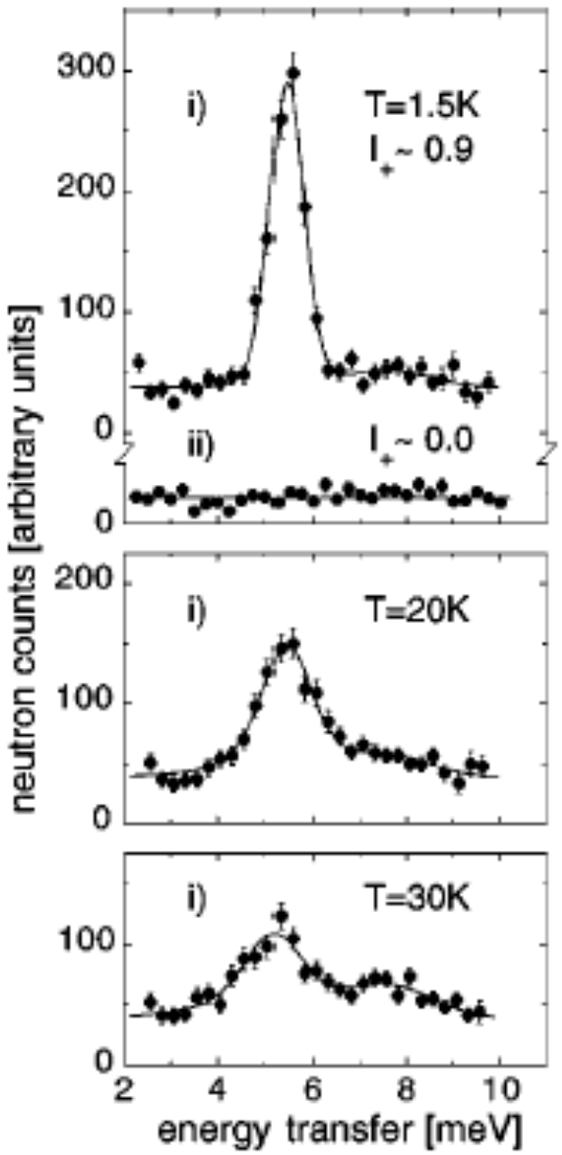
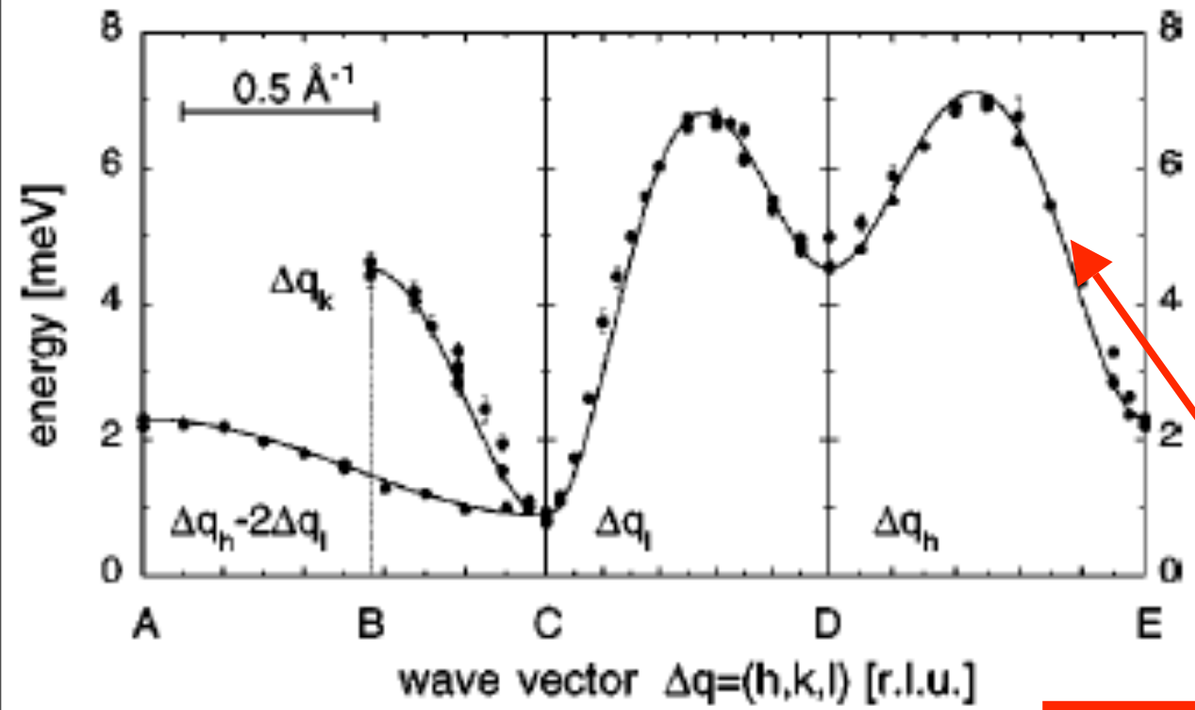
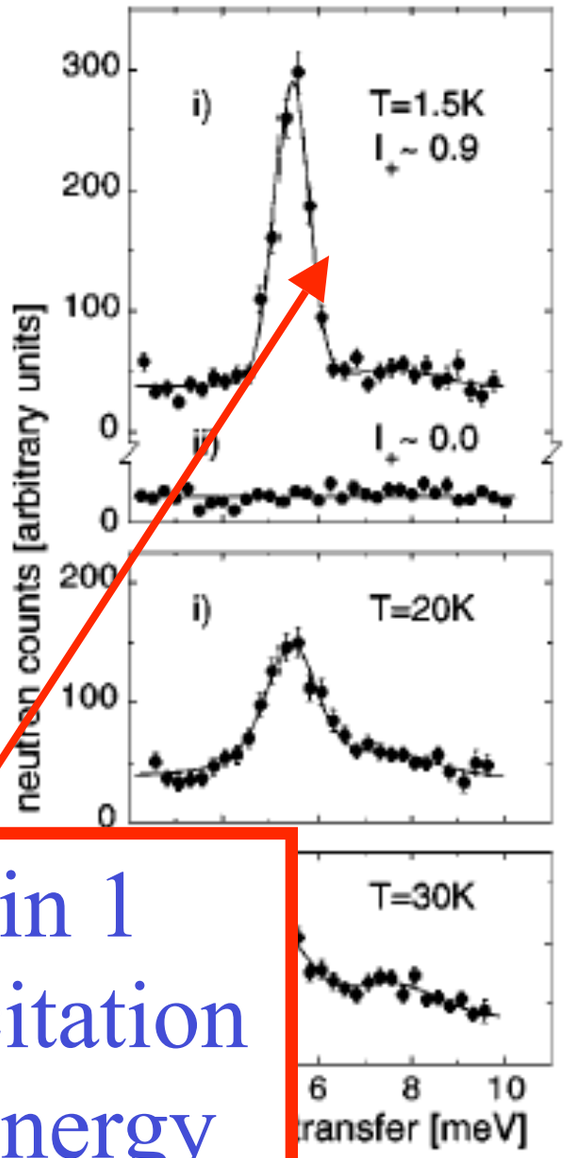


FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

$TlCuCl_3$ at ambient pressure



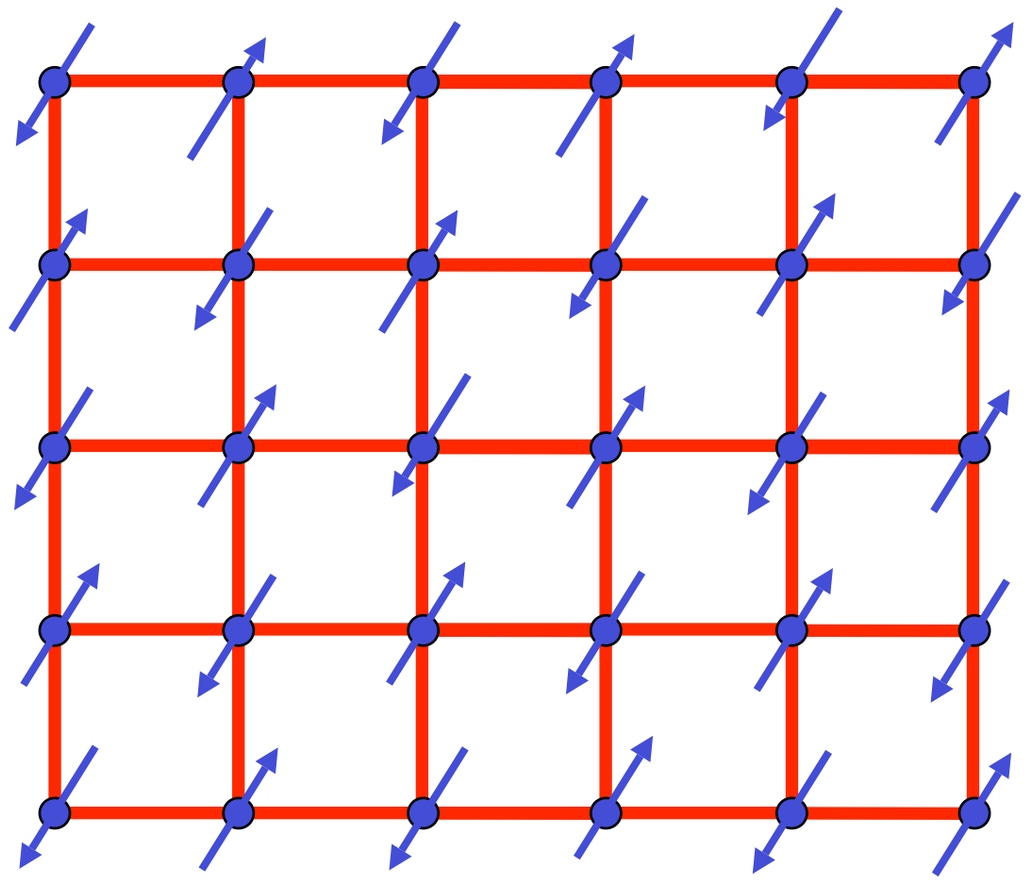


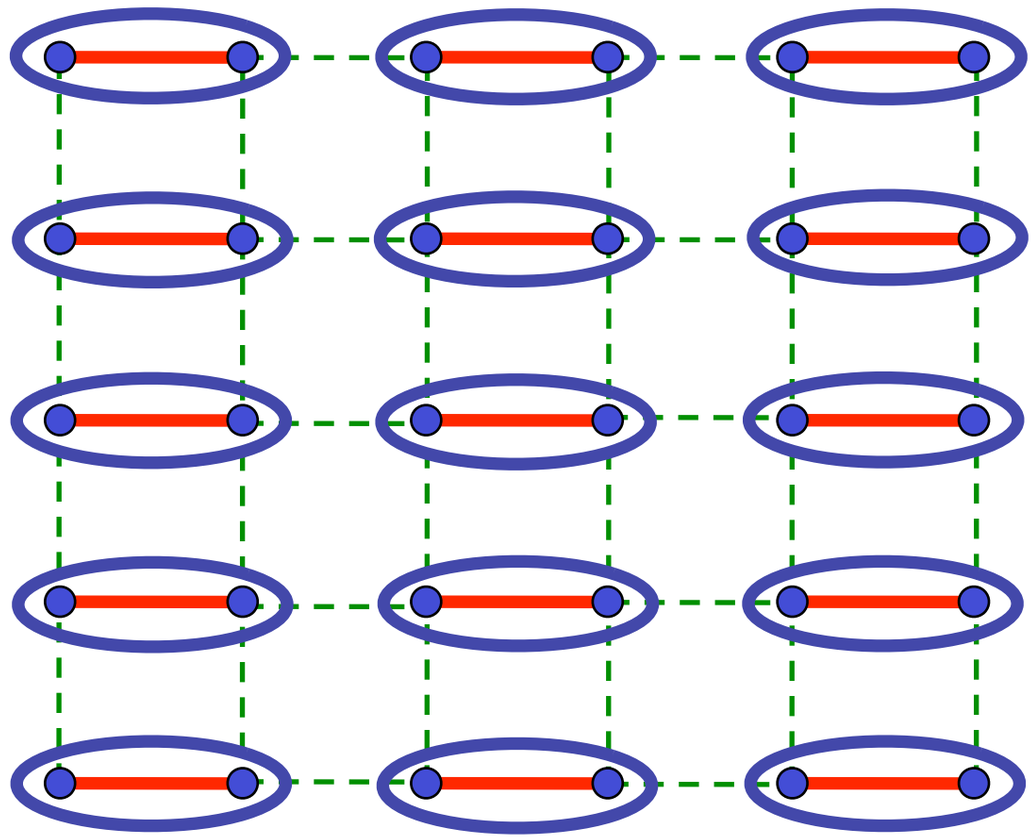
Sharp spin 1 particle excitation above an energy gap (spin gap)

les in the a^*c^* plane of $TlCuCl_3$ r.l.u]. The spectrum at $T = 1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

Excitation spectrum in the Néel phase





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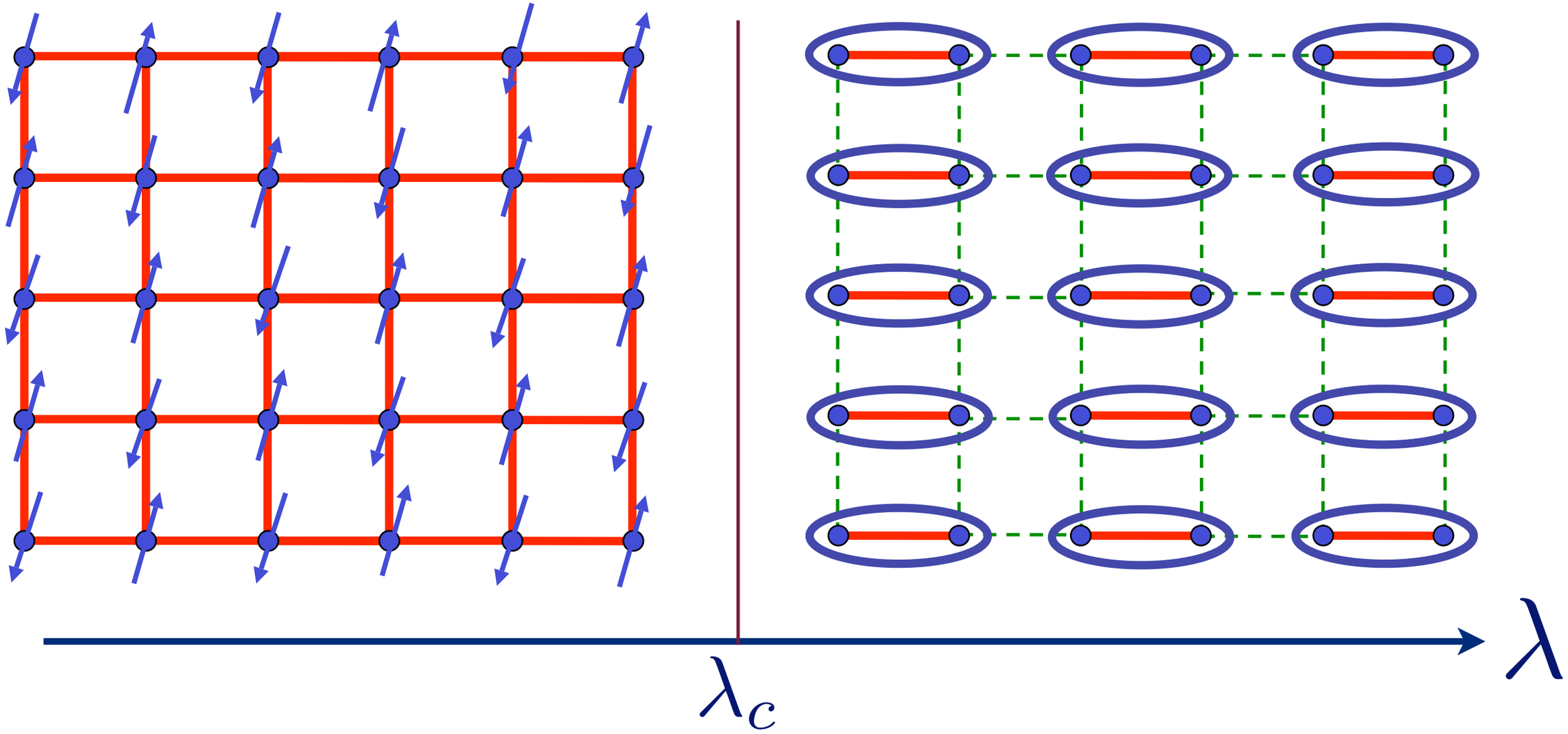
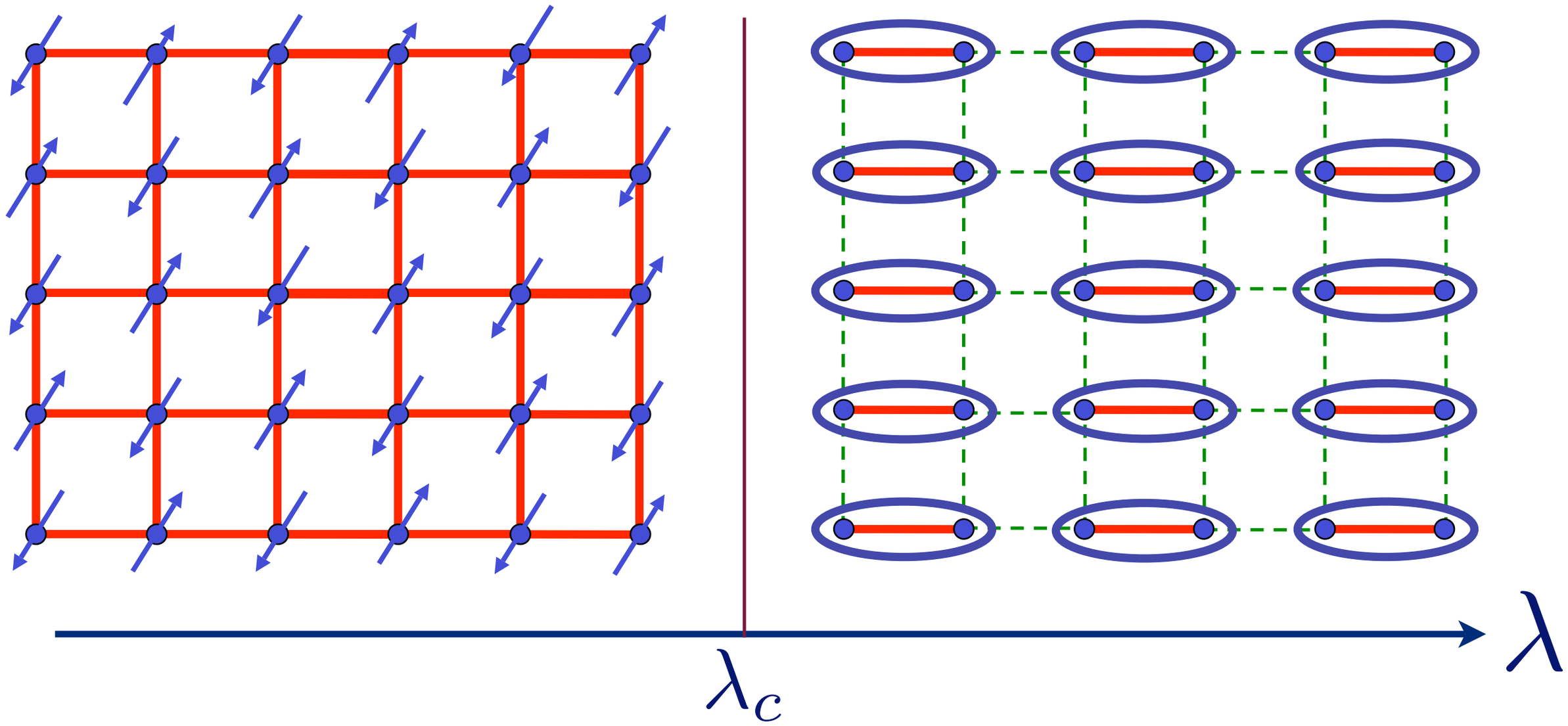


Spin waves

Excitation spectrum in the Néel phase



Spin waves

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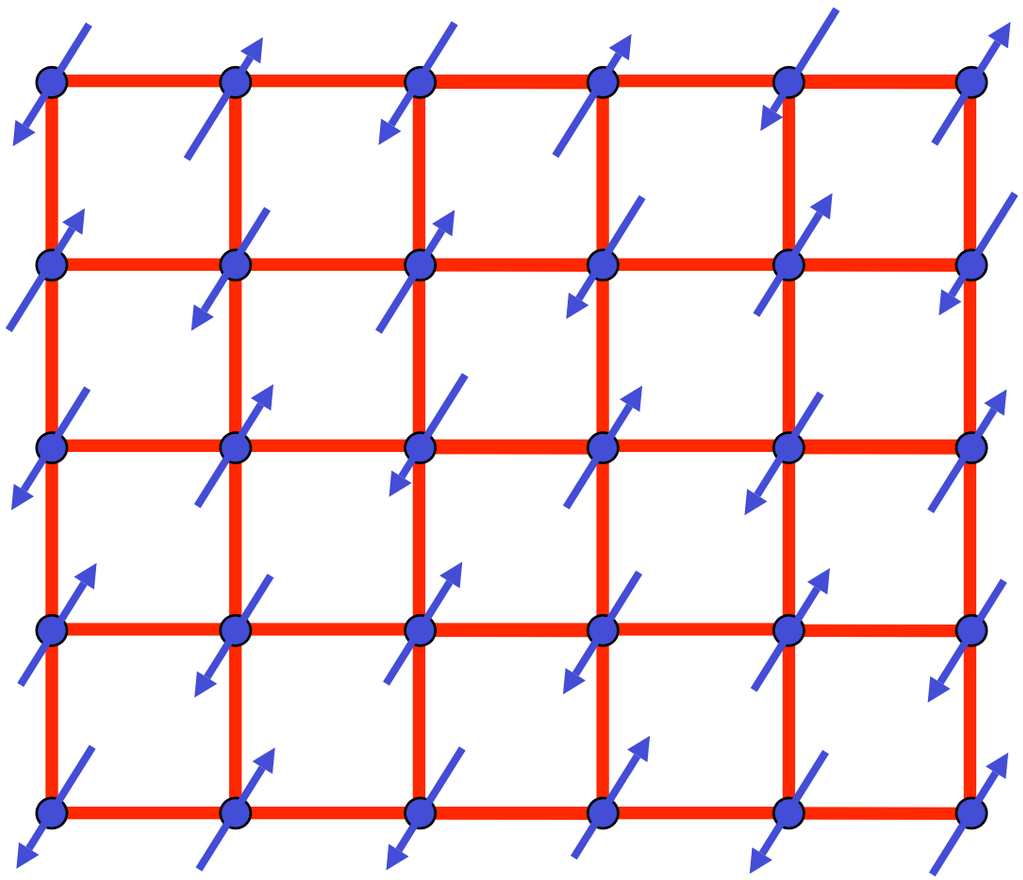
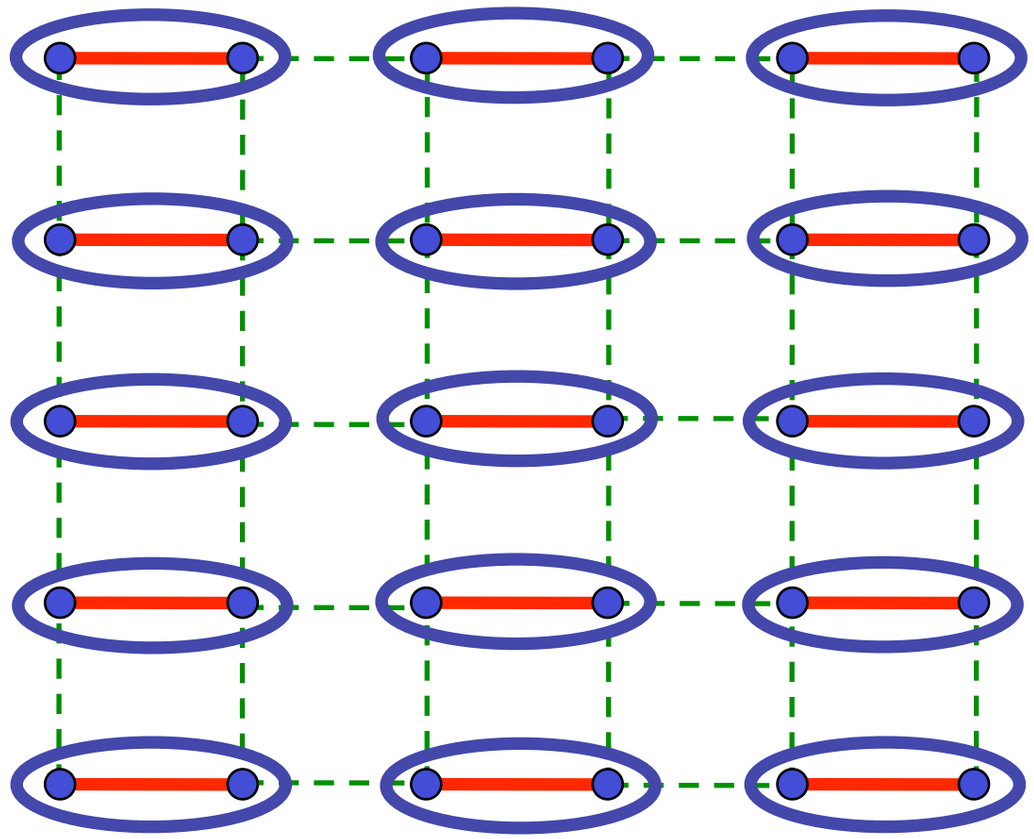
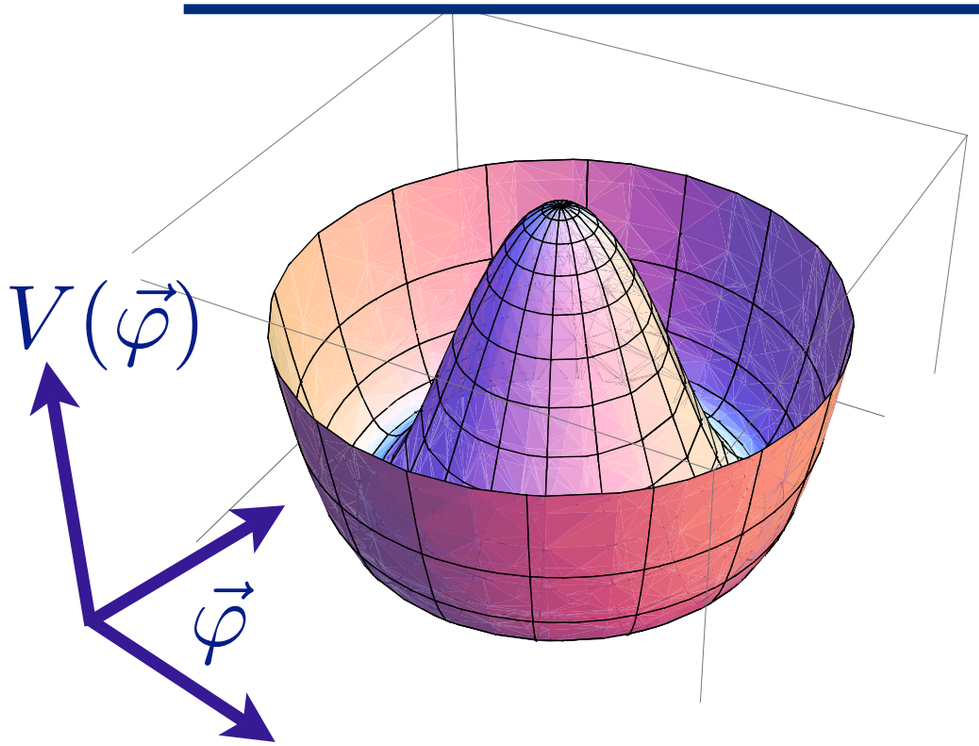




λ

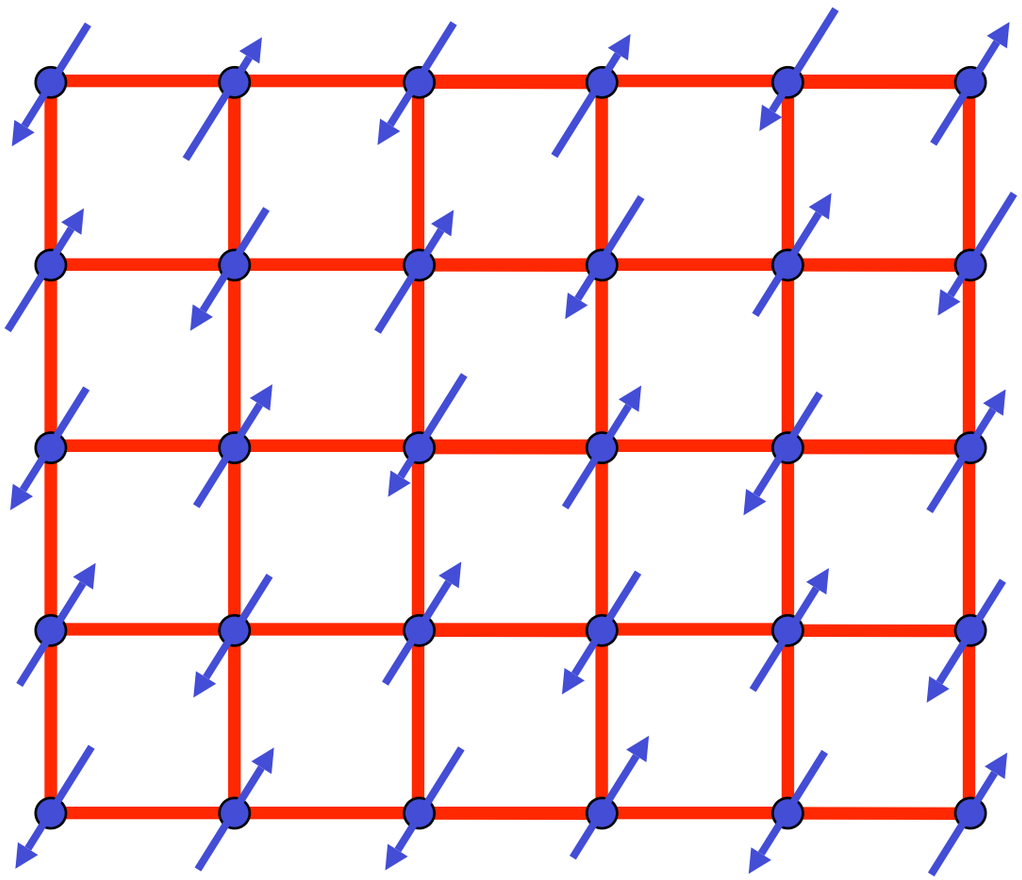
λ_c

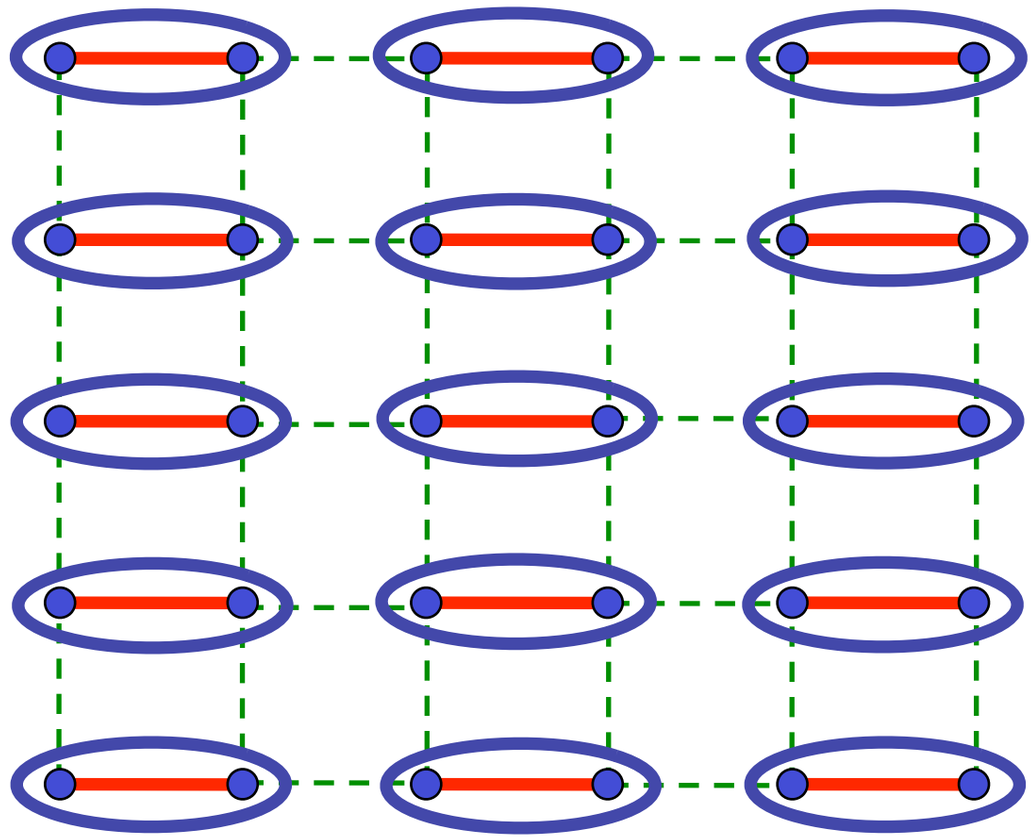


$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda < \lambda_c$$

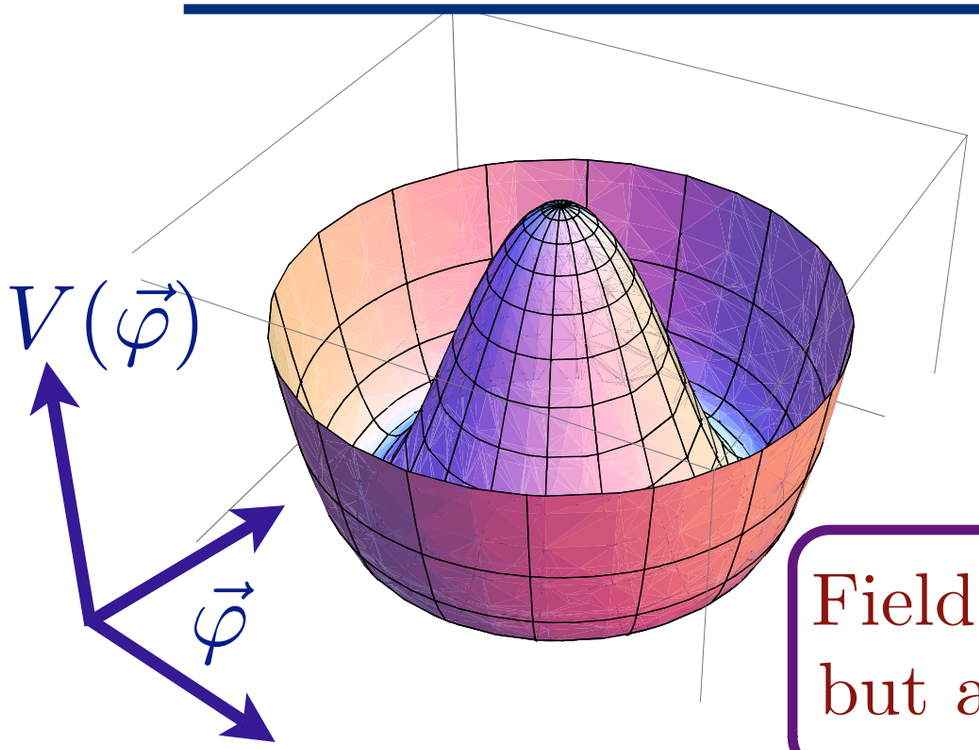
Excitation spectrum in the Néel phase





λ

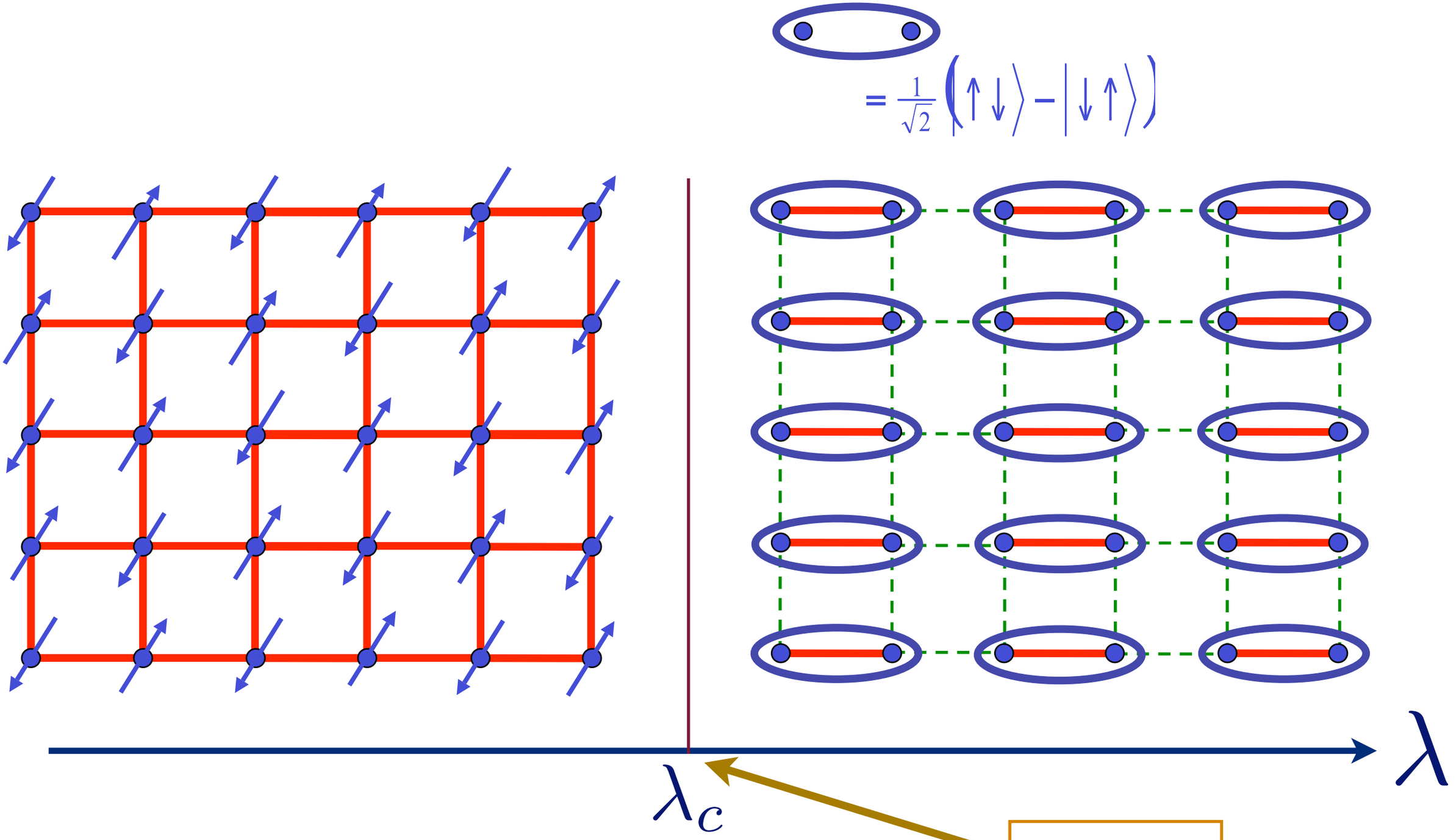
λ_c



$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda < \lambda_c$$

Field theory yields spin waves ("Goldstone" modes) but also an additional longitudinal "Higgs" particle



O(3) order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2r d\tau \left[(\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2 \right]$$

Quantum Monte Carlo - critical exponents

Table IV: Fit results for the critical exponents ν, β/ν, and η. We summarize results including a variation of the critical point within its error bar. For the ladder model (top group of values) fit results and quality of fits are also given at the previous best estimate of α_c. The bottom group are results for the plaquette model. Numbers in [. . .] brackets denote the χ^2/d.o.f. For comparison relevant reference values for the 3D $O(3)$ universality class are given in the last line.

α_c	ν^a	β/ν^b	η^c
$1.9096 - \sigma$	0.712(4) [1.8]	0.516(2) [0.5]	0.026(2) [0.2]
1.9096	0.711(4) [1.8]	0.518(2) [1.1]	0.029(5) [0.8]
$1.9096 + \sigma$	0.710(4) [1.8]	0.519(3) [2.5]	0.032(7) [1.4]
1.9107^d	0.709(3) [1.7]	0.525(8) [15.3]	0.051(10) [12]
$1.8230 - \sigma$	0.708(4) [0.99]	0.515(2) [0.84]	0.025(4) [0.15]
1.8230	0.706(4) [1.04]	0.516(2) [0.40]	0.028(3) [0.31]
$1.8230 + \sigma$	0.706(4) [1.10]	0.517(2) [1.6]	0.031(5) [0.80]
Ref. 49	0.7112(5)	0.518(1)	0.0375(5)

[a] $L > 12$.
[b] $L > 16$.
[c] $L > 20$.
[d] Previous best estimate of Ref. 19.

S. Wenzel and W. Janke, arXiv:0808.1418

M. Troyer, M. Imada, and K. Ueda, *J. Phys. Soc. Japan* (1997)

Quantum Monte Carlo - critical exponents

Table IV: Fit results for the critical exponents ν, β/ν, and η. We summarize results including a variation of the critical point within its error bar. For the ladder model (top group of values) fit results and quality of fits are also given at the previous best estimate of α_c. The bottom group are results for the plaquette model. Numbers in [. . .] brackets denote the χ^2/d.o.f. For comparison relevant reference values for the 3D $O(3)$ universality class are given in the last line.

α_c	ν^a	β/ν^b	η^c
$1.9096 - \sigma$	0.712(4) [1.8]	0.516(2) [0.5]	0.026(2) [0.2]
1.9096	0.711(4) [1.8]	0.518(2) [1.1]	0.029(5) [0.8]
$1.9096 + \sigma$	0.710(4) [1.8]	0.519(3) [2.5]	0.032(7) [1.4]
1.9107^d	0.709(3) [1.7]	0.525(8) [15.3]	0.051(10) [12]
$1.8230 - \sigma$	0.708(4) [0.99]	0.515(2) [0.84]	0.025(4) [0.15]
1.8230	0.706(4) [1.04]	0.516(2) [0.40]	0.028(3) [0.31]
$1.8230 + \sigma$	0.706(4) [1.10]	0.517(2) [1.6]	0.031(5) [0.80]
Ref. 49	0.7112(5)	0.518(1)	0.0375(5)

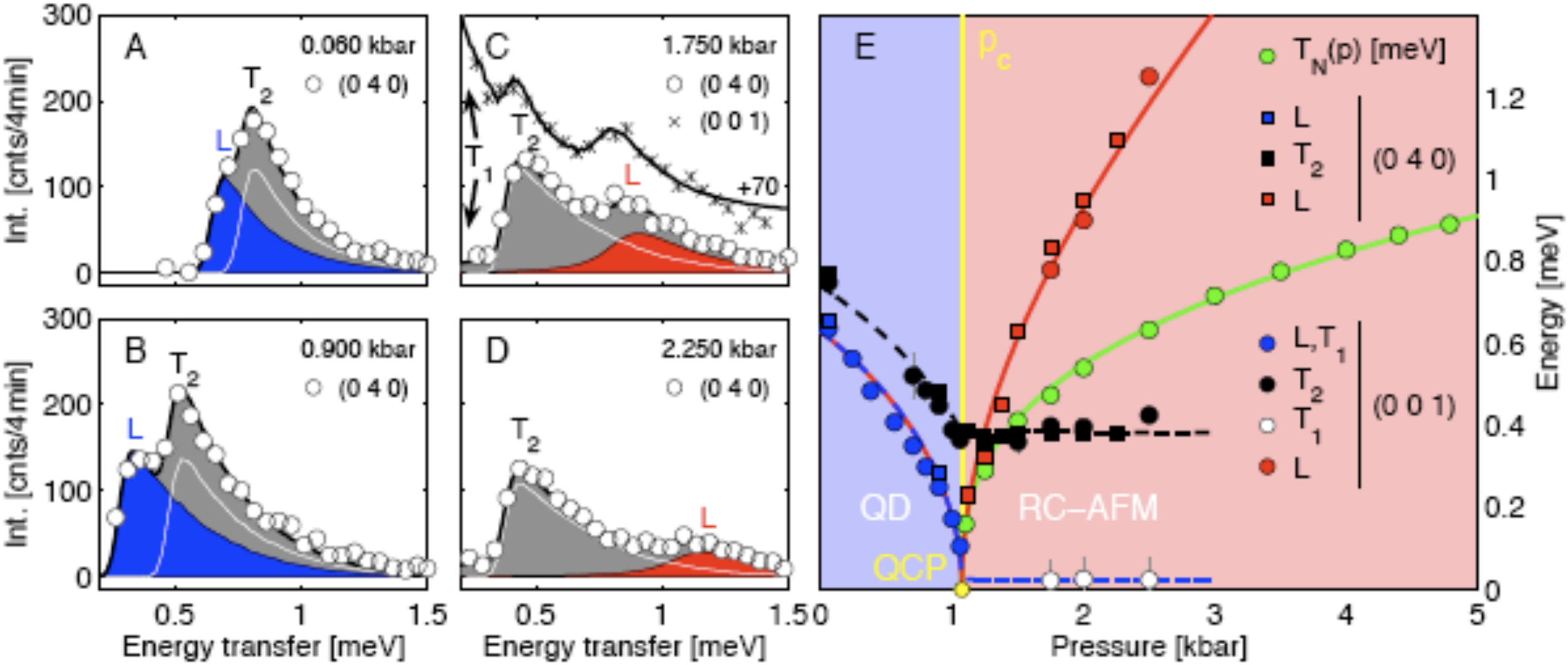
[a] $L > 12$.
[b] $L > 16$.
[c] $L > 20$.
[d] Previous best estimate of Ref. 19.

Field-theoretic RG of CFT3 E. Vicari *et al.*

S. Wenzel and W. Janke, arXiv:0808.1418

M. Troyer, M. Imada, and K. Ueda, *J. Phys. Soc. Japan* (1997)

$TlCuCl_3$ with varying pressure



Observation of $3 \rightarrow 2$ low energy modes,
emergence of new Higgs particle in the Néel phase,
and vanishing of Néel temperature at the quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer,
Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya,
Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Prediction of quantum field theory

Potential for $\vec{\varphi}$ fluctuations: $V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$

Paramagnetic phase, $\lambda > \lambda_c$

Expand about $\vec{\varphi} = 0$:

$$V(\vec{\varphi}) \approx (\lambda - \lambda_c)\vec{\varphi}^2$$

Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$

Néel phase, $\lambda < \lambda_c$

Expand $\vec{\varphi} = \left(0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}\,\right) + \vec{\varphi}_1$:

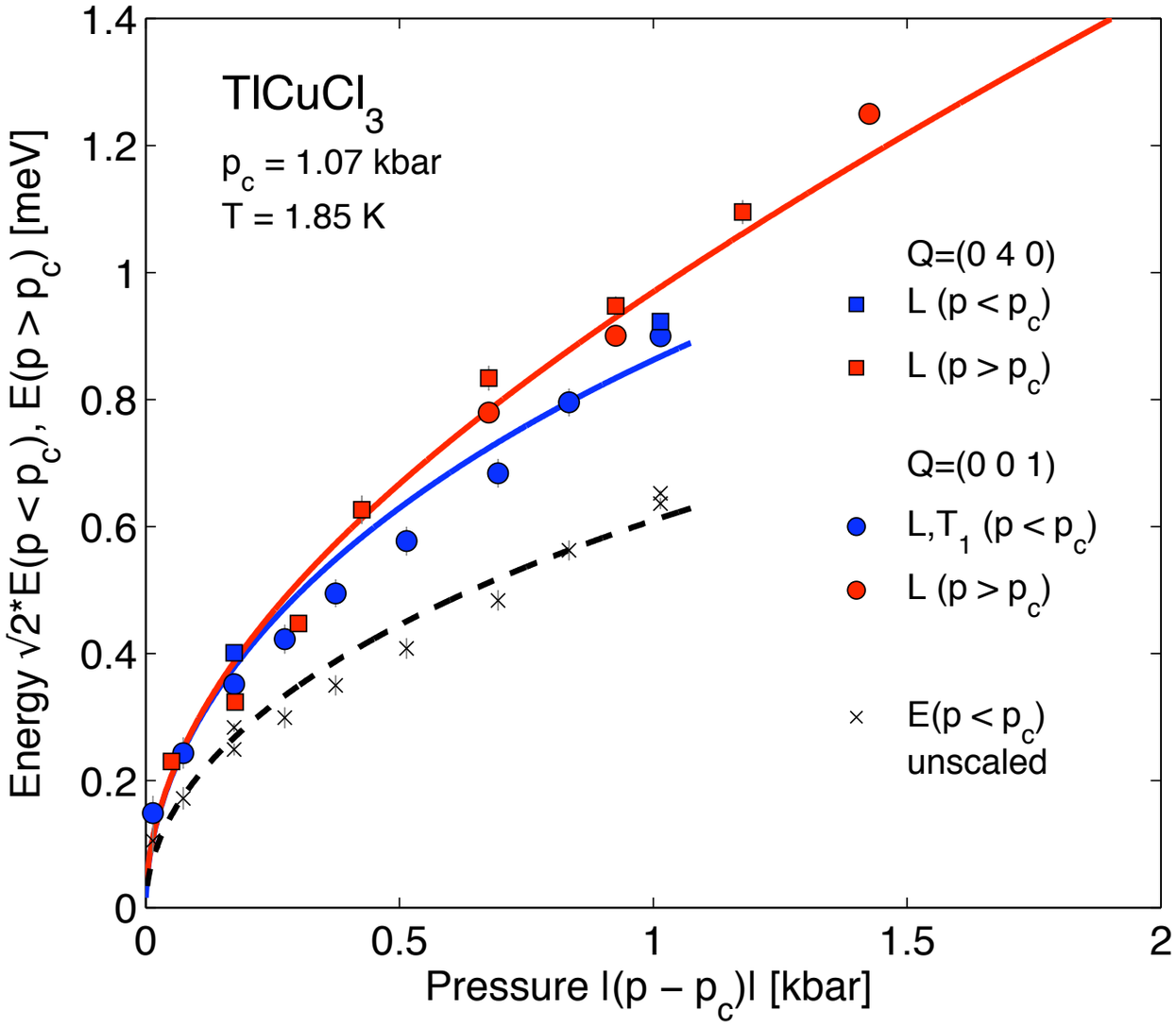
$$V(\vec{\varphi}) \approx 2(\lambda_c - \lambda)\varphi_{1z}^2$$

Yields 2 gapless spin waves and one Higgs particle with energy gap $\sim \sqrt{2(\lambda_c - \lambda)}$

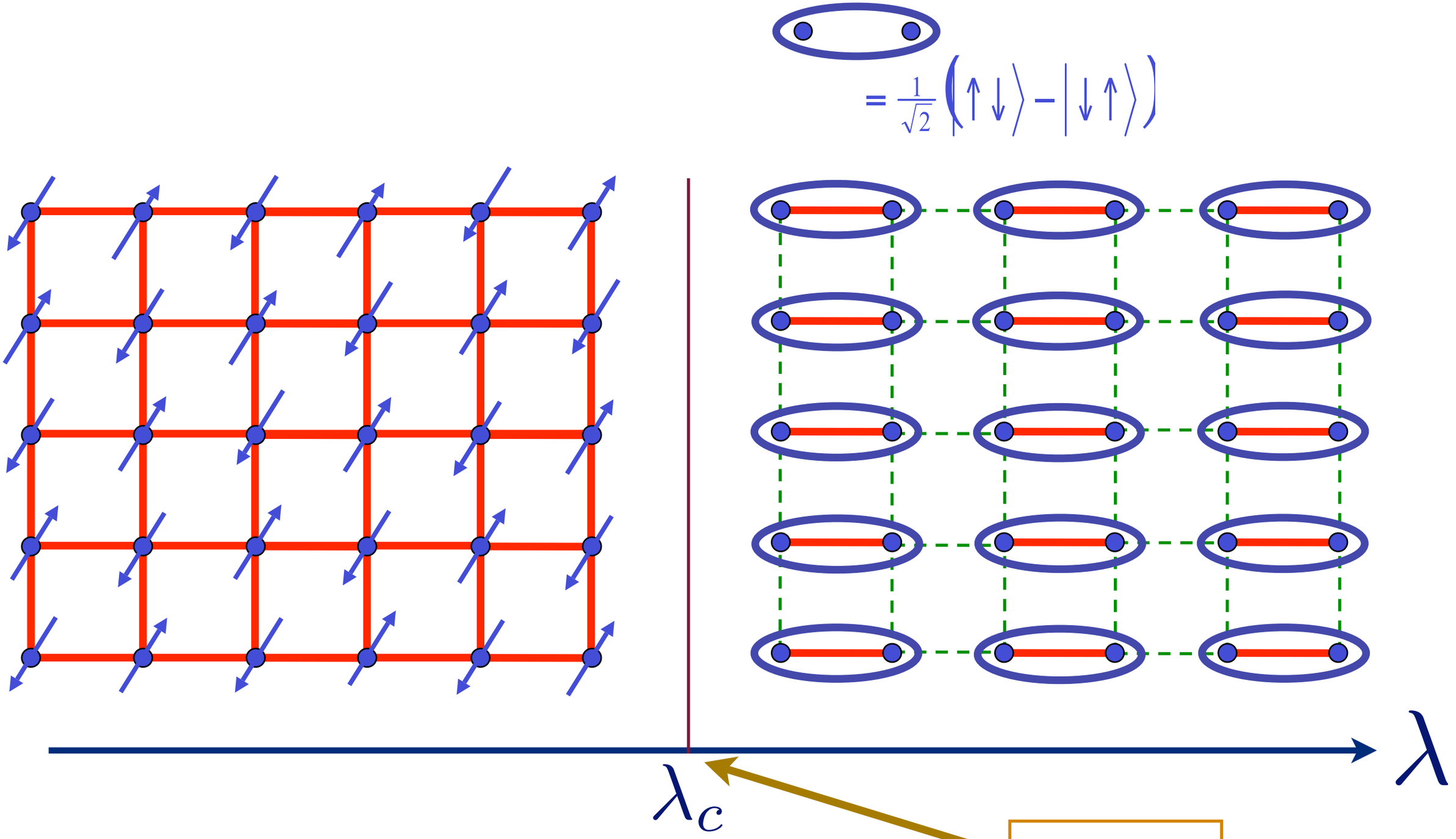
Prediction of quantum field theory

$$\frac{\text{Energy of ``Higgs'' particle}}{\text{Energy of triplon}} = \sqrt{2}$$

$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

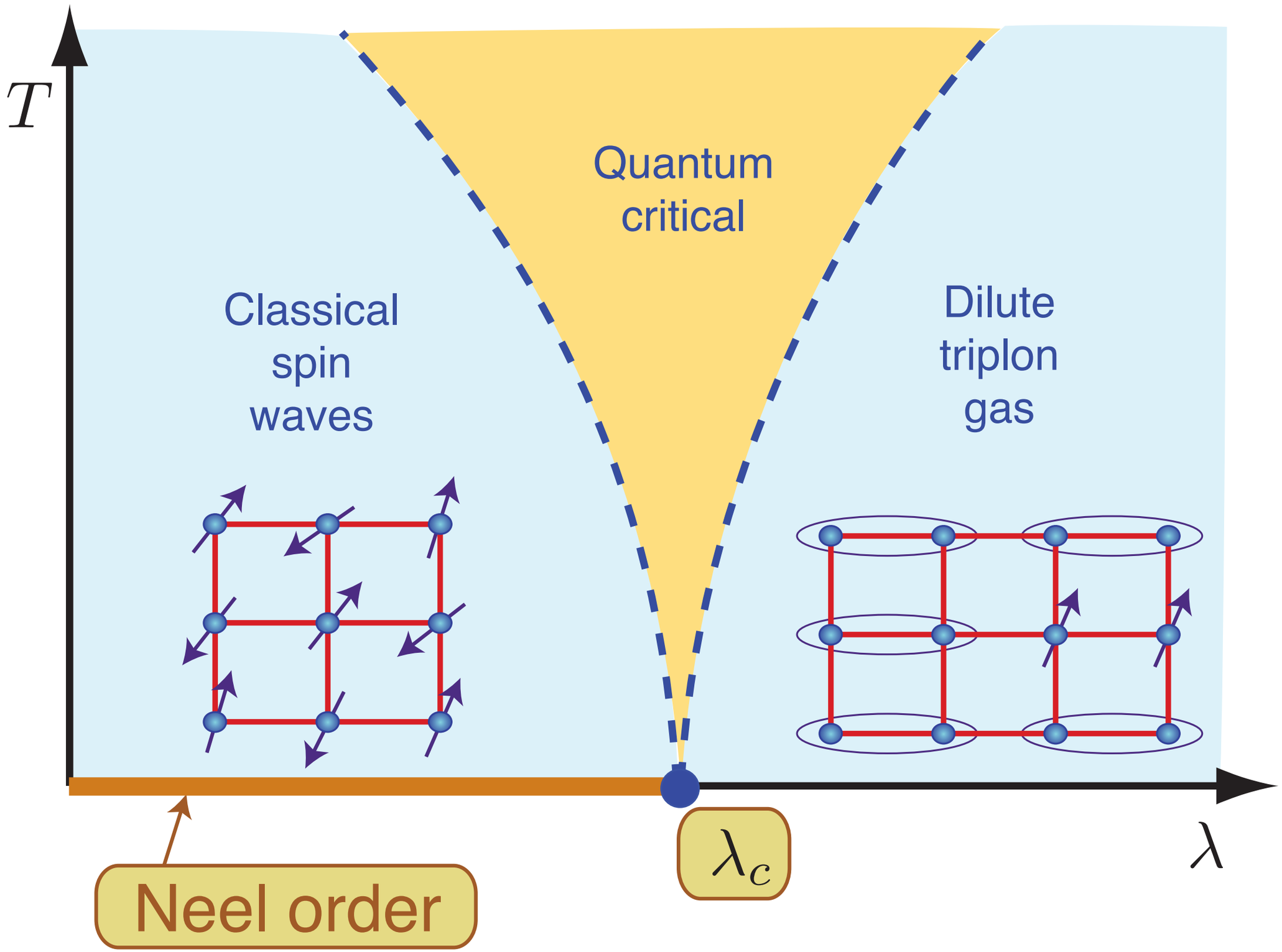


Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)



O(3) order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + s\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2 \right]$$


T
Quantum
critical
Classical
spin
waves
Dilute
triplon
gas
Neel order
λ_c
λ

Classical dynamics of spin waves

T

Quantum critical

Classical spin waves

Dilute triplon gas

λ_c

λ

Neel order

Classical Boltzmann equation for S=1 particles

T

Quantum critical

Classical spin waves

Dilute triplon gas

λ_c

λ

Neel order

CFT3 at $T>0$

Quantum critical

Classical spin waves

Dilute triplon gas

T

λ

λ_c

Neel order

Outline

A. "Relativistic" field theories of quantum phase transitions

1. Coupled dimer antiferromagnets

2. Triangular lattice antiferromagnets

3. Graphene

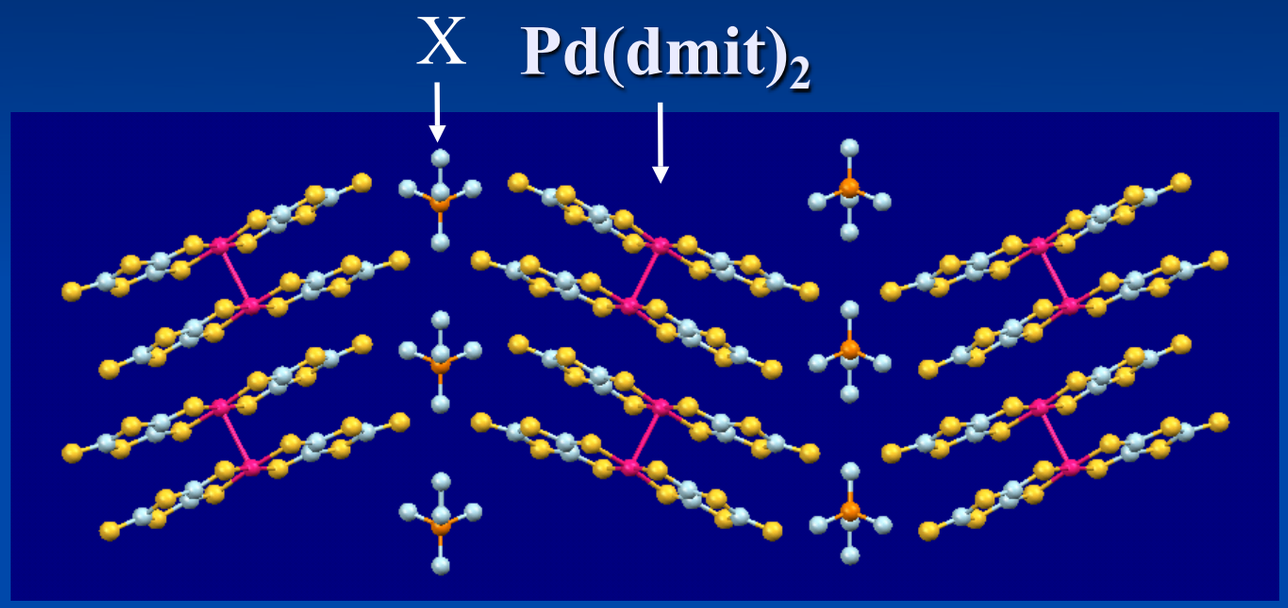
4. AdS/CFT and quantum critical transport

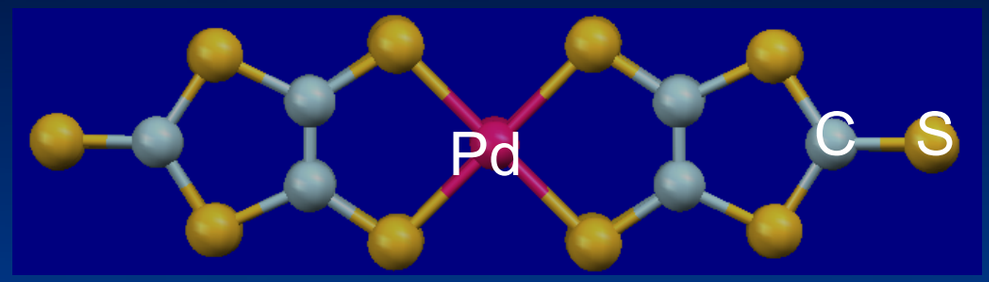
Outline

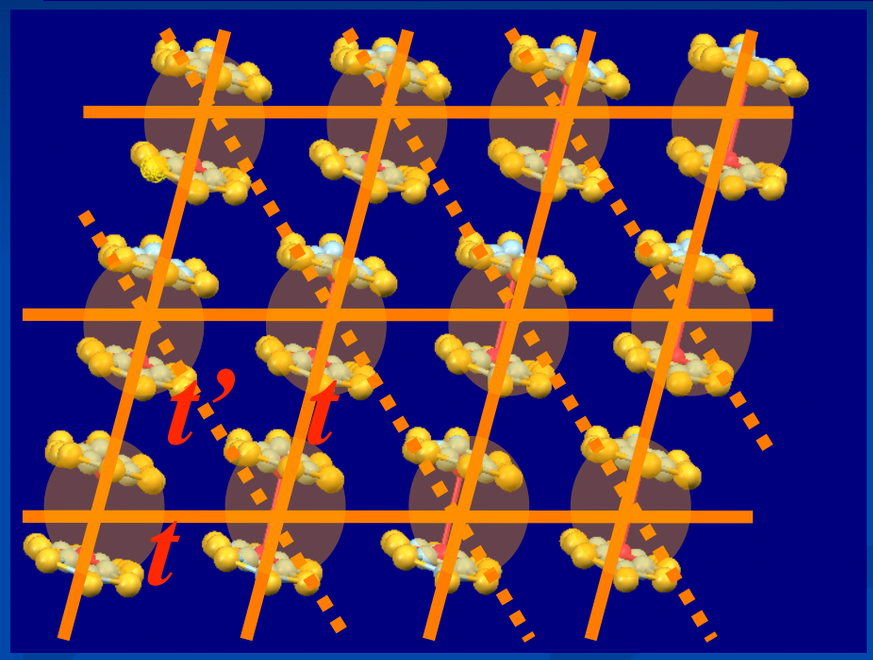
A. "Relativistic" field theories of quantum phase transitions

1. Coupled dimer antiferromagnets
2. Triangular lattice antiferromagnets
3. Graphene
4. AdS/CFT and quantum critical transport

$X[Pd(dmit)_2]_2$







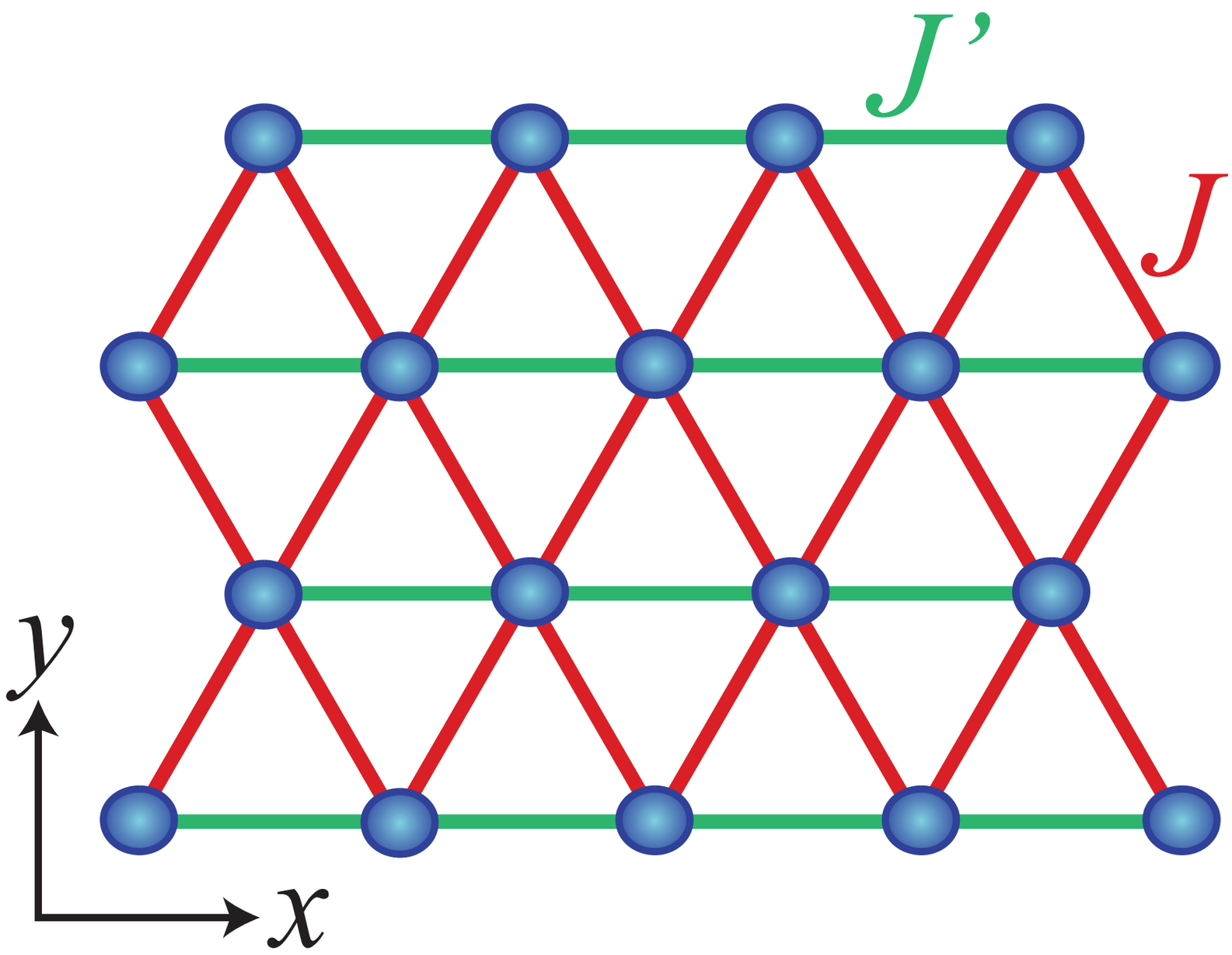
Half-filled band → Mott insulator with spin S = 1/2

Triangular lattice of $[Pd(dmit)_2]_2$
→ frustrated quantum spin system

Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \ldots$$

$\vec{S}_i \;\Rightarrow\;$ spin operator with $S = 1/2$



$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \ldots$$

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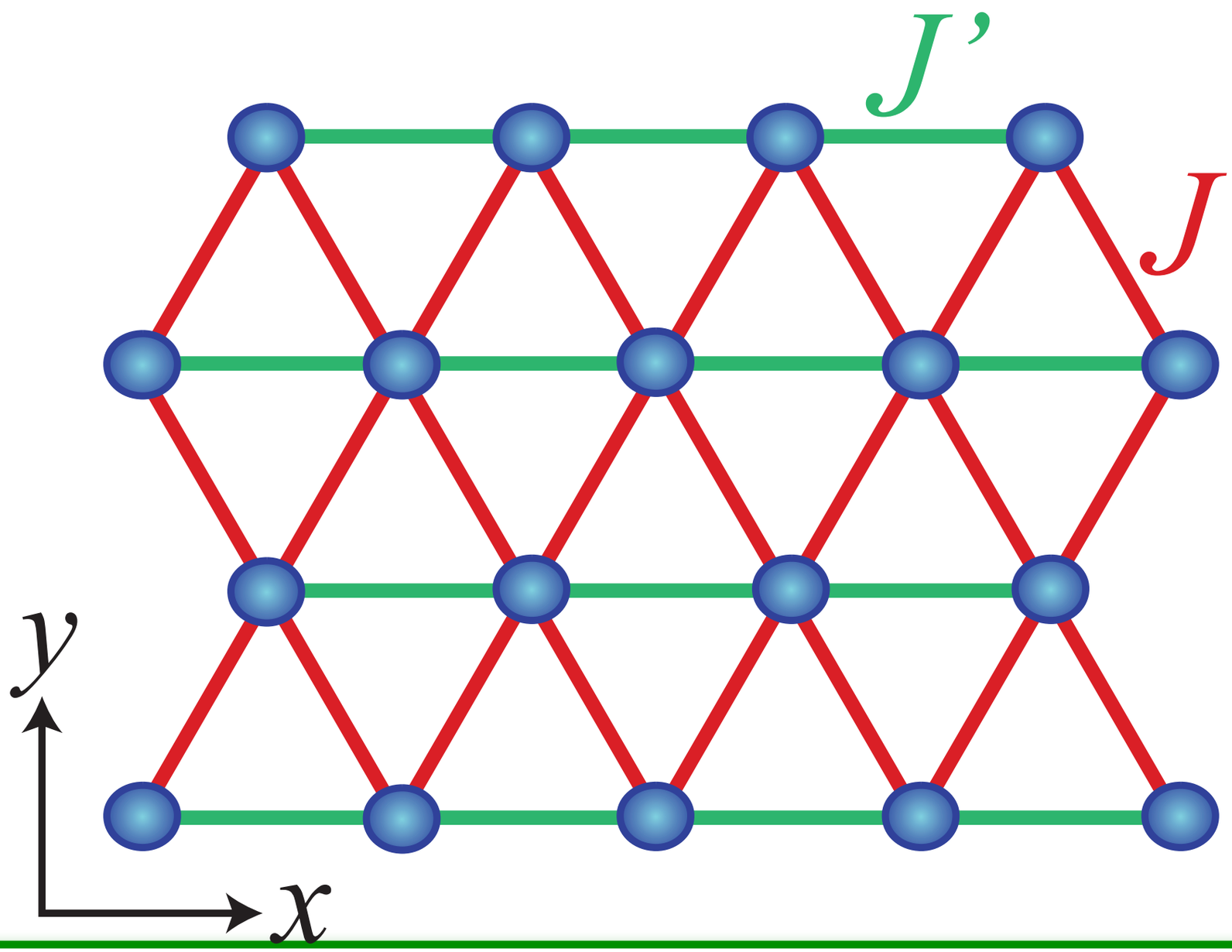
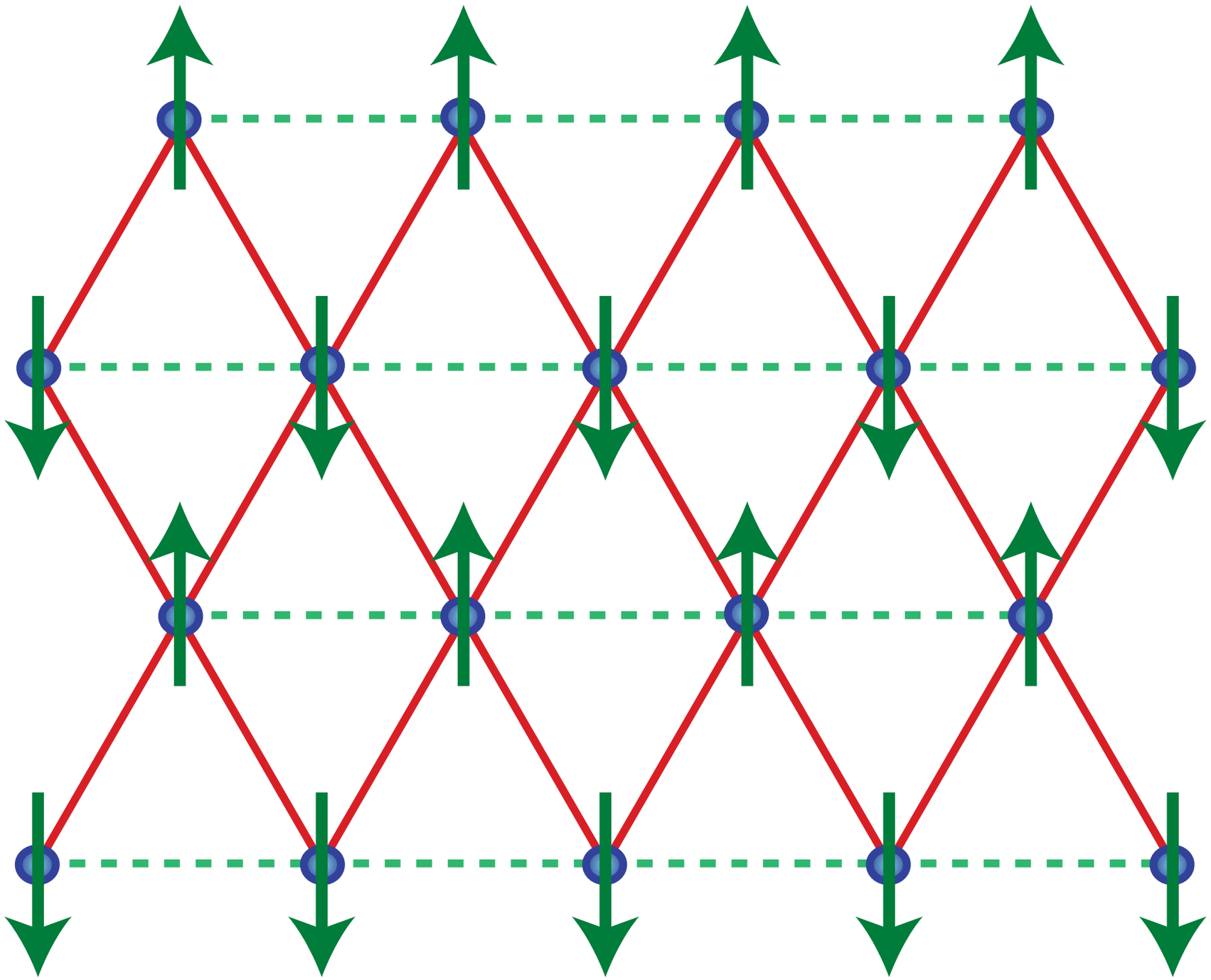


What is the ground state as a function of J'/J ?

Anisotropic triangular lattice antiferromagnet

Broken spin rotation symmetry



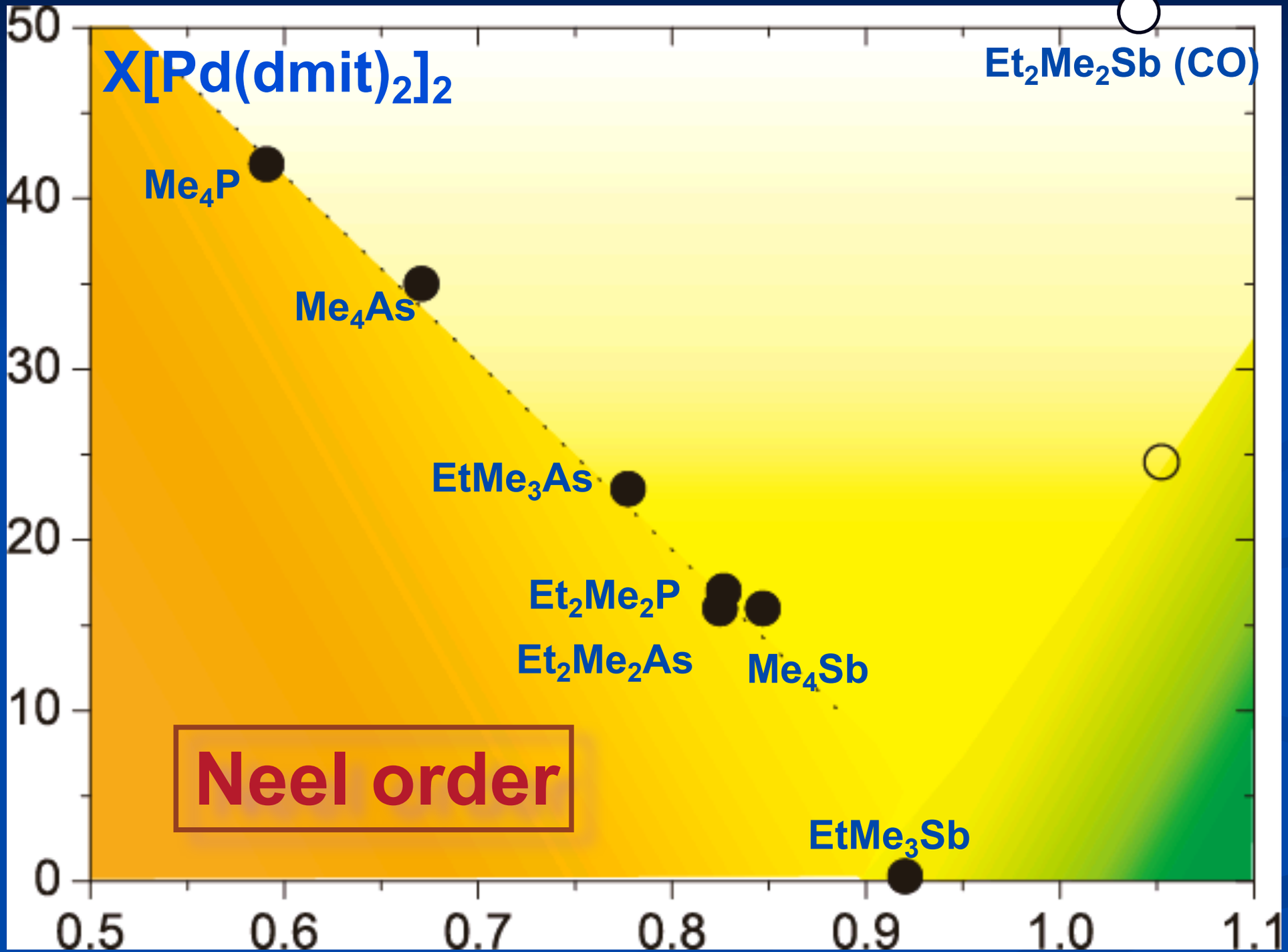
Neel ground state for small J'/J

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

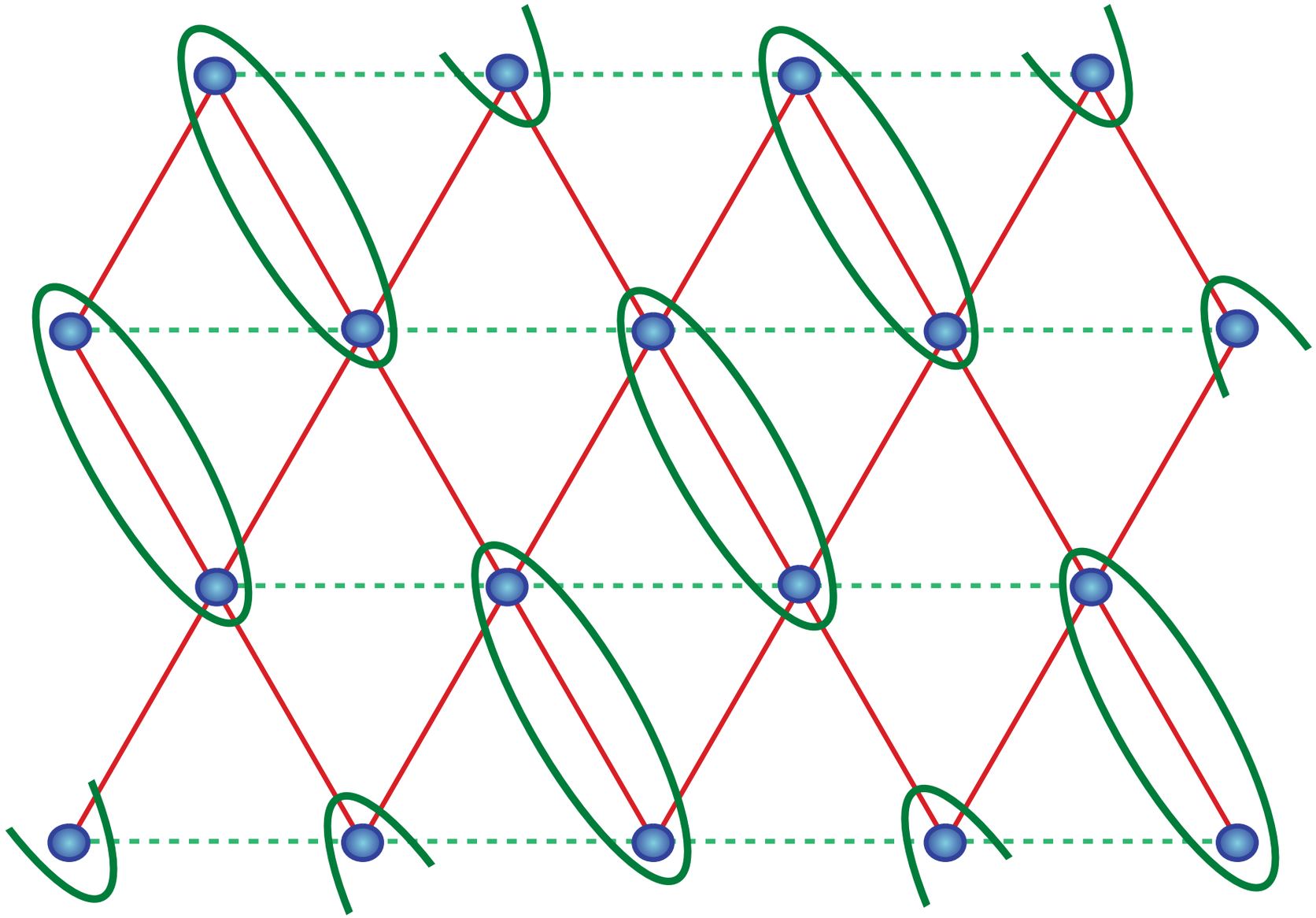
- Néel antiferromagnetic LRO

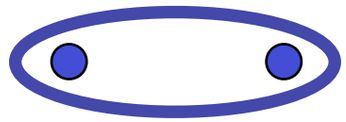
Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Anisotropic triangular lattice antiferromagnet





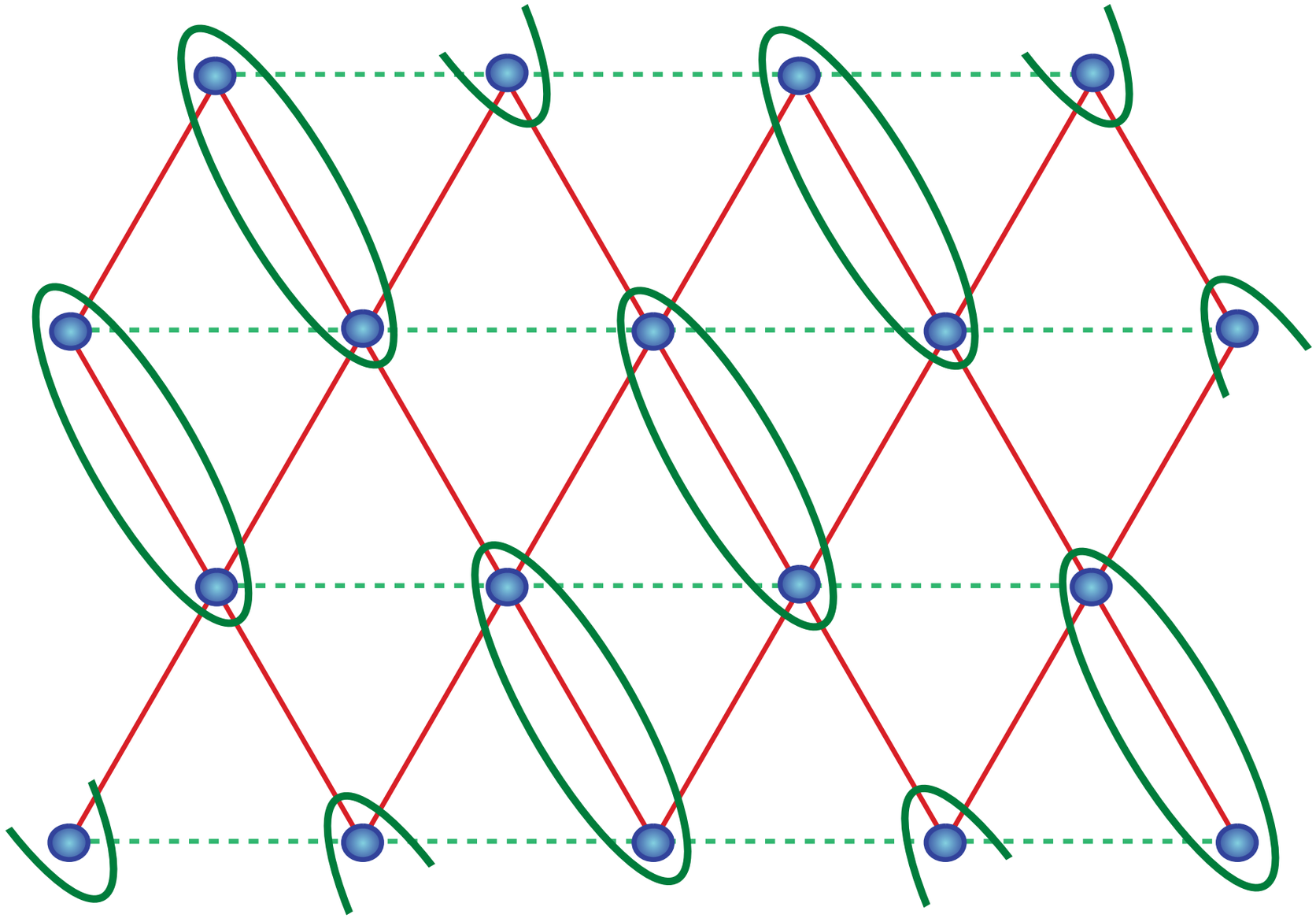
$$= \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$

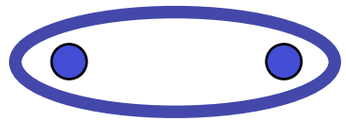
Possible ground state for intermediate J'/J

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)

Anisotropic triangular lattice antiferromagnet

Broken lattice space group symmetry





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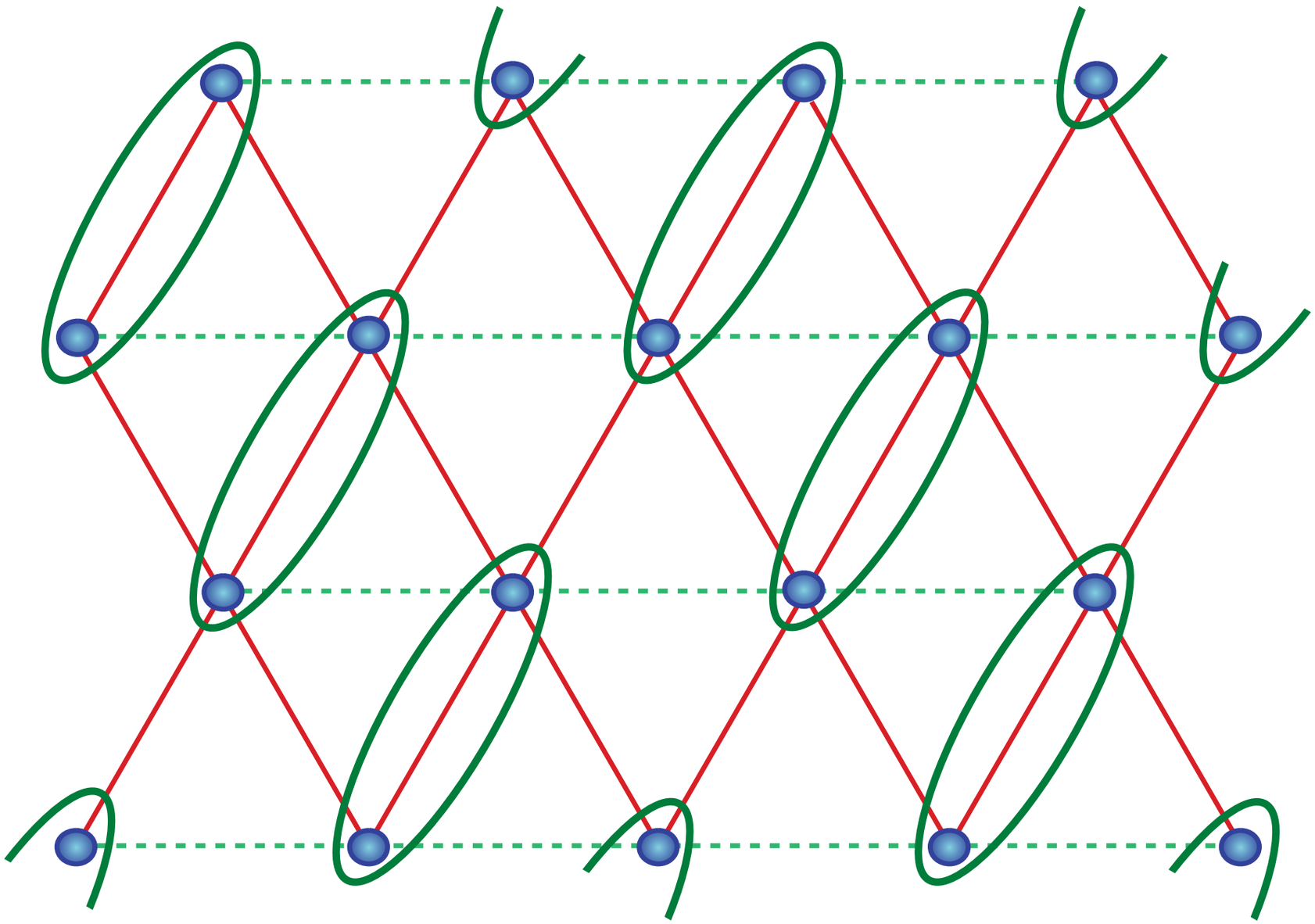
Valence bond solid (VBS)

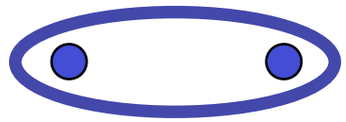
Possible ground state for intermediate J'/J

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)

Anisotropic triangular lattice antiferromagnet

Broken lattice space group symmetry





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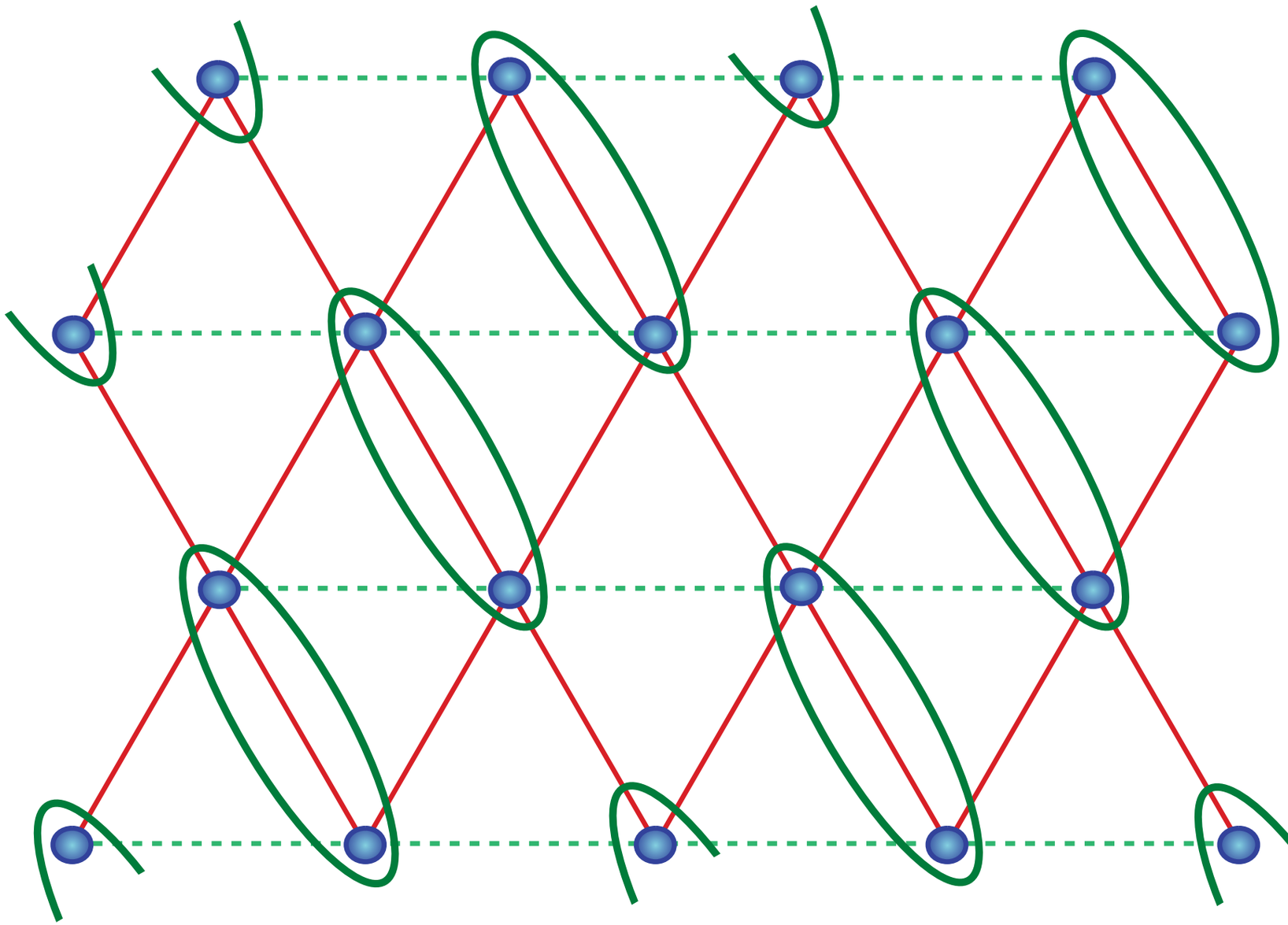

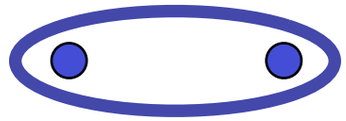

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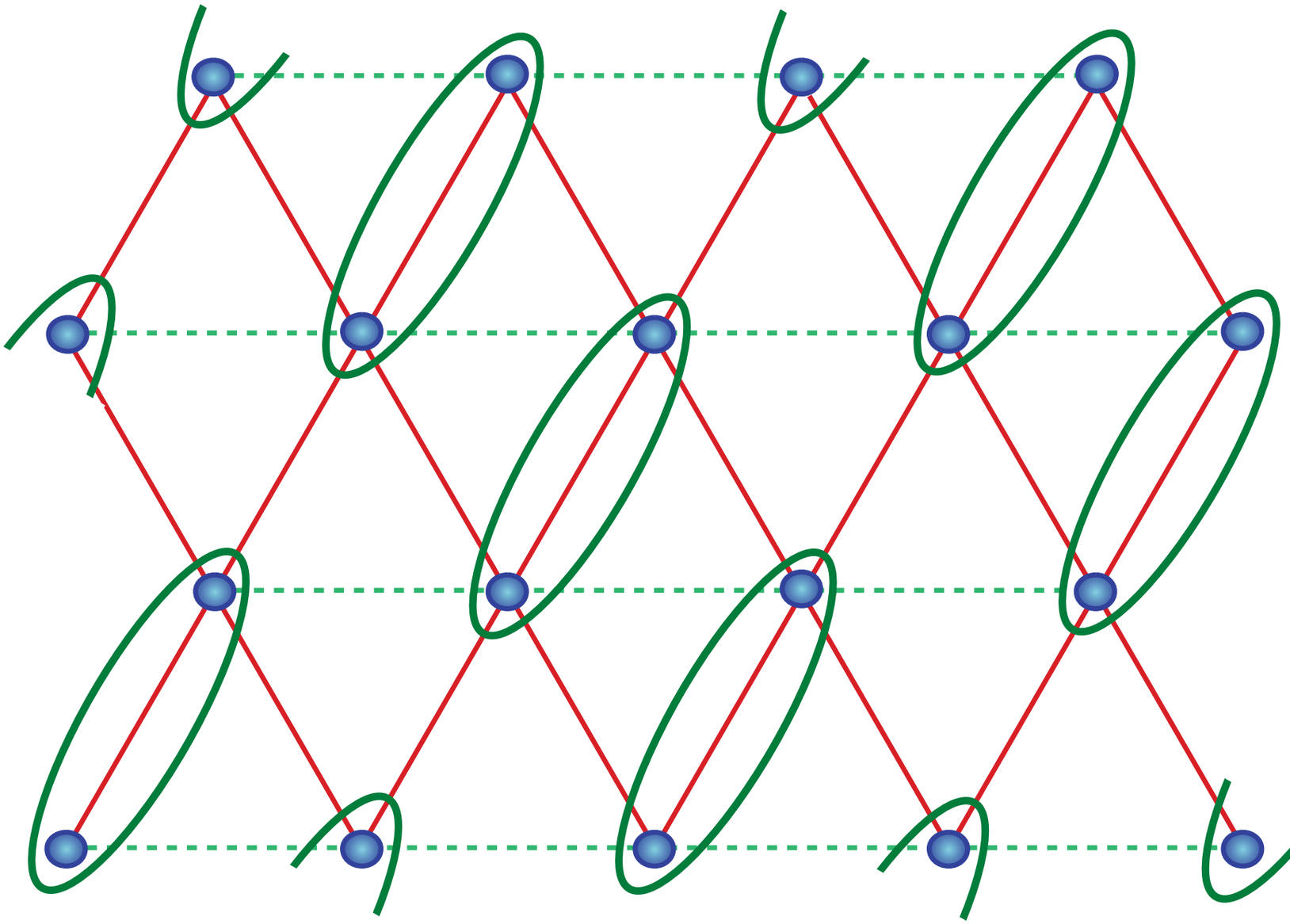

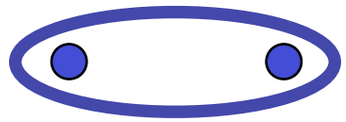

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Valence bond solid (VBS)

Possible ground state for intermediate J'/J

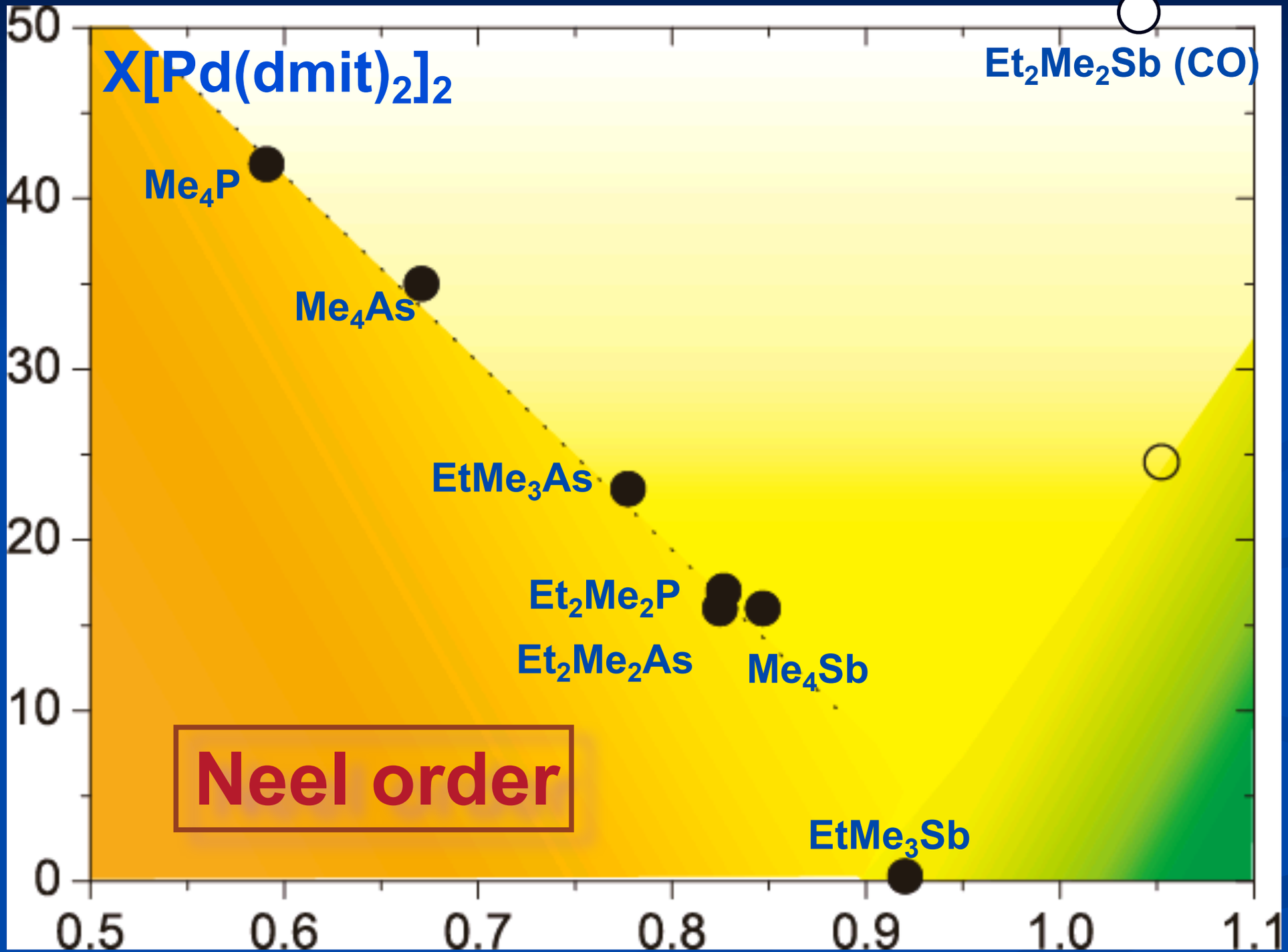
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

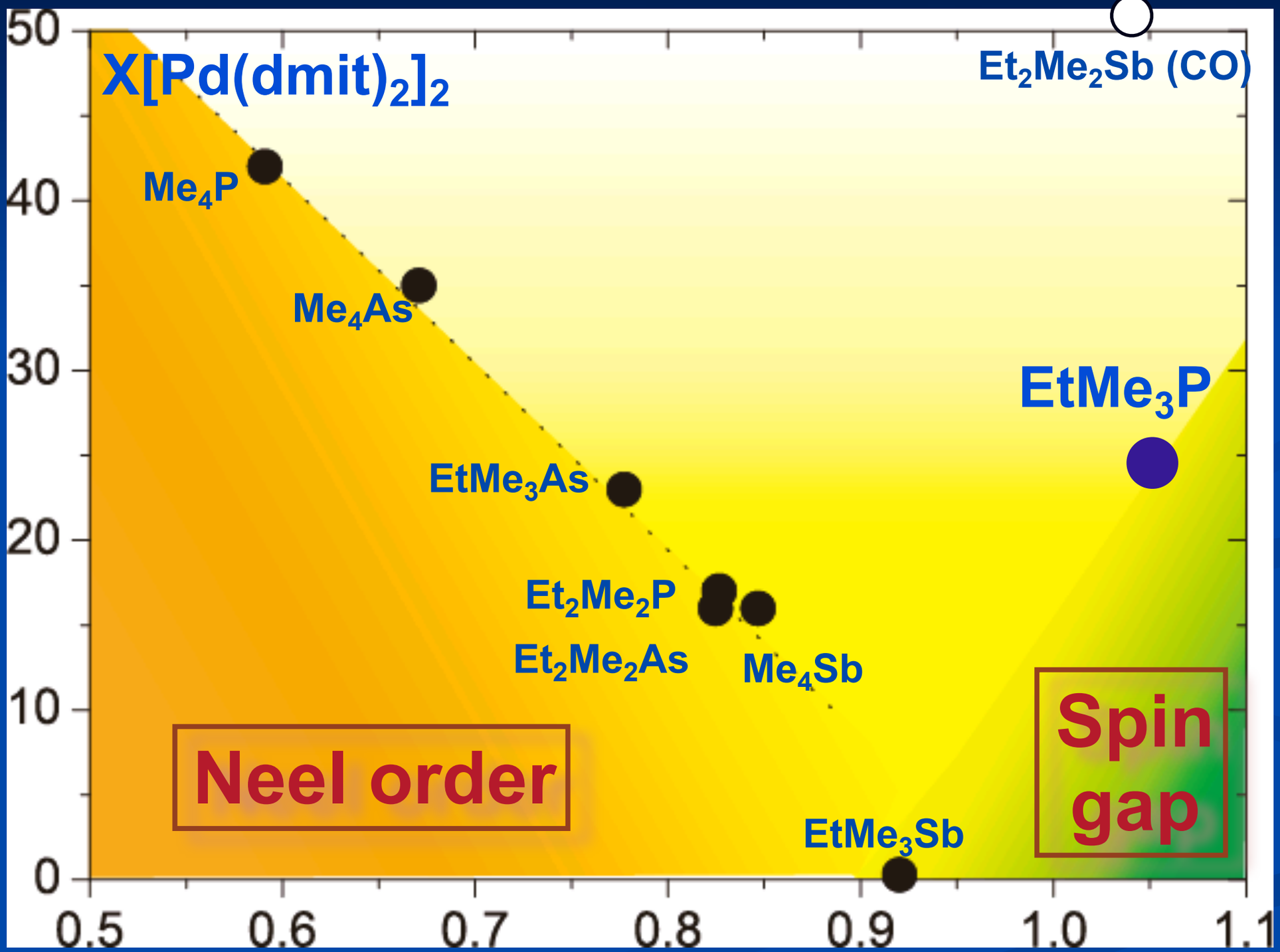
- Néel antiferromagnetic LRO
- Valence bond solid

Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Magnetic Criticality



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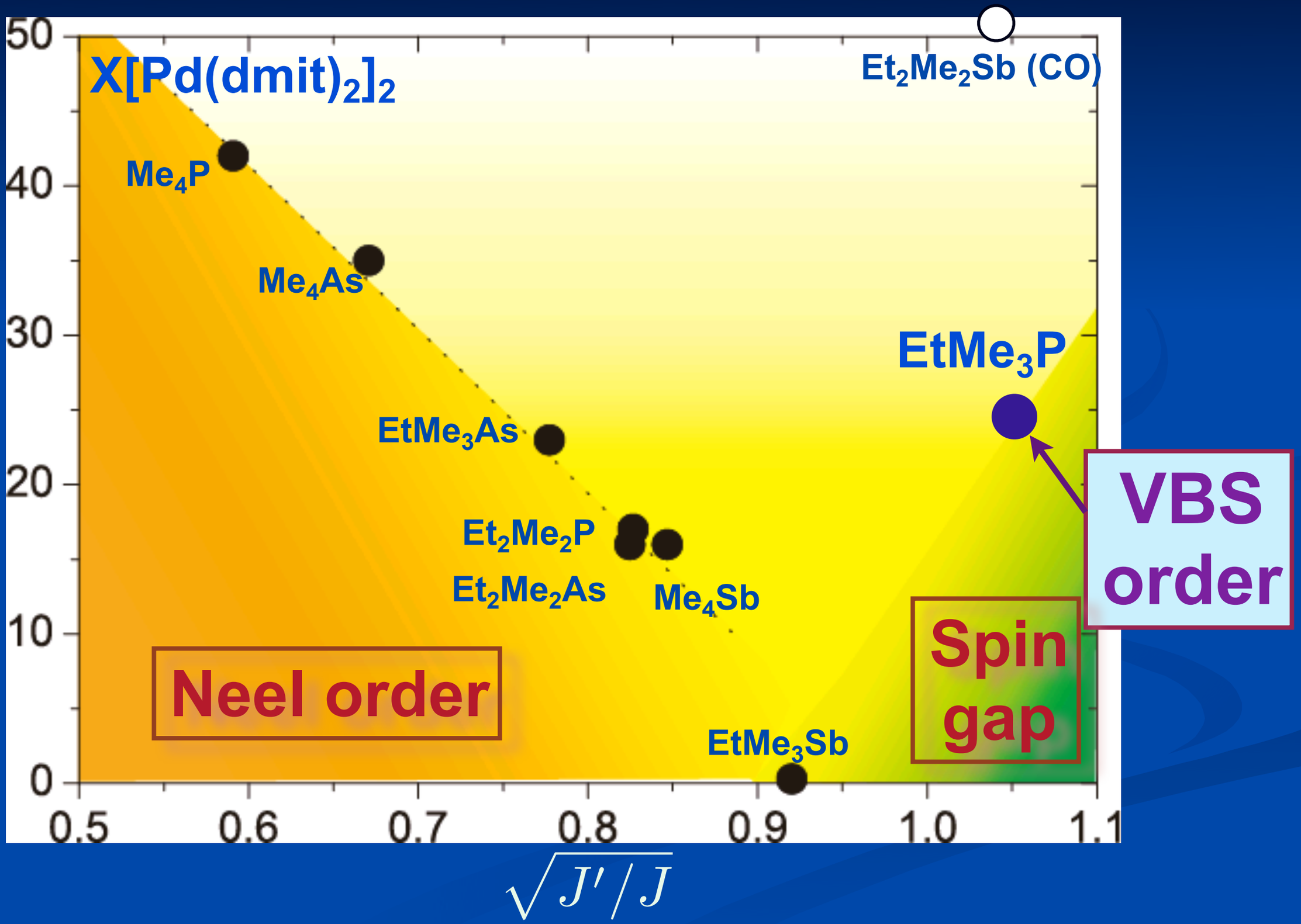
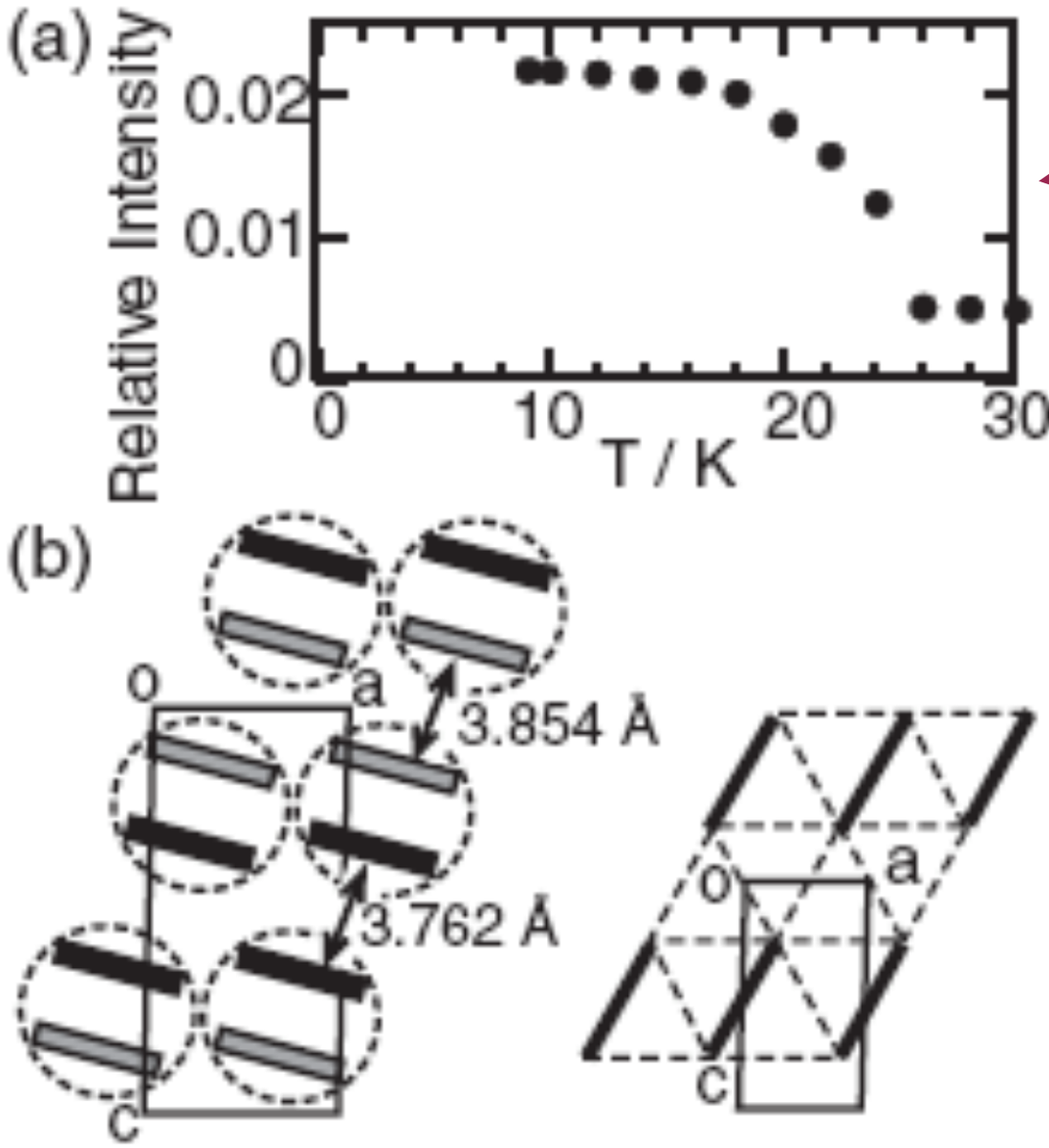
Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Observation of a valence bond solid (VBS) in $ETMe_3P[Pd(dmit)_2]_2$

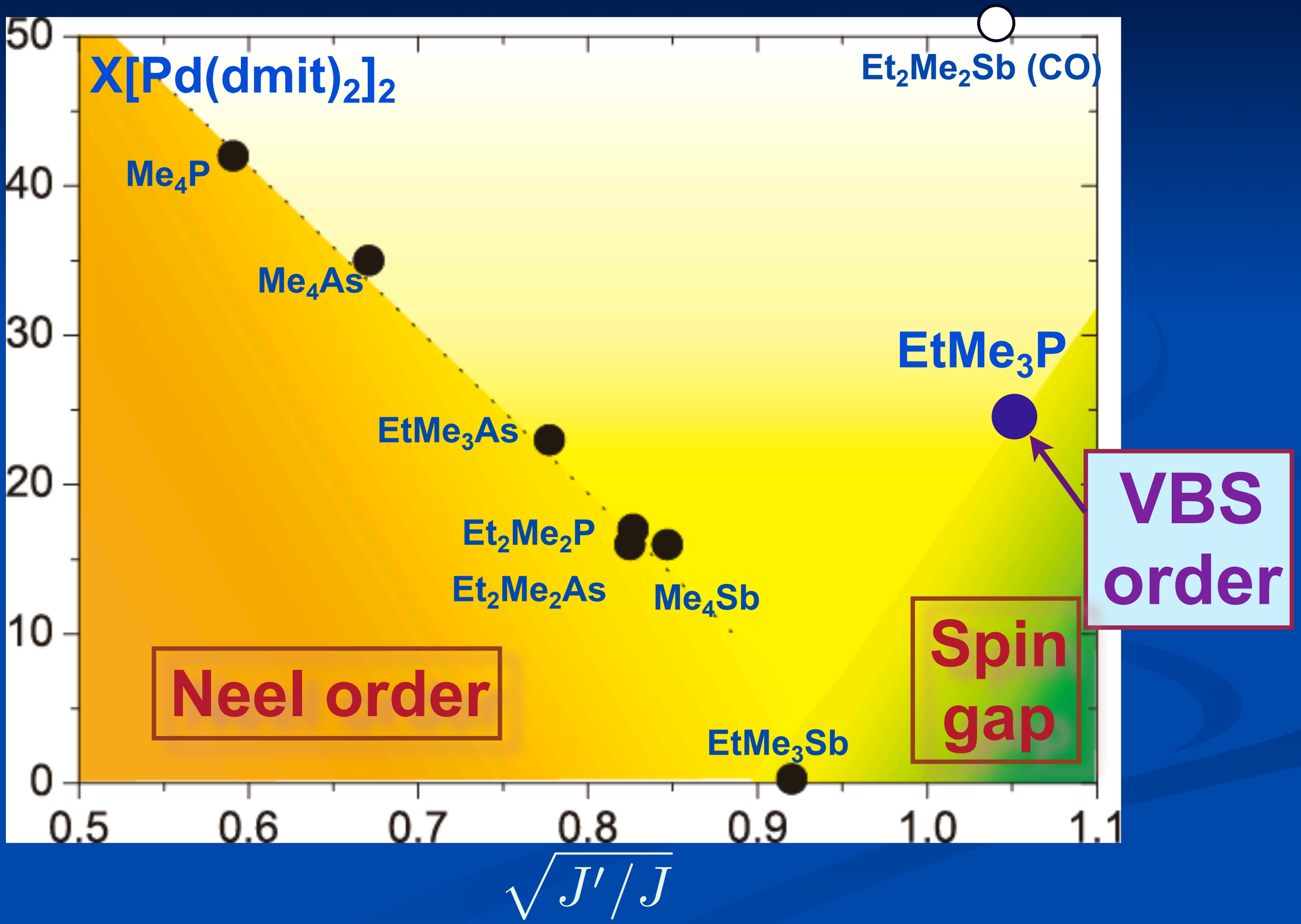


X-ray scattering

Spin gap ~ 40 K
J ~ 250 K

M. Tamura, A. Nakao and R. Kato, *J. Phys. Soc. Japan* **75**, 093701 (2006)
Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *Phys. Rev. Lett.* **99**, 256403 (2007)

Magnetic Criticality



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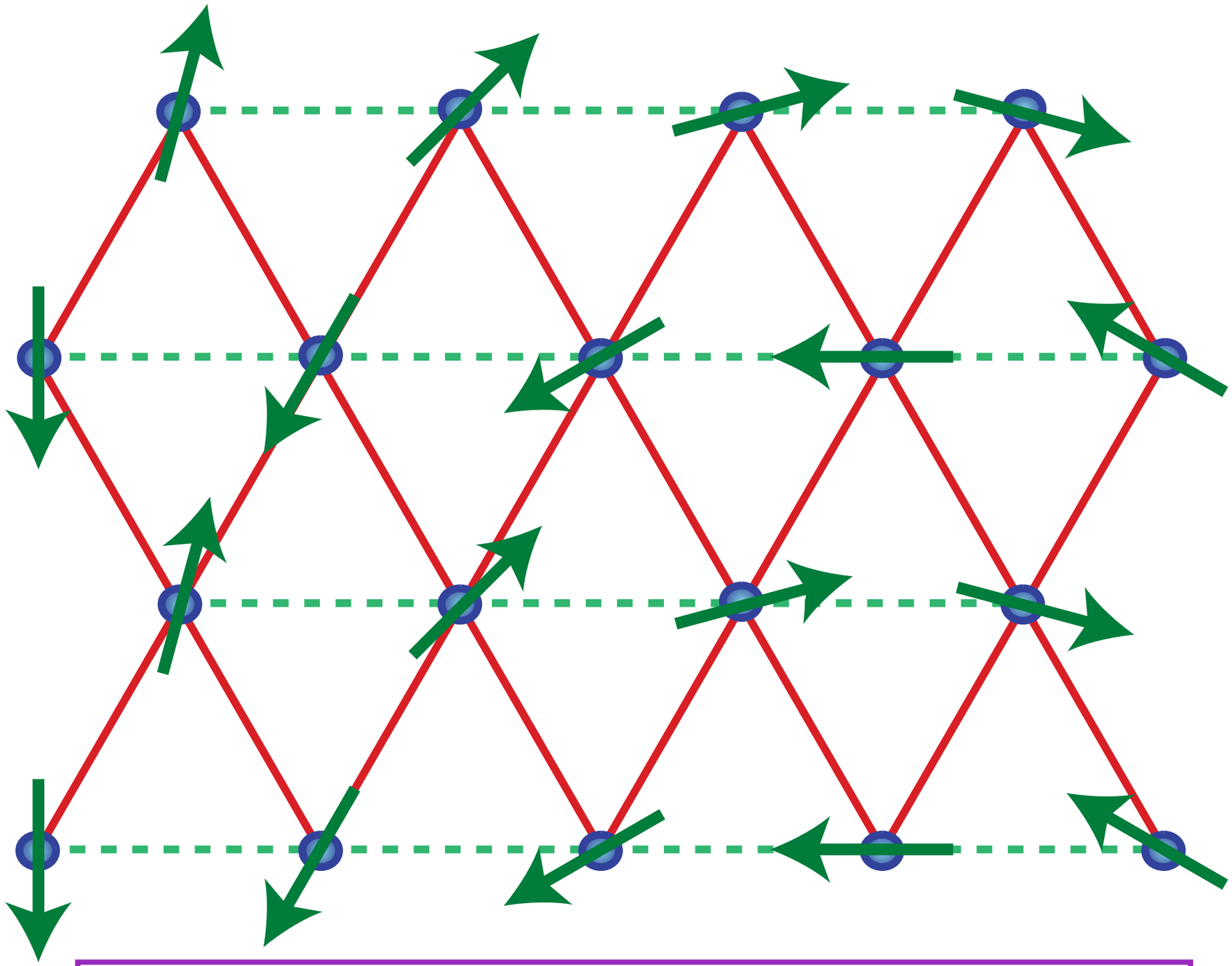
Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid

Anisotropic triangular lattice antiferromagnet



Classical ground state for large J'/J

Found in Cs_2CuCl_4

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid
- Spiral LRO

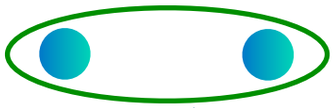
Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

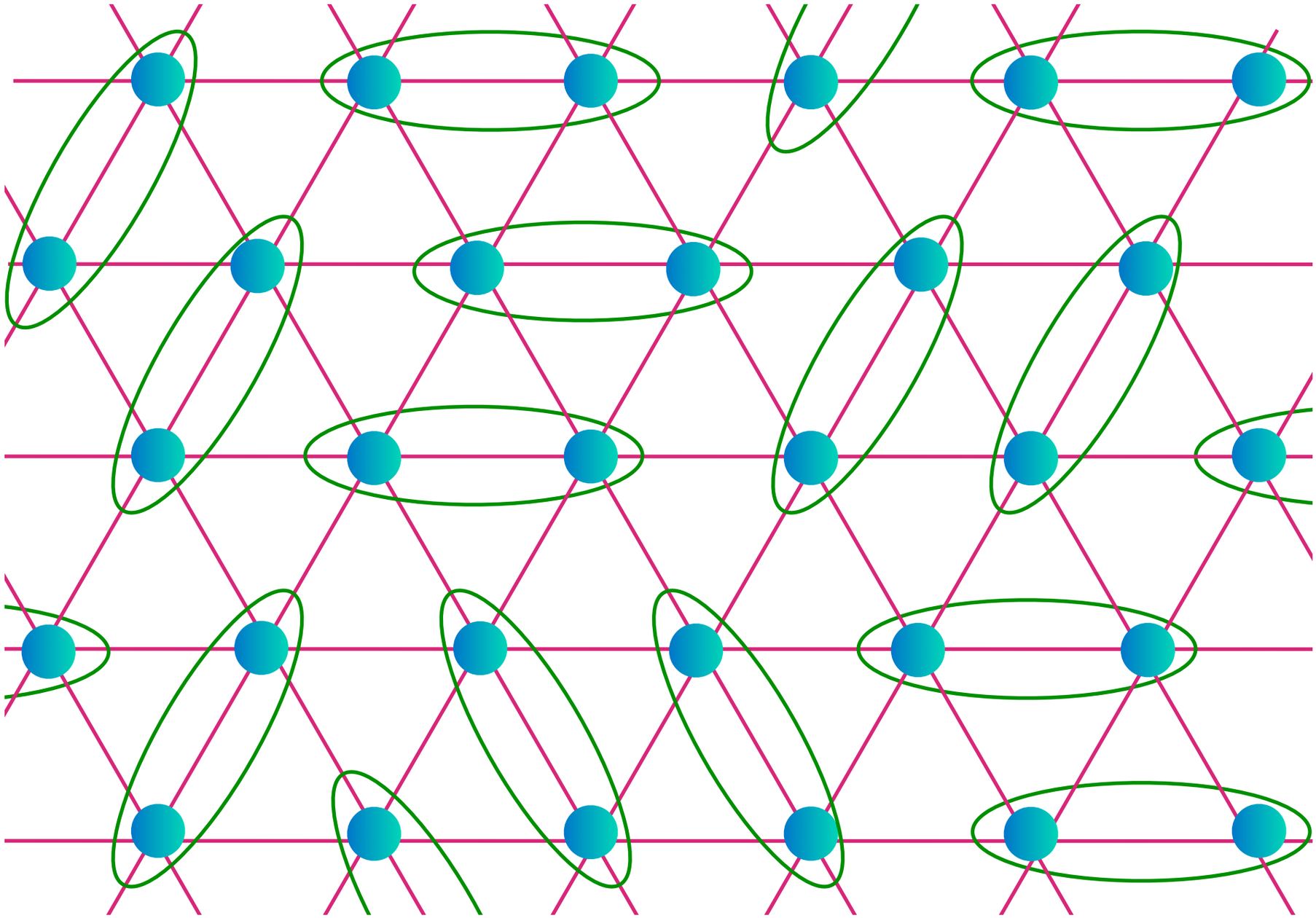
- Néel antiferromagnetic LRO
- Valence bond solid
- Spiral LRO
- Z_2 spin liquid: preserves all symmetries of Hamiltonian

Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with $S=1/2$ per unit cell



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Triangular lattice antiferromagnet

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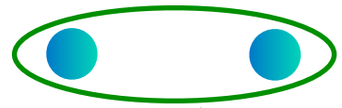

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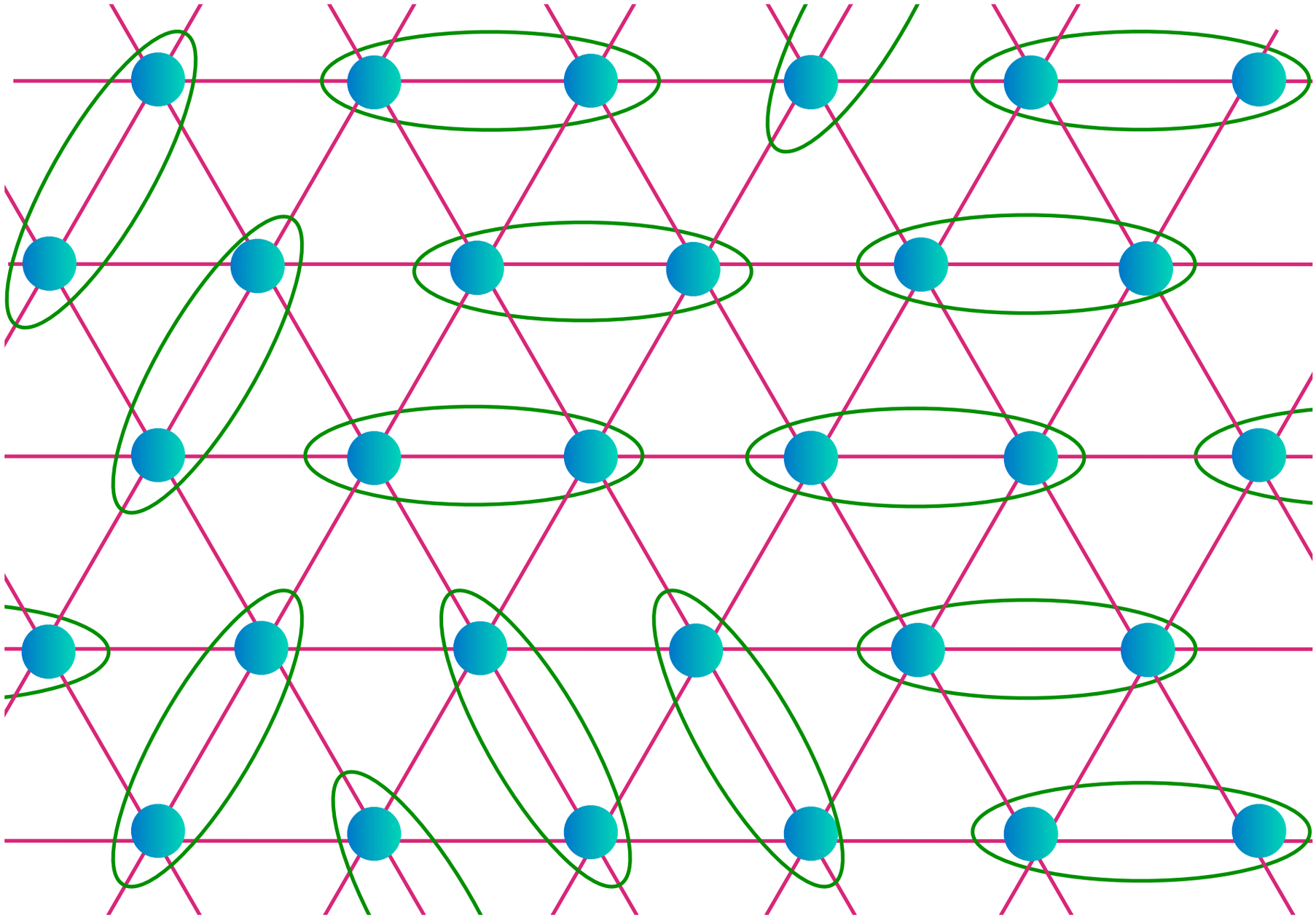

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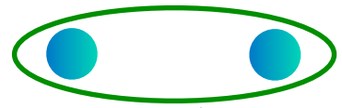

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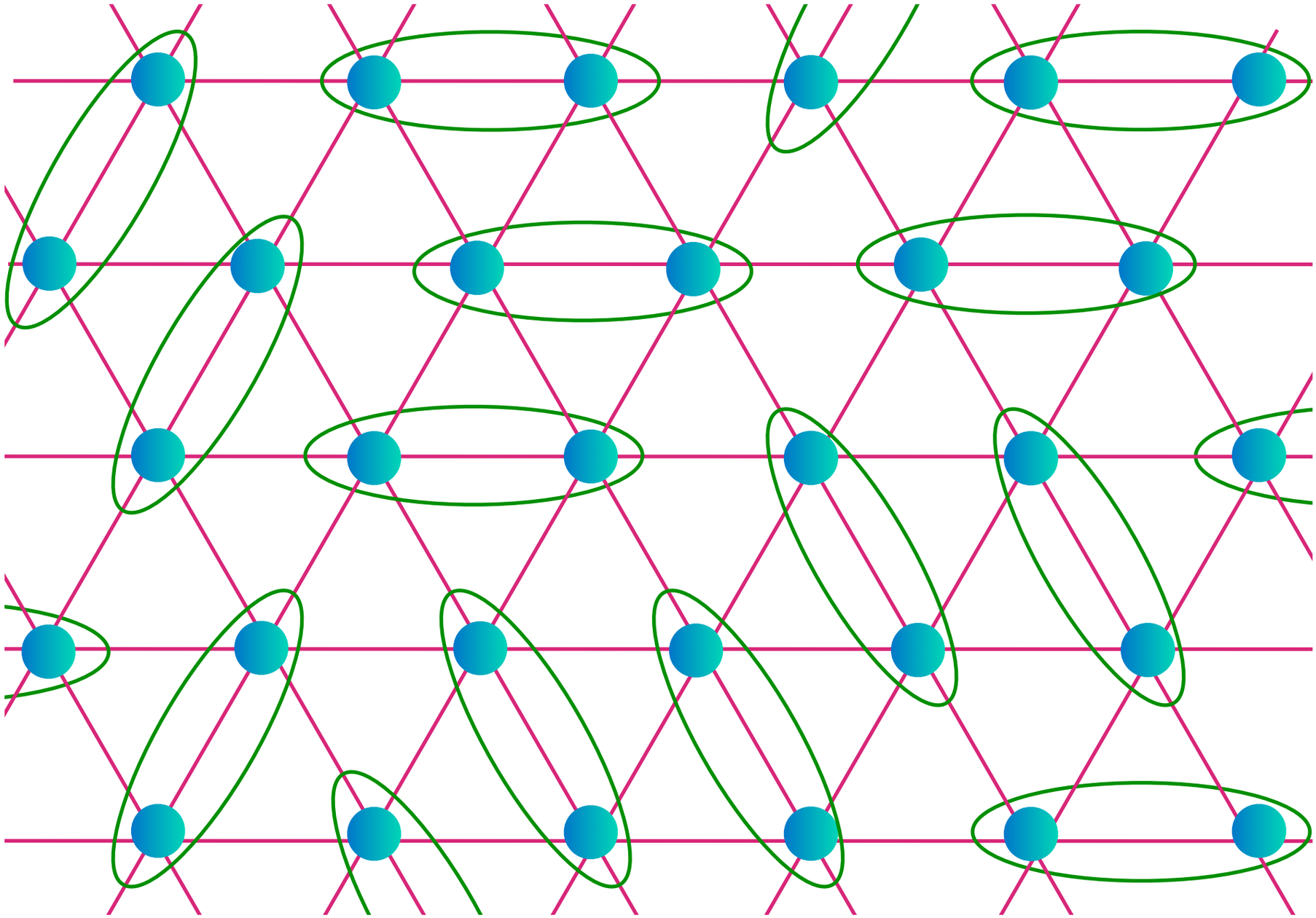

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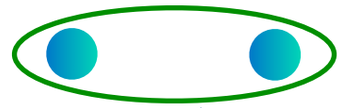

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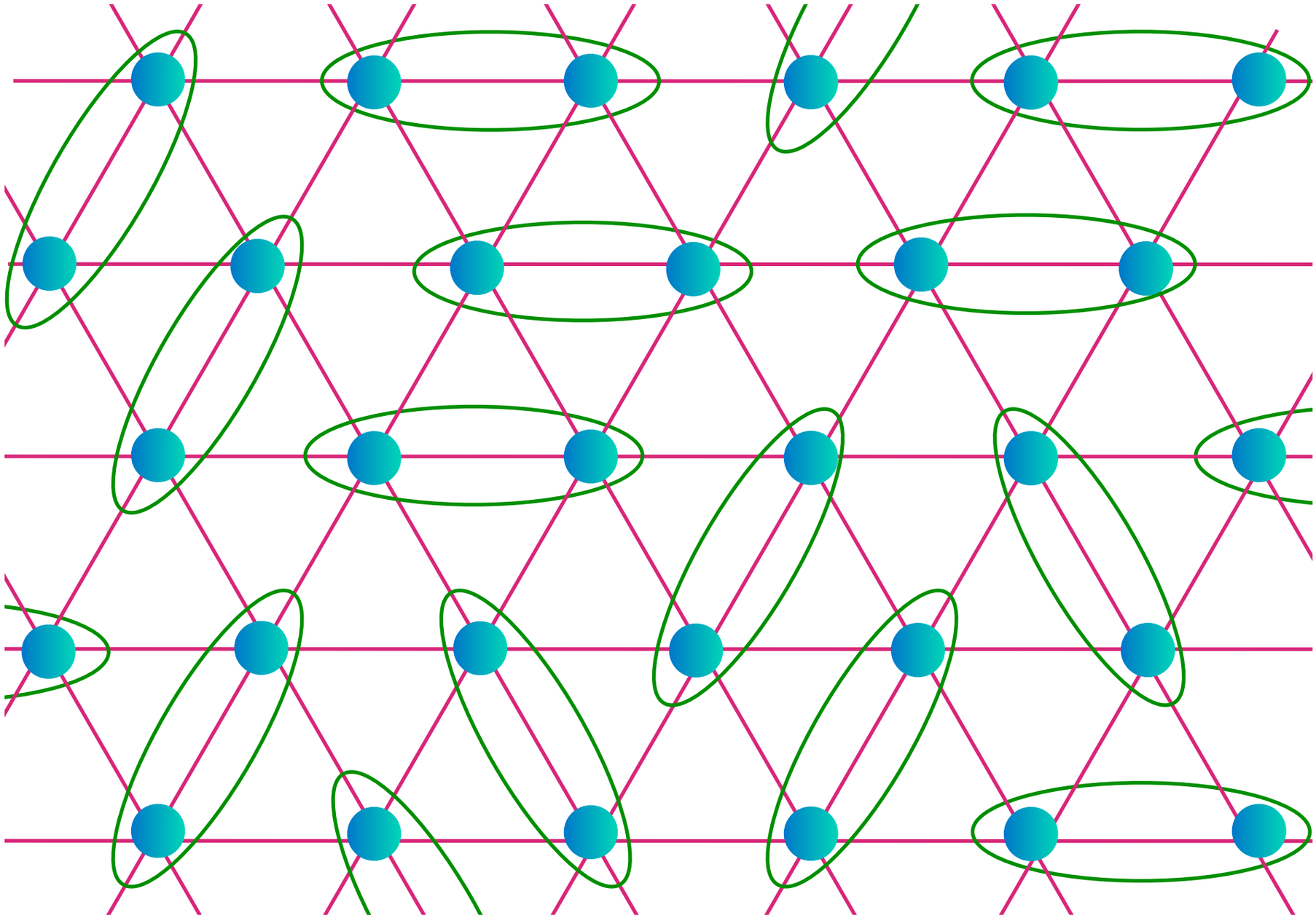

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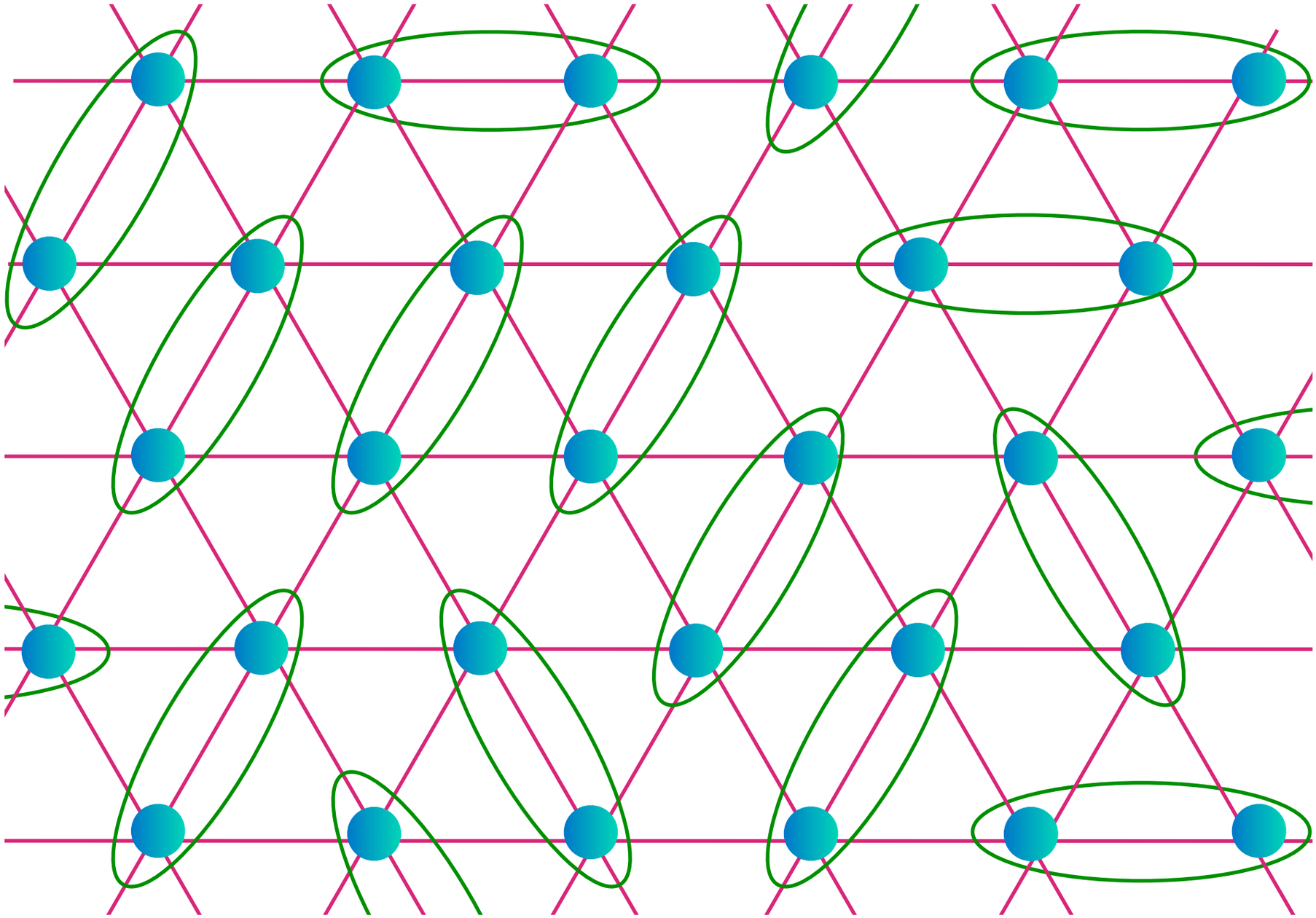

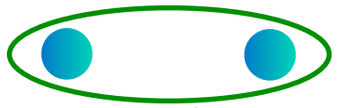

$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Triangular lattice antiferromagnet

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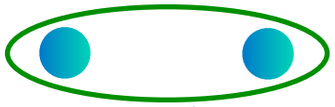

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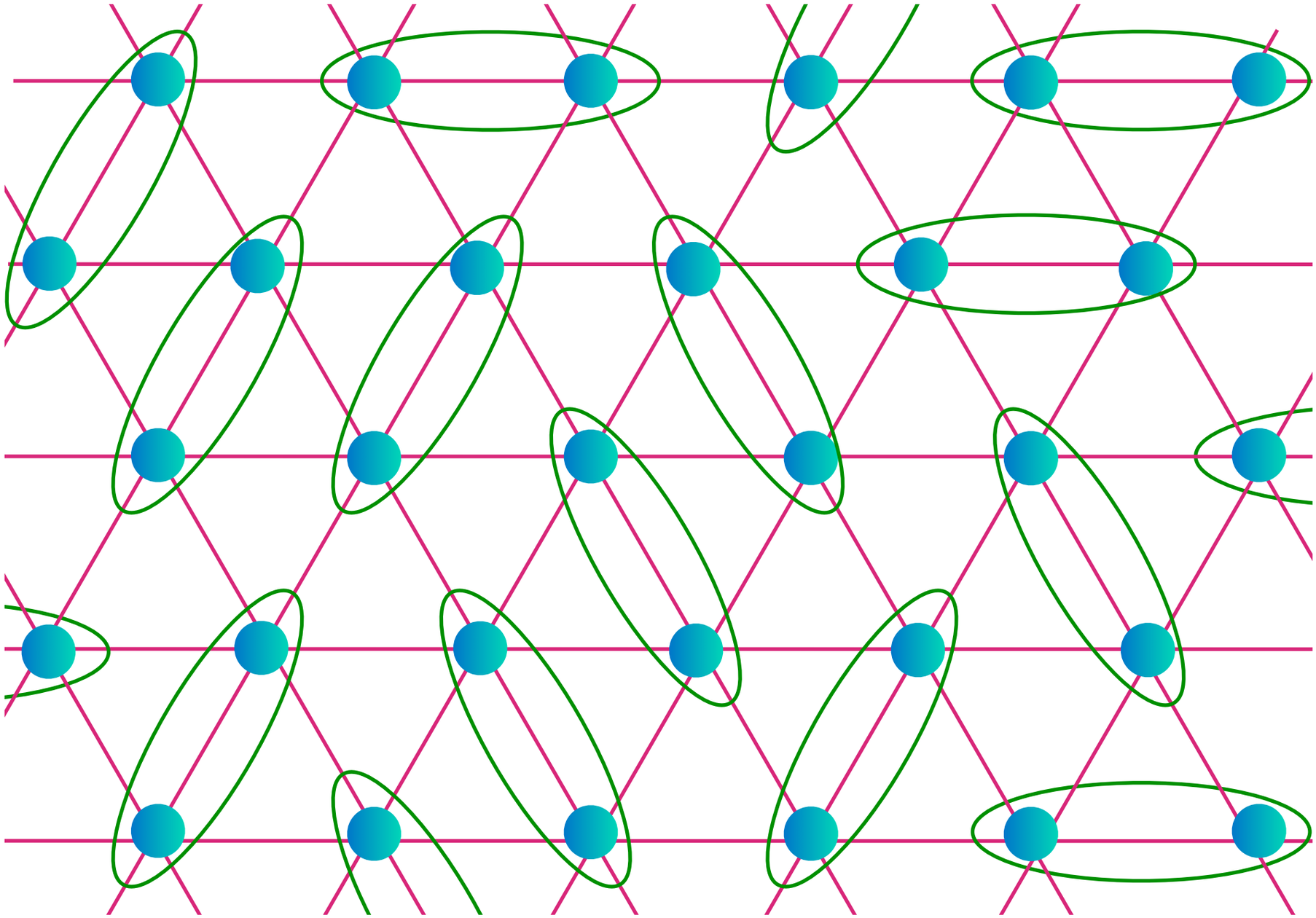

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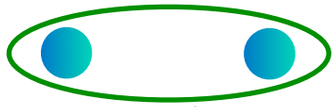
Excitations of the Z_2 Spin liquid



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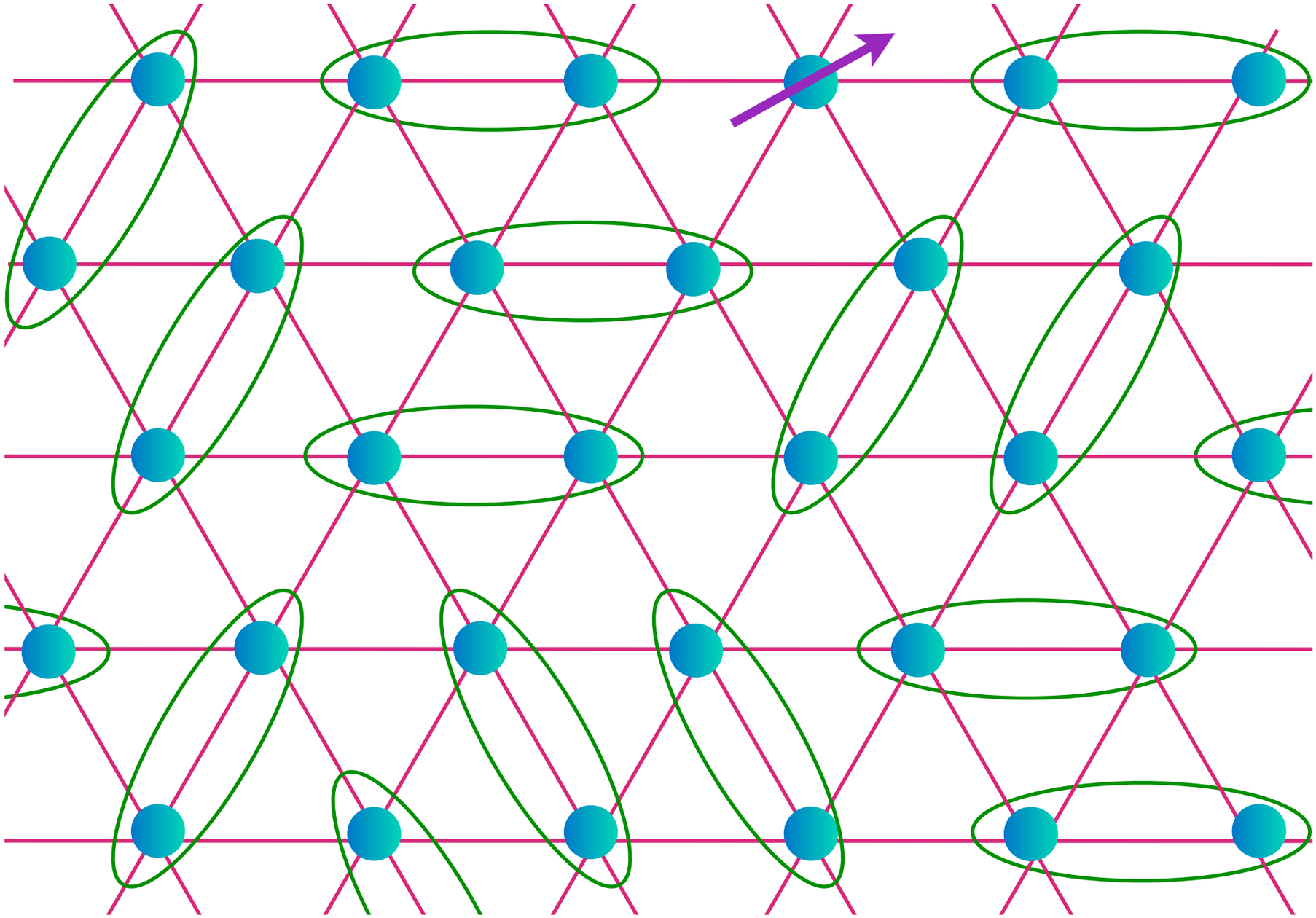
A spinon



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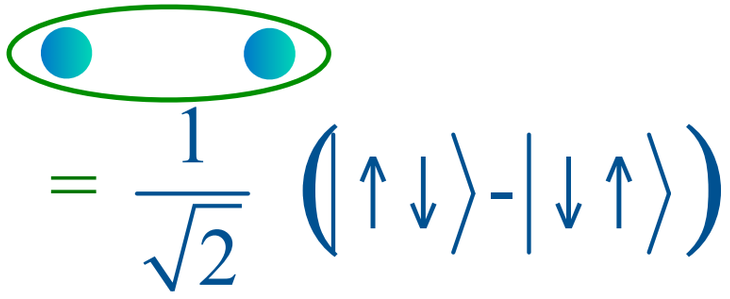

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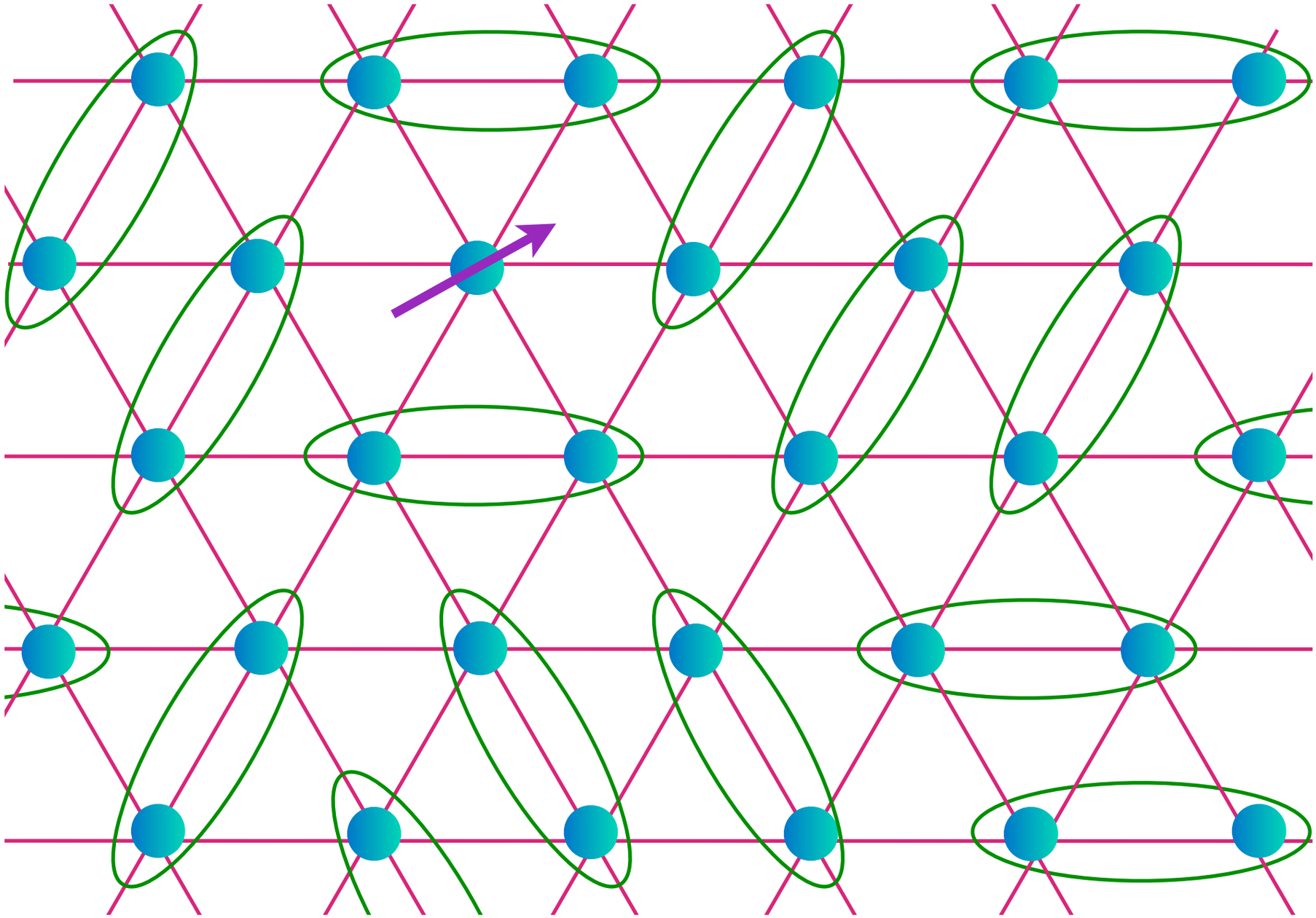

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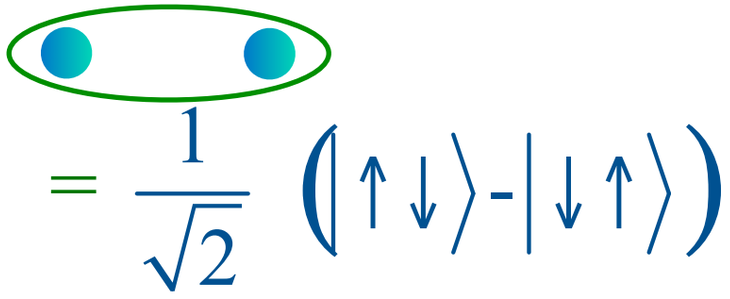

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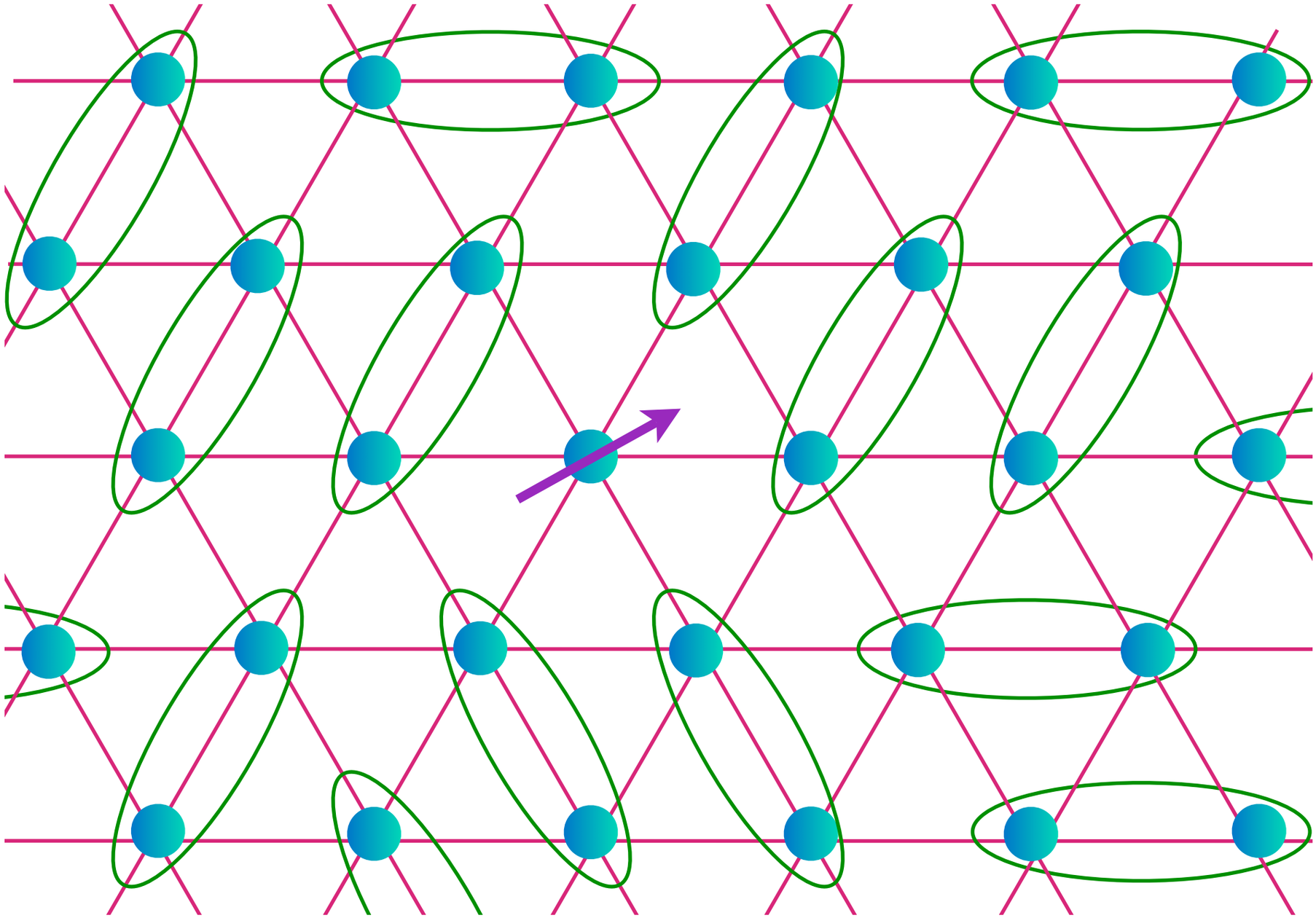

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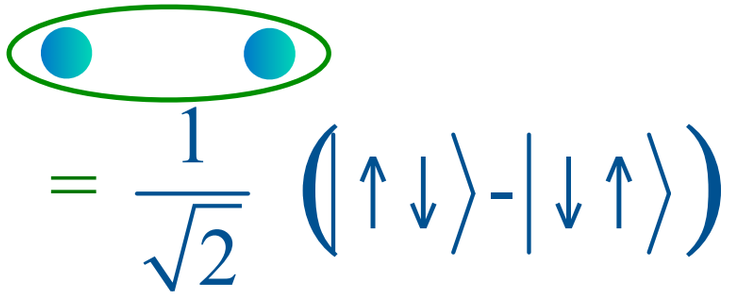

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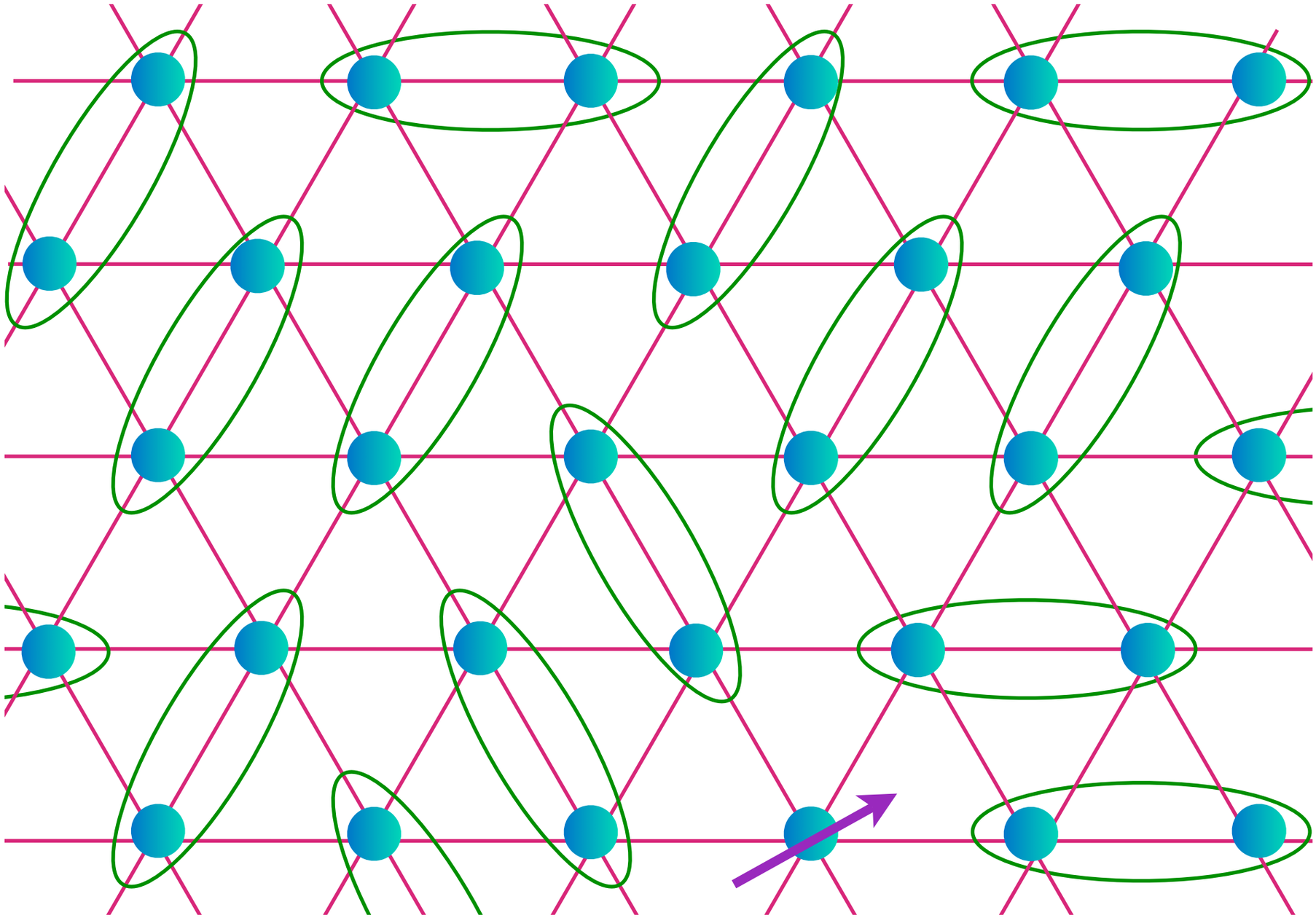

Excitations of the Z_2 Spin liquid

A spinon

The spinon annihilation operator is a spinor z_α, where $\alpha = \uparrow, \downarrow$.

The Néel order parameter, $\vec{\varphi}$ is a composite of the spinons:

$$\vec{\varphi} = z^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} z_{i\beta}$$

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where $\vec{\sigma}$ are Pauli matrices

The theory for quantum phase transitions is expressed in terms of fluctuations of z_α, and *not* the order parameter $\vec{\varphi}$.

Effective theory for z_α must be invariant under the U(1) gauge transformation

$$z_{i\alpha} \to e^{i\theta} z_{i\alpha}$$

Excitations of the Z_2 Spin liquid

A vison

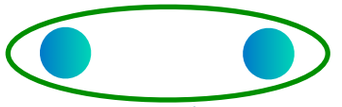
- A characteristic property of a Z_2 spin liquid is the presence of a spinon pair condensate

- A vison is an Abrikosov vortex in the pair condensate of spinons

- Visons are are the **dark matter** of spin liquids: they likely carry most of the energy, but are very hard to detect because they do not carry charge or spin.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

Excitations of the Z_2 Spin liquid



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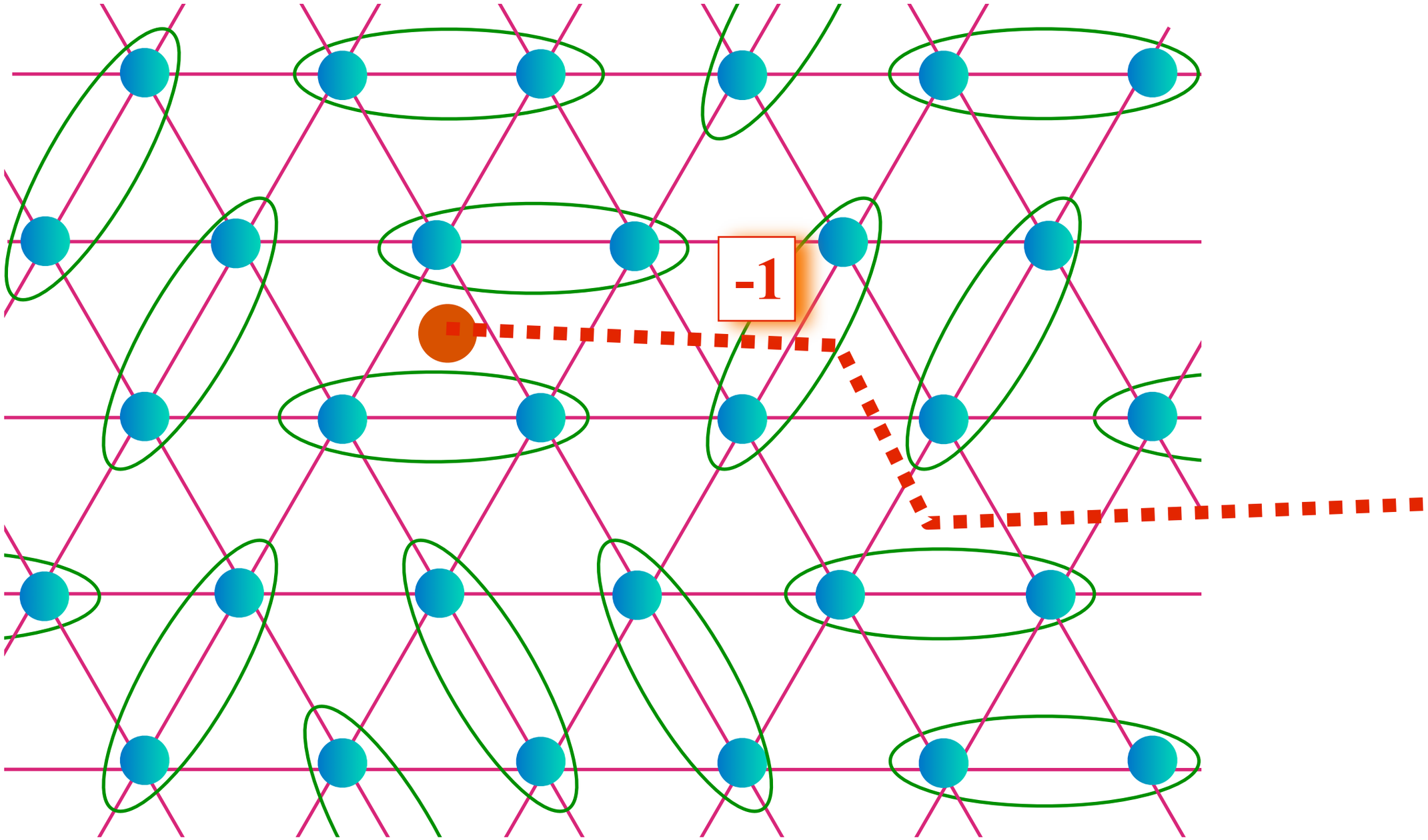
A vison



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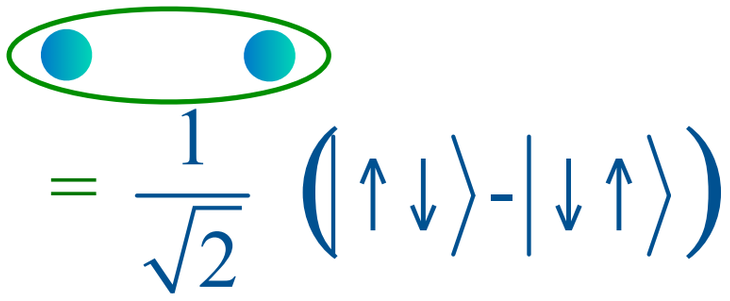

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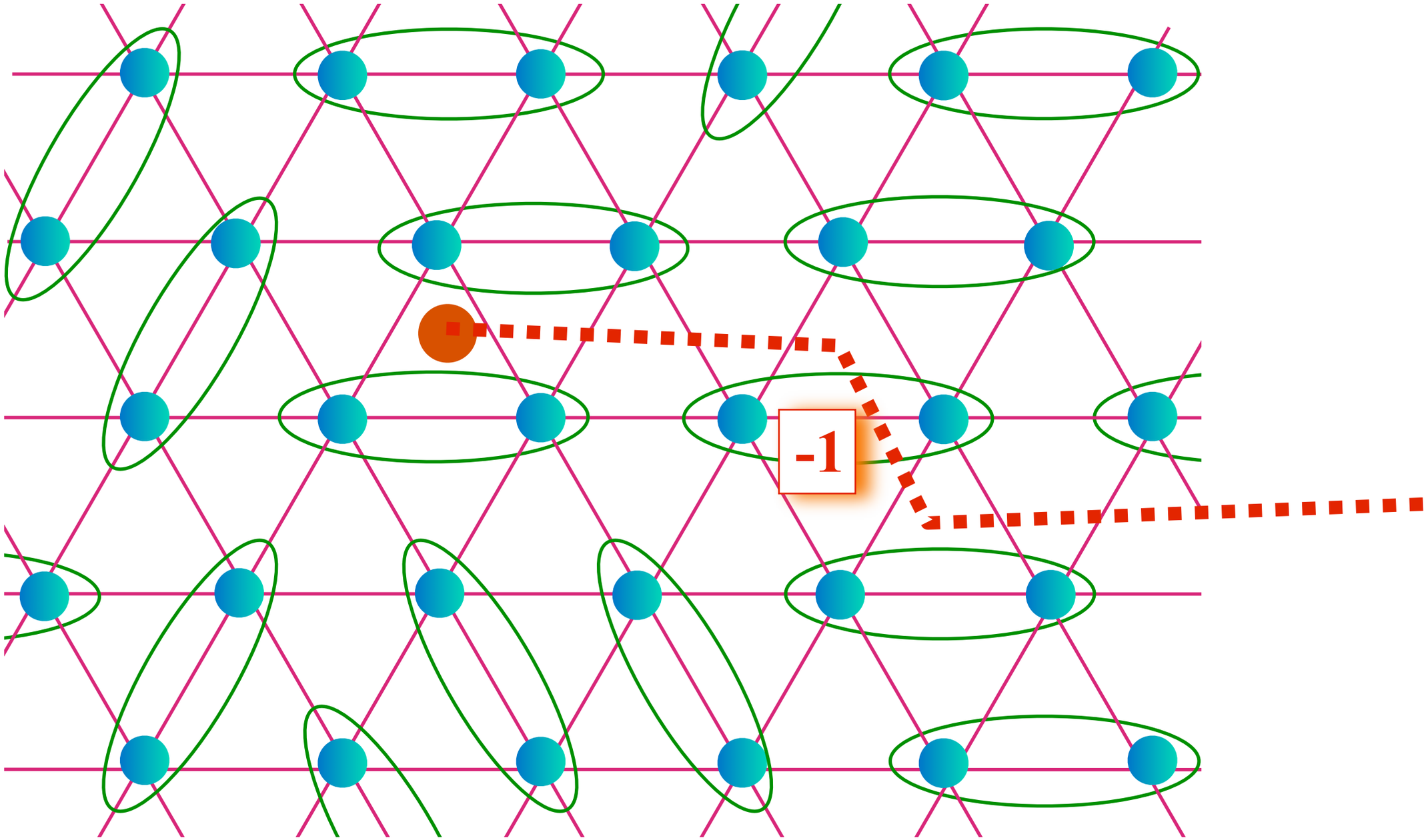

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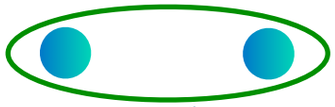

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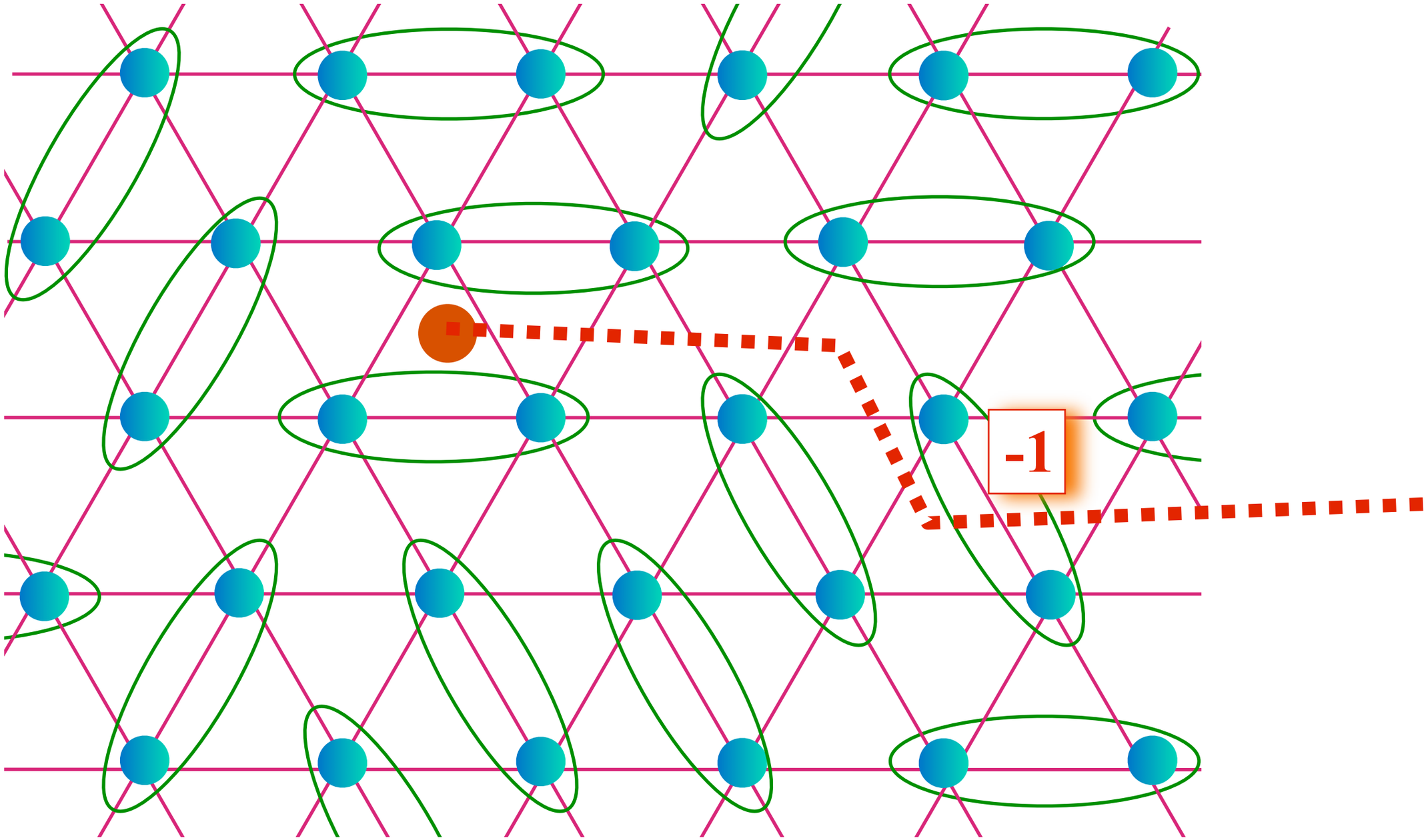

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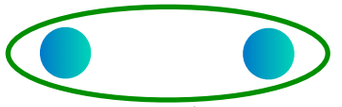

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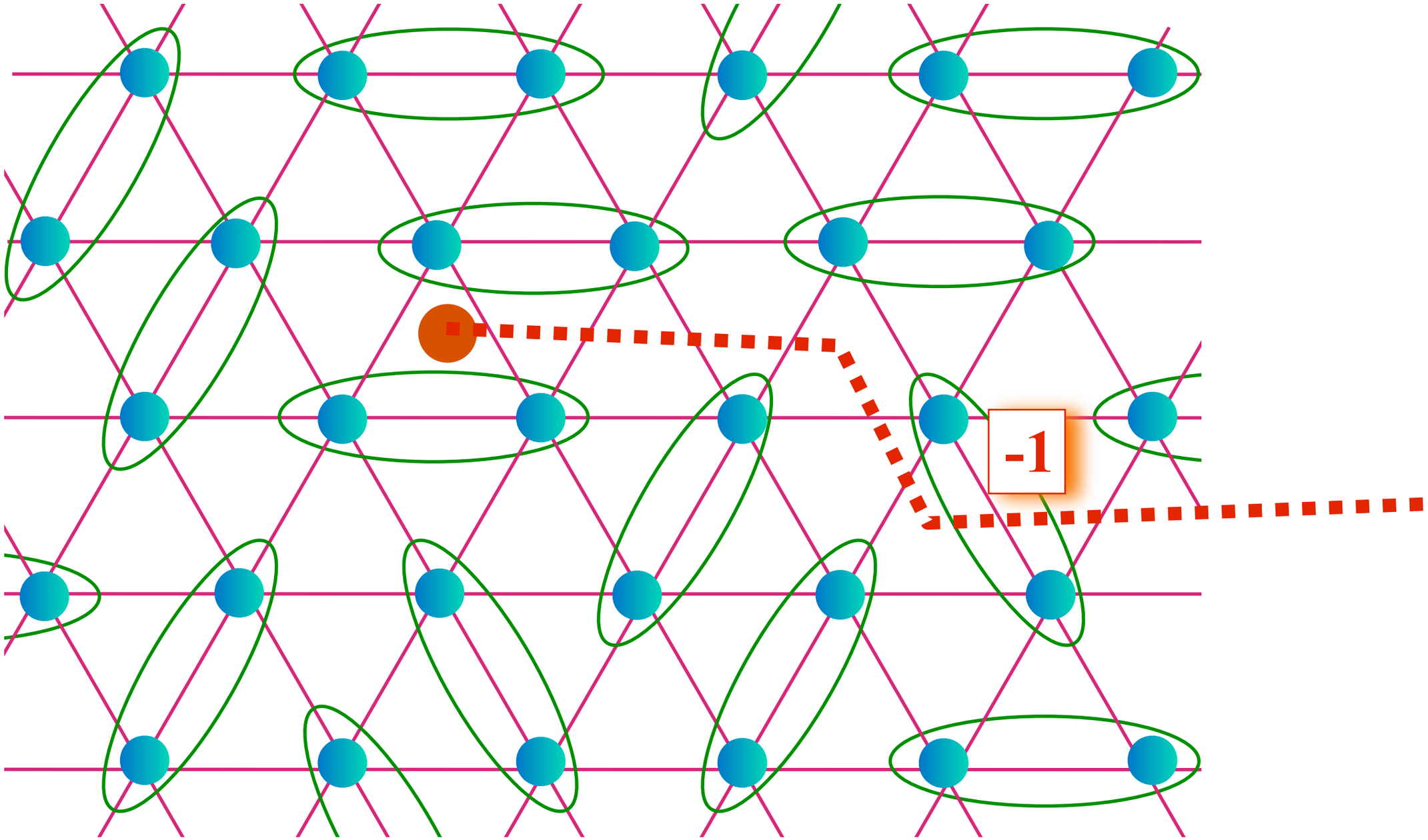

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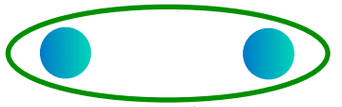

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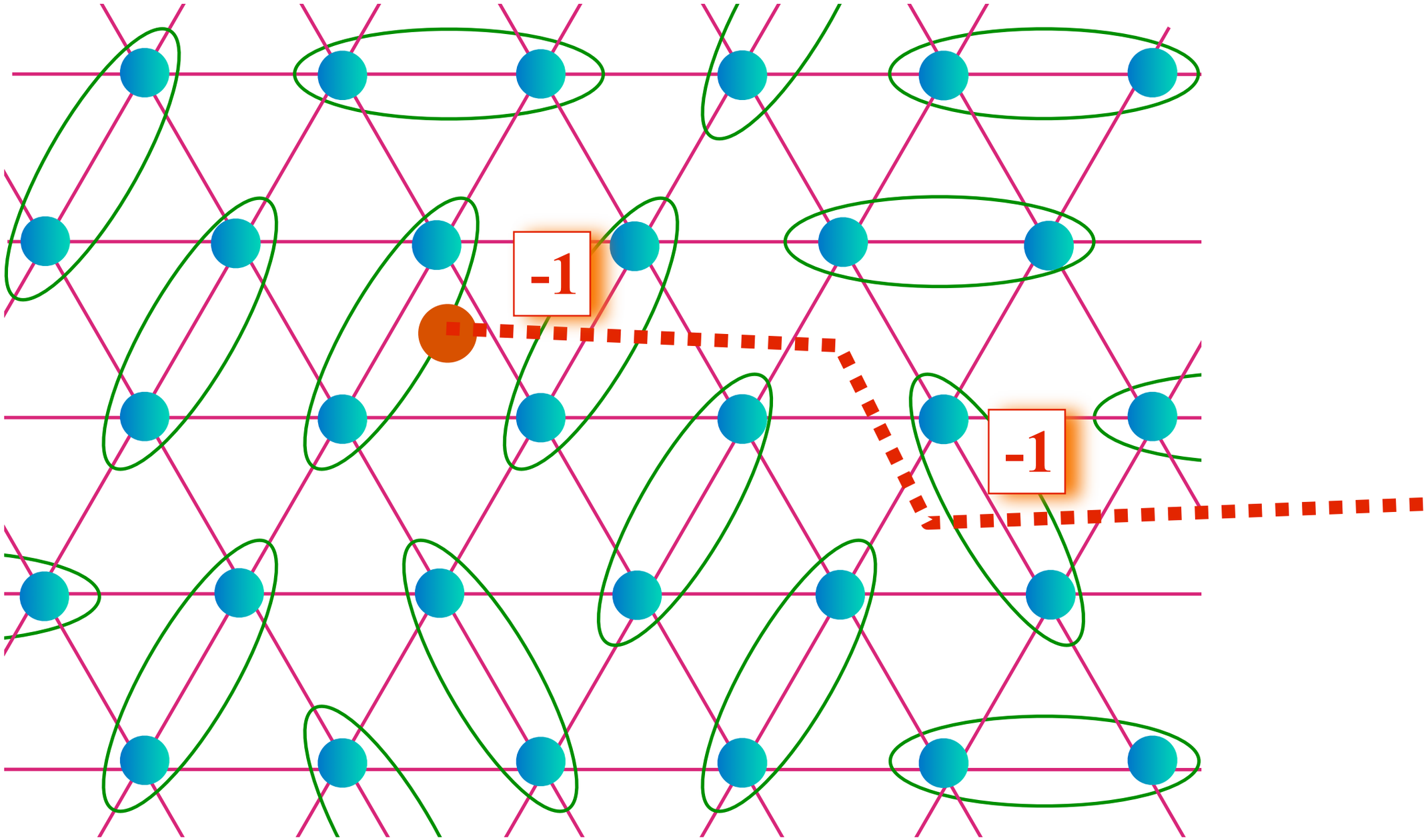

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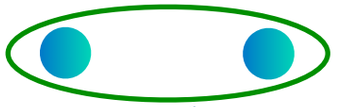

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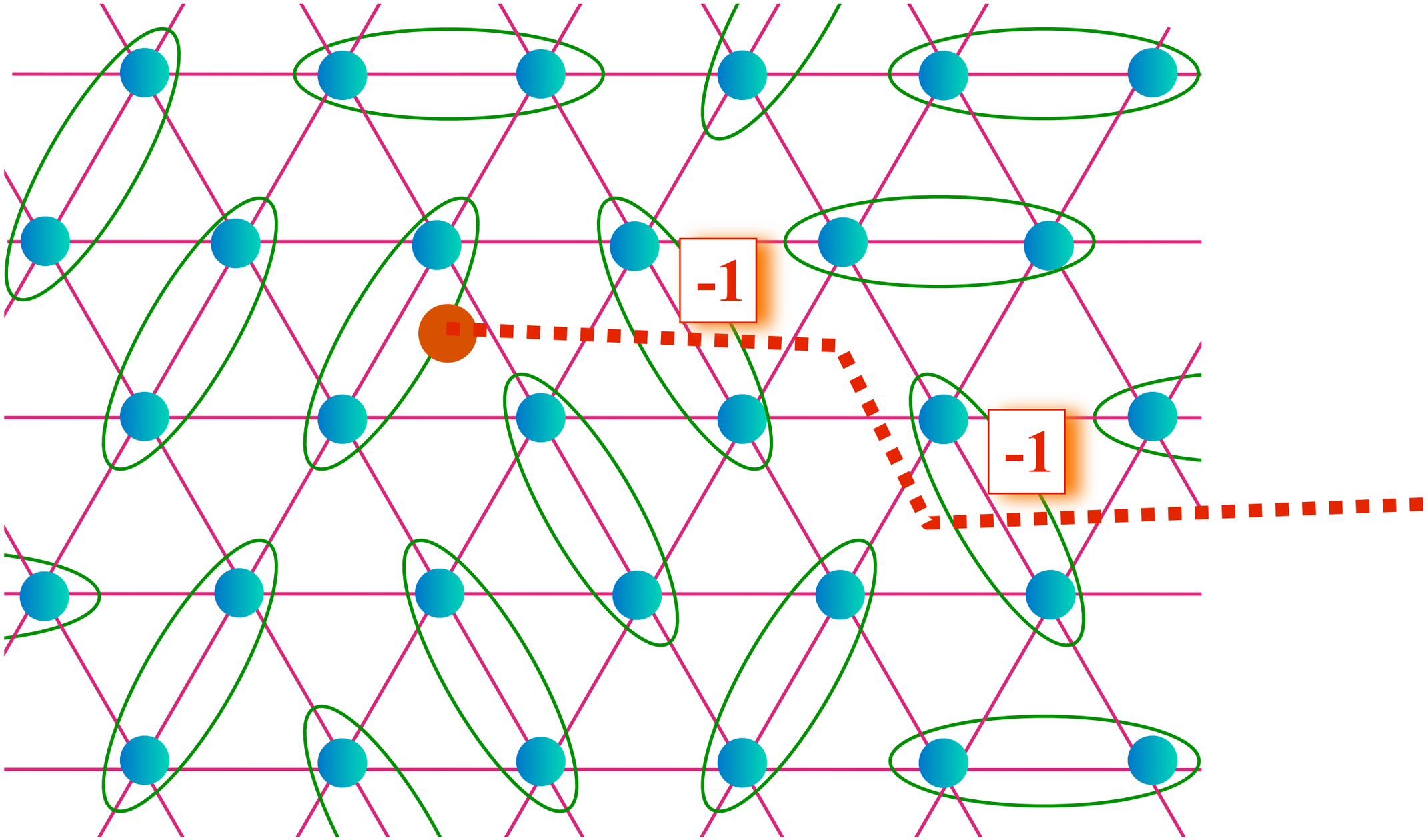

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

Mutual Chern-Simons Theory

Express theory in terms of the physical excitations of the Z_2 spin liquid: the spinons, z_α, and the visons. After accounting for Berry phase effects, the visons can be described by complex fields v_a, which transforms non-trivially under the square lattice space group operations.

A related Berry phase is the phase of -1 acquired by a spinon encircling a vortex. This is implemented in the following "mutual Chern-Simons" theory at $k = 2$:

$$\begin{aligned}
\mathcal{L} &= \sum_{\alpha=1}^{2} \left\{ |(\partial_\mu - i a_\mu) z_\alpha|^2 + s_z |z_\alpha|^2 + u_z (|z_\alpha|^2)^2 \right\} \\
&+ \sum_{a=1}^{N_v} \left\{ |(\partial_\mu - i b_\mu) v_a|^2 + s_v |v_a|^2 + u_v (|v_a|^2)^2 \right\} \\
&+ \frac{ik}{2\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu b_\lambda + \cdots
\end{aligned}$$

Cenke Xu and S. Sachdev, arXiv:0811.1220

s_z

$\langle v_a v_b \rangle \neq 0$

$\langle v_a^* v_b \rangle \neq \delta_{ab}$

Z_2 spin liquid

M

s_v

$\langle z_\alpha^* z_\beta \rangle \neq \delta_{\alpha\beta}$

$\langle v_a^* v_b \rangle \neq \delta_{ab}$

$\langle z_\alpha z_\beta \rangle \neq 0$

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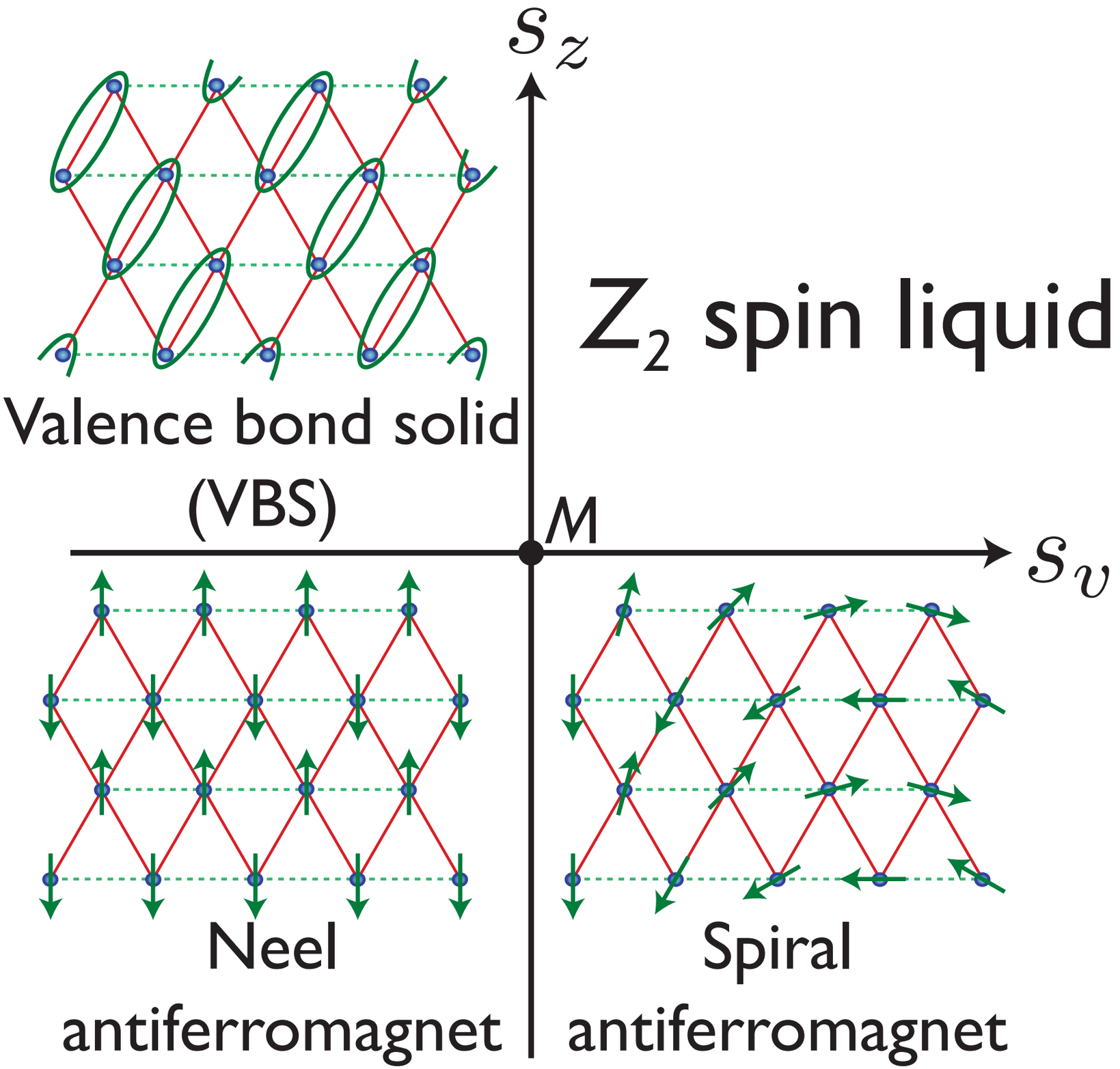
Cenke Xu and S. Sachdev, arXiv:0811.1220

Theoretical global phase diagram



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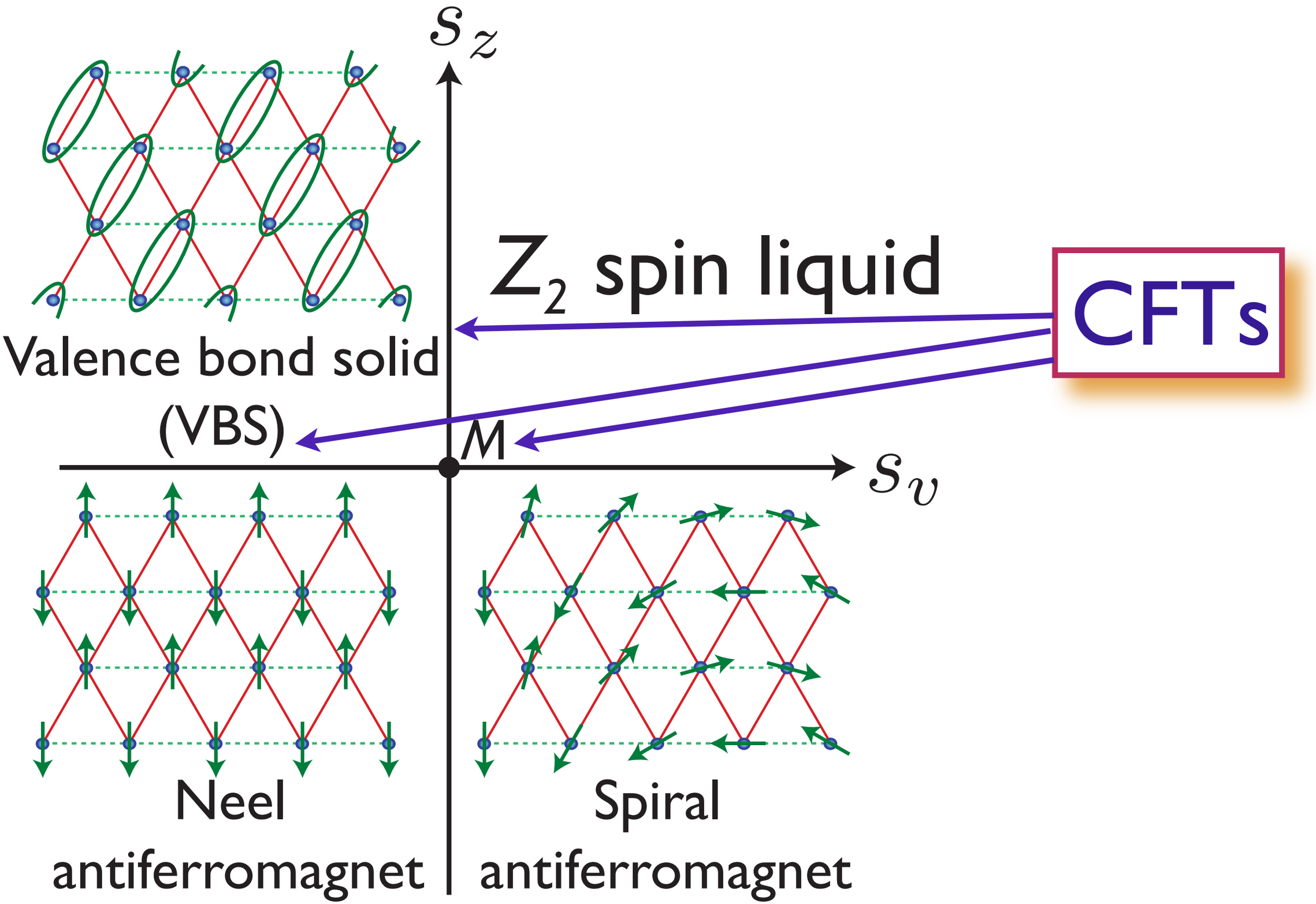

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From quantum antiferromagnets to string theory

A direct generalization of the CFT of the multicritical point M ($s_z = s_v = 0$) to $\mathcal{N} = 4$ supersymmetry and the U(N) gauge group was shown by O. Aharony, O. Bergman, D. L. Jafferis, J. Maldacena, JHEP **0810**, 091 (2008) to be dual to a theory of quantum gravity (M theory) on $\mathrm{AdS}_4 \times \mathrm{S}_7/Z_k$.

$$
\begin{aligned}
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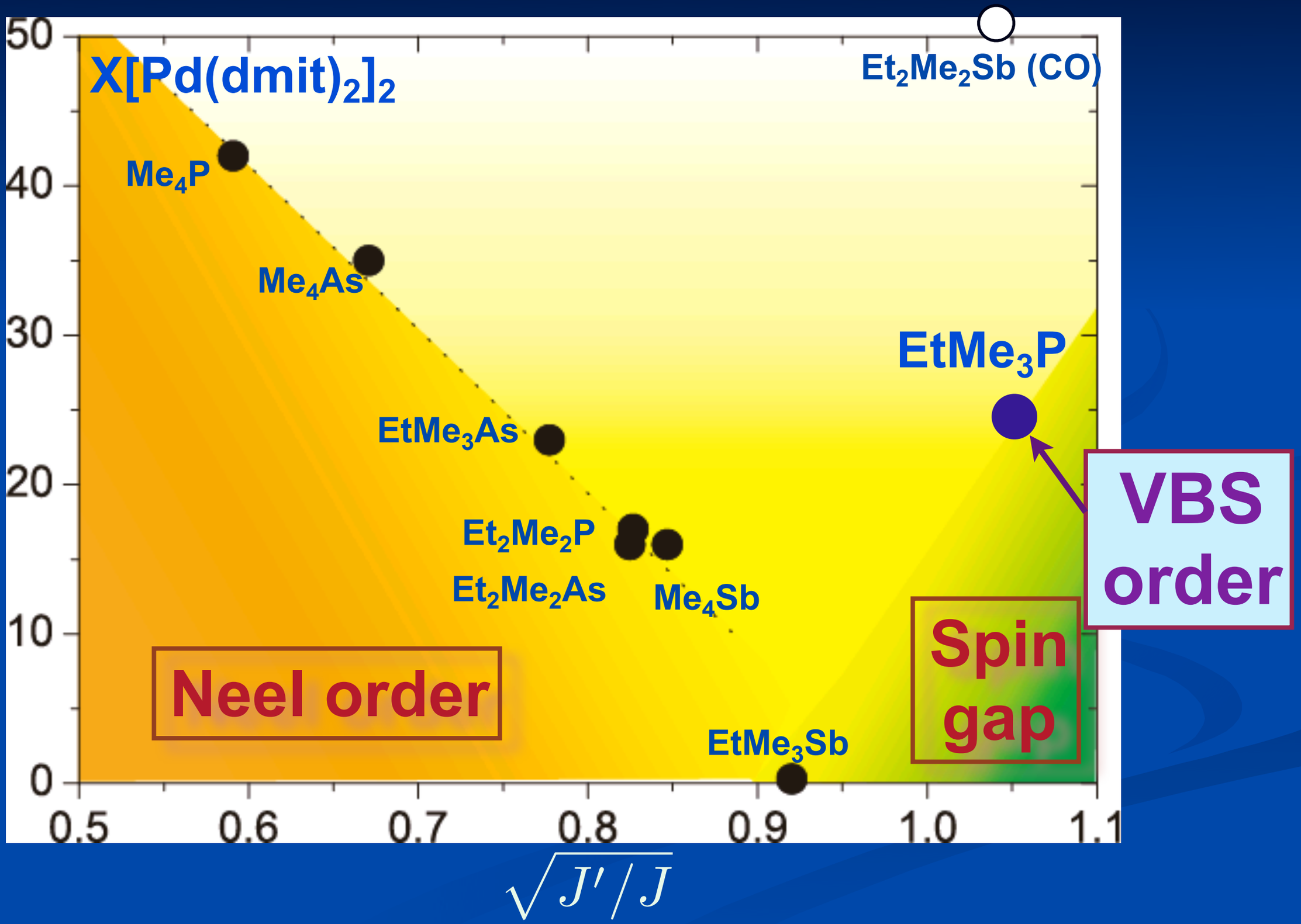
$$

Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

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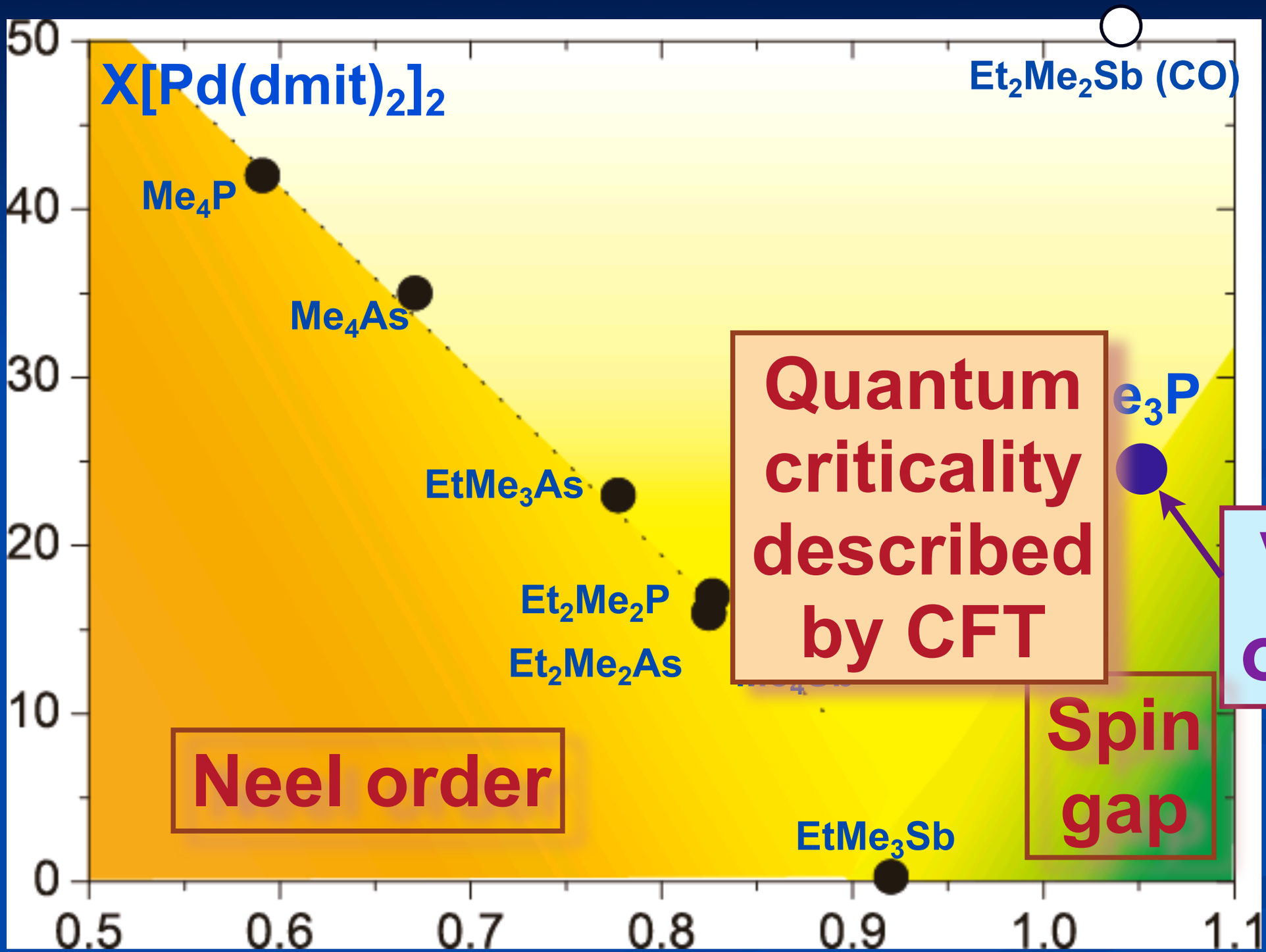

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Outline

A. "Relativistic" field theories of quantum phase transitions

1. Coupled dimer antiferromagnets

2. Triangular lattice antiferromagnets

3. Graphene

4. AdS/CFT and quantum critical transport

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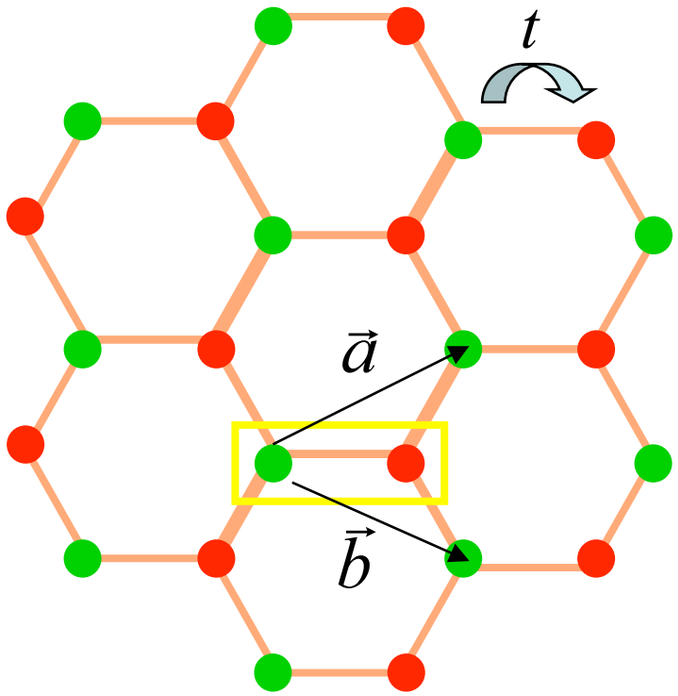

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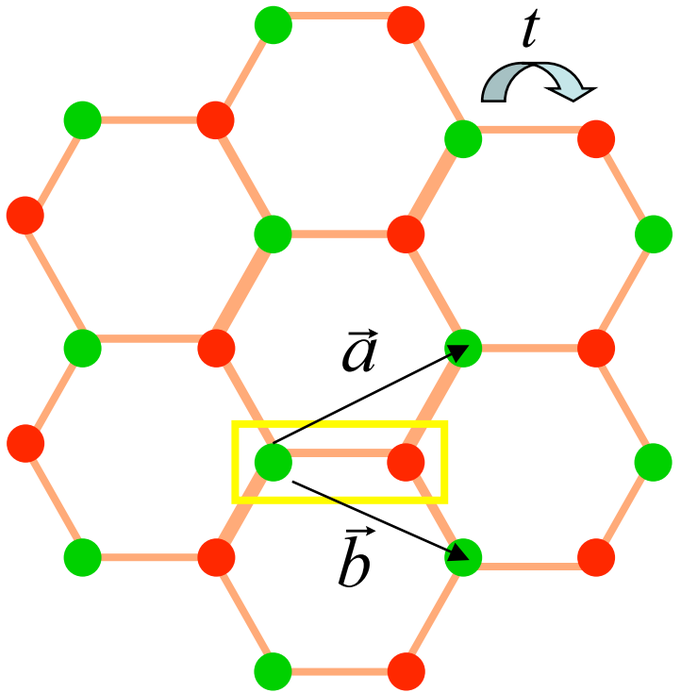
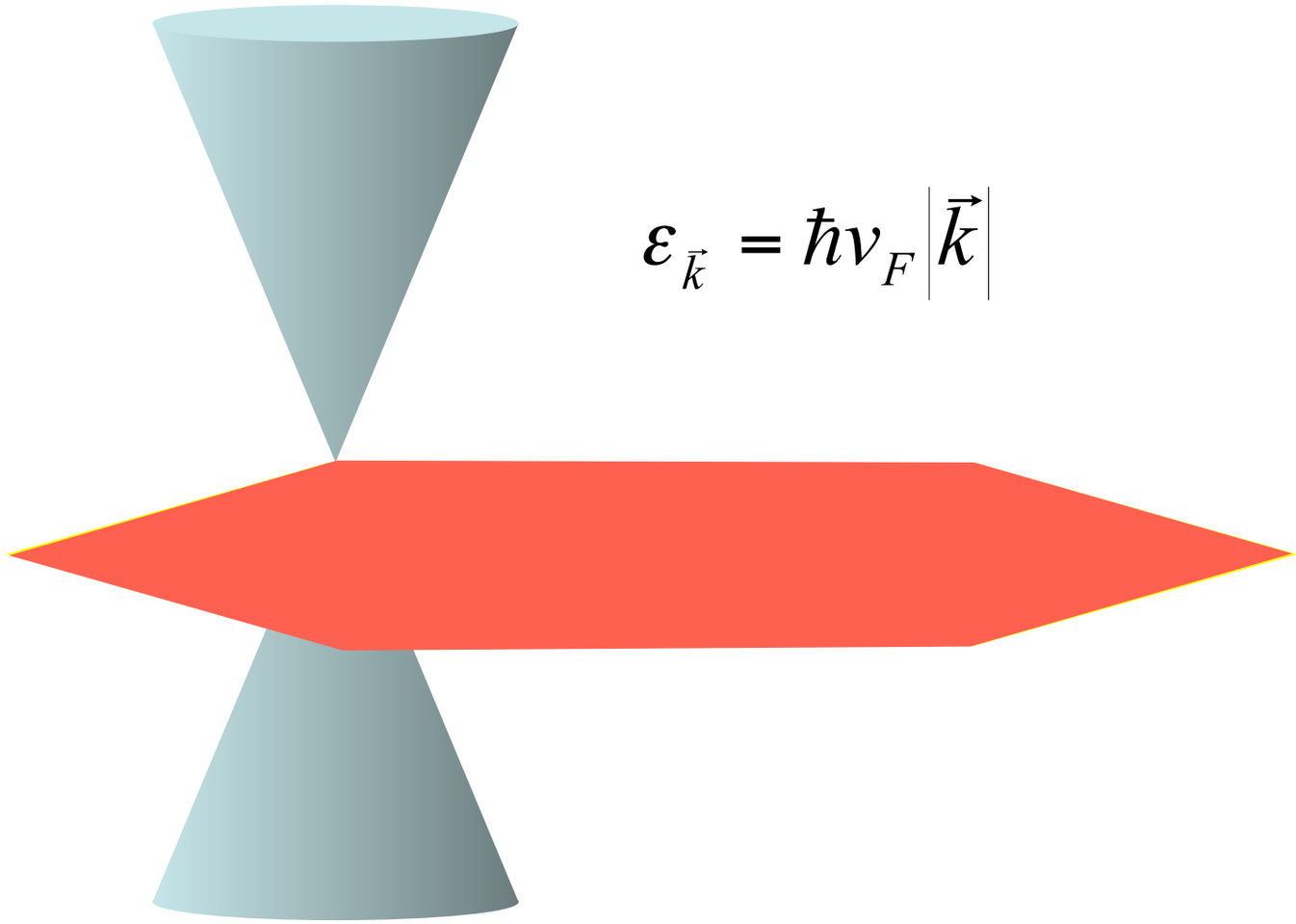
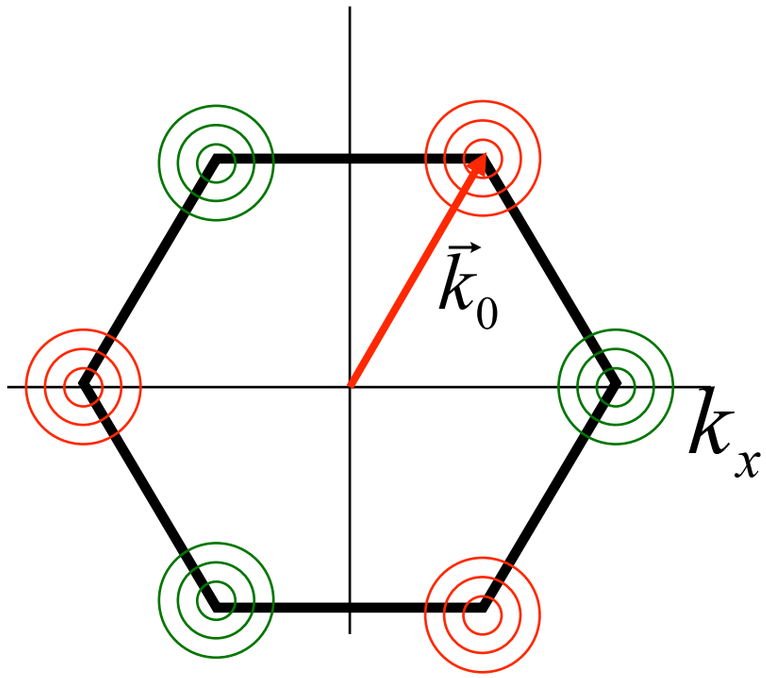
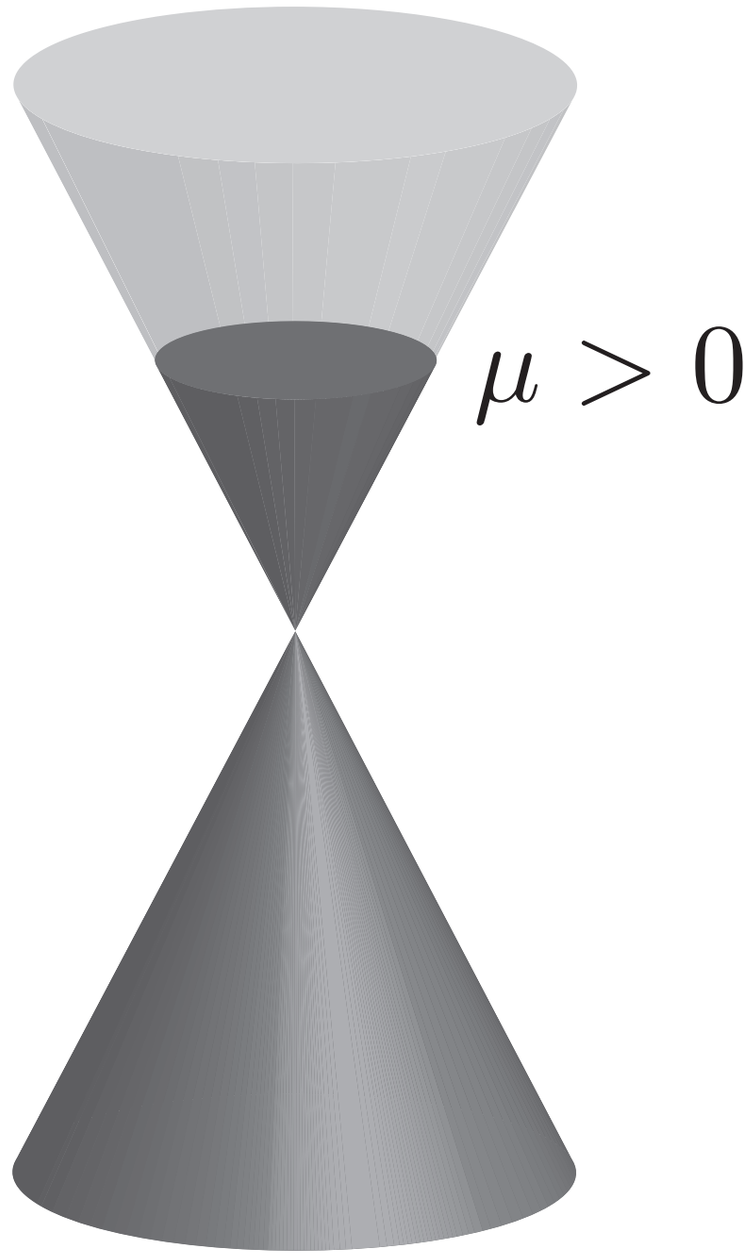






Conical Dirac dispersion

Quantum phase transition in graphene tuned by a bias voltage



Electron
Fermi surface

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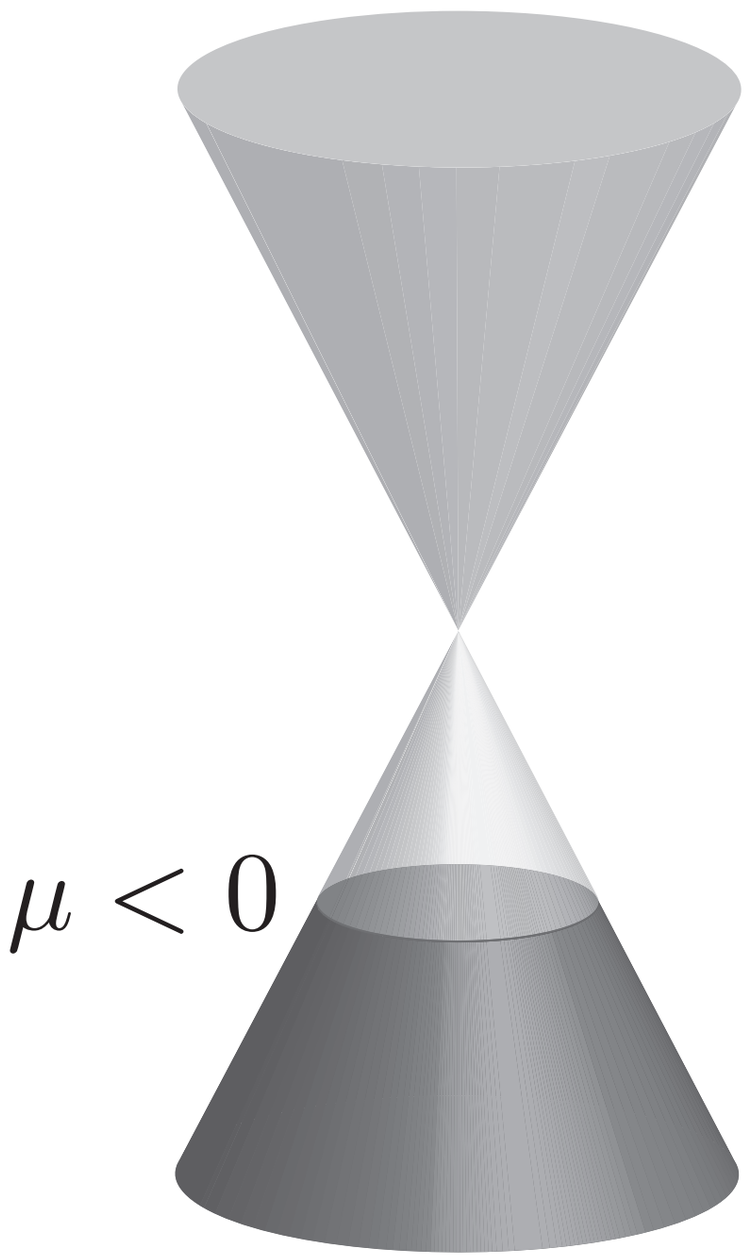
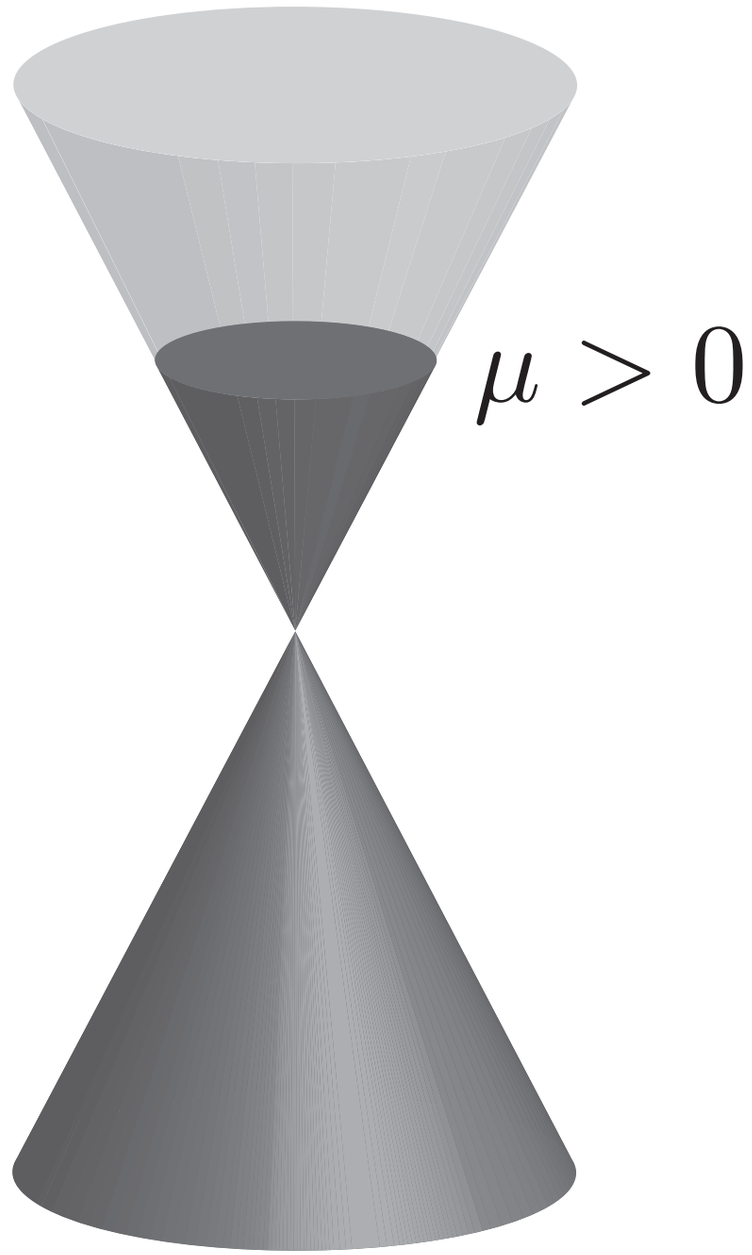


Hole
Fermi surface



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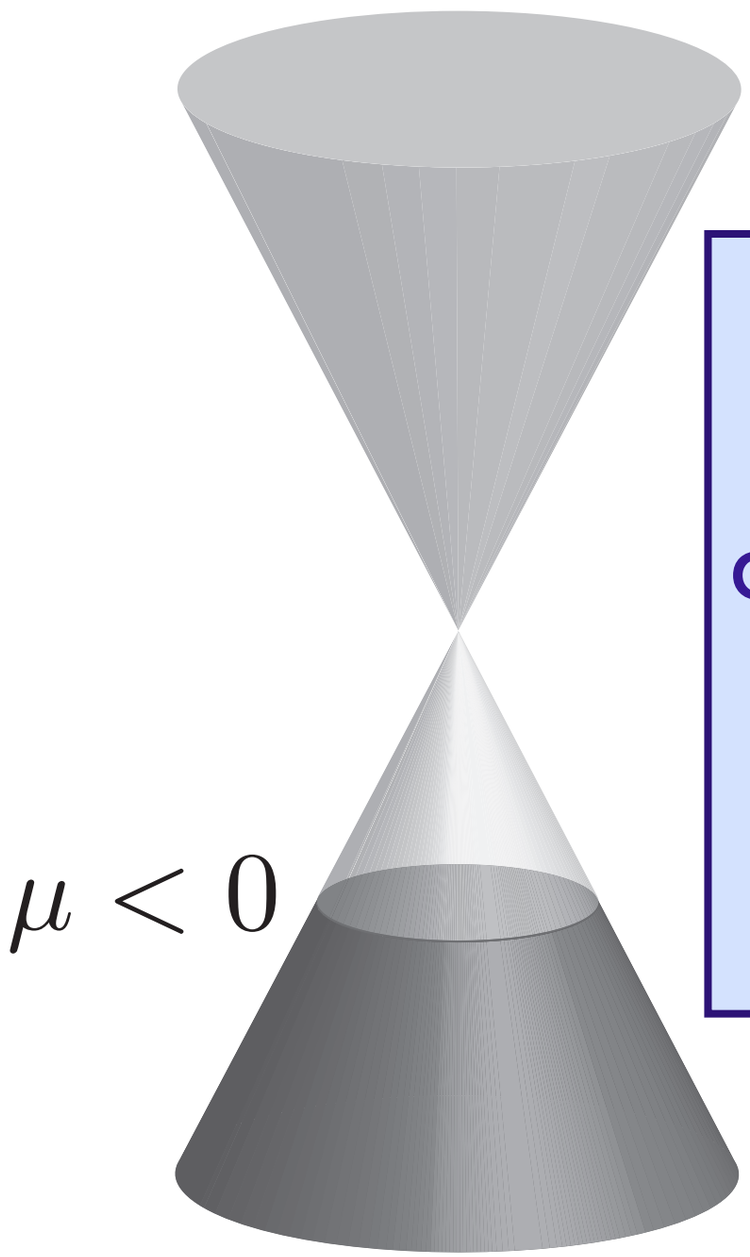
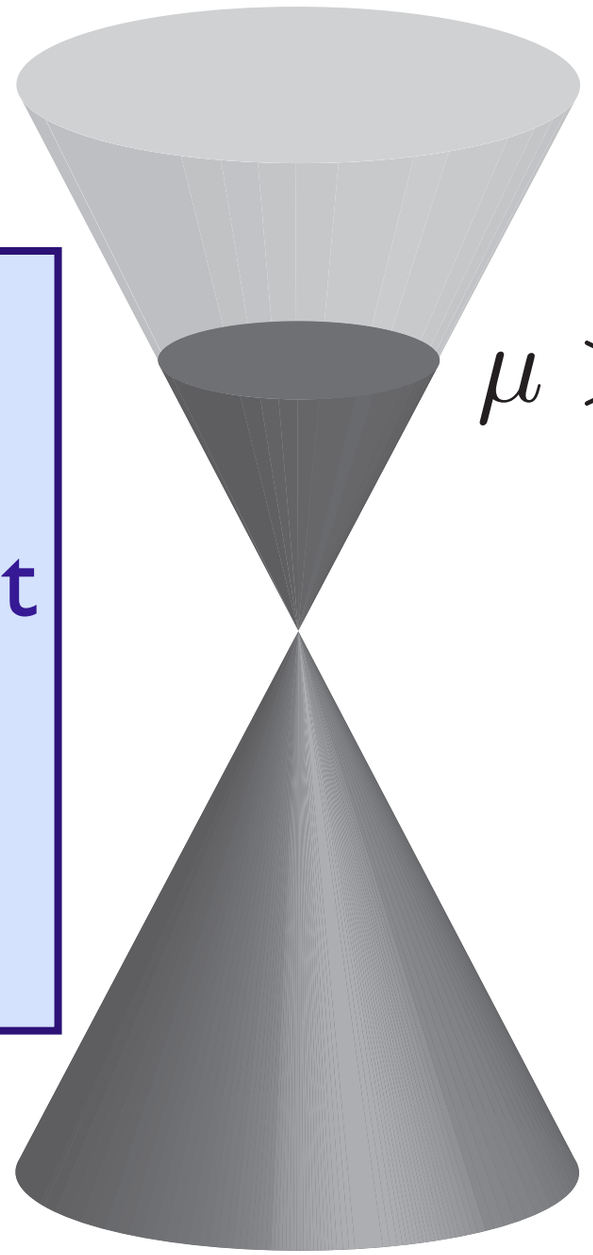


$\mu < 0$

Hole
Fermi surface

There must be an intermediate quantum critical point where the Fermi surfaces reduce to a Dirac point



$\mu > 0$

Electron
Fermi surface

Quantum critical graphene

Low energy theory has 4 two-component Dirac fermions, ψ_σ, $\sigma = 1 \ldots 4$, interacting with a $1/r$ Coulomb interaction

$$\begin{aligned} \mathcal{S} &= \int d^2 r d\tau \psi_\sigma^\dagger \left(\partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\sigma \\ &\quad + \frac{e^2}{2} \int d^2 r d^2 r' d\tau \psi_\sigma^\dagger \psi_\sigma(r) \frac{1}{|r - r'|} \psi_{\sigma'}^\dagger \psi_{\sigma'}(r') \end{aligned}$$

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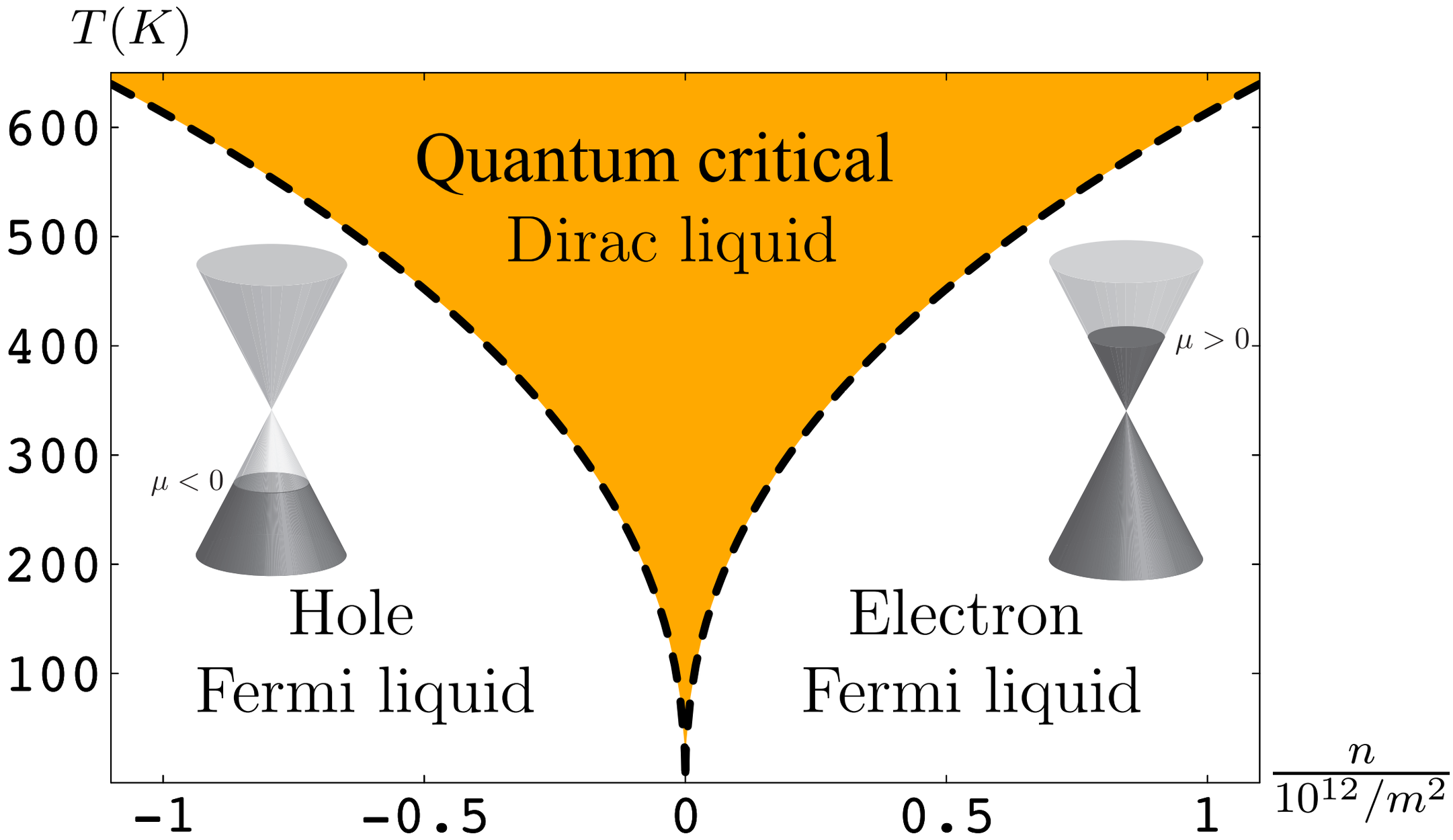
Dimensionless "fine-structure" constant $\alpha = e^2/(\hbar v_F)$.
RG flow of α:

$$\frac{d\alpha}{d\ell} = -\alpha^2 + \ldots$$

Behavior is similar to a conformal field theory (CFT) in 2+1 dimensions with $\alpha \sim 1/\ln(\text{scale})$

Quantum phase transition in graphene



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Quantum critical transport

Quantum *"perfect fluid"* with shortest possible relaxation time, τ_R

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

$$\sigma = \frac{e^2}{h} \times [\text{Universal constant } \mathcal{O}(1) \ ]$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Density correlations in CFTs at $T>0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k\to 0} \frac{-i\omega}{k^2}\chi(k,\omega)$

For *all* CFT2s, at all $\hbar\omega/k_BT$

$$\chi(k,\omega) \quad = \quad \frac{4e^2}{h} K \frac{vk^2}{v^2k^2-\omega^2} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h}\frac{Kv}{-i\omega}$$

where K is a universal number characterizing the CFT2 (the level number), and v is the velocity of "light".

This follows from the conformal mapping of the plane to the cylinder, which relates correlators at $T=0$ to those at $T>0$.

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No hydrodynamics in CFT2s.

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For *all* CFT3s, at $\hbar\omega \gg k_B T$

$$\chi(k,\omega) \quad = \quad \frac{4e^2}{h} K \frac{k^2}{\sqrt{v^2k^2-\omega^2}} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h} K$$

where K is a universal number characterizing the CFT3, and v is the velocity of "light".

Density correlations in CFTs at $T>0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k\to 0} \dfrac{-i\omega}{k^2}\chi(k,\omega)$

However, for *all* CFT3s, at $\hbar\omega \ll k_B T$, we have the Einstein relation

$$\chi(k,\omega) \;\; = \;\; 4e^2\chi_c \frac{Dk^2}{Dk^2 - i\omega} \;\; ; \;\; \sigma(\omega) = 4e^2 D\chi_c = \frac{4e^2}{h}\Theta_1\Theta_2$$

where the **compressibility**, χ_c, and the **diffusion constant** D obey

$$\chi = \frac{k_B T}{(hv)^2}\Theta_1 \;\; ; \;\; D = \frac{hv^2}{k_B T}\Theta_2$$

with Θ_1 and Θ_2 universal numbers characteristic of the CFT3

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Density correlations in CFTs at $T>0$

In CFT3s collisions are "phase" randomizing, and lead to relaxation to local thermodynamic equilibrium. So there is a crossover from collisionless behavior for $\hbar\omega \gg k_B T$, to hydrodynamic behavior for $\hbar\omega \ll k_B T$.

$$\sigma(\omega) = \begin{cases} \dfrac{4e^2}{h} K & , \quad \hbar\omega \gg k_B T \\ \\ \dfrac{4e^2}{h} \Theta_1 \Theta_2 \equiv \sigma_Q & , \quad \hbar\omega \ll k_B T \end{cases}$$

and in general we expect $K \neq \Theta_1 \Theta_2$ (verified for Wilson-Fisher fixed point).

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

SU(N) SYM3 with $\mathcal{N} = 8$ supersymmetry

- Has a single dimensionful coupling constant, e_0, which flows to a strong-coupling fixed point $e_0 = e_0^*$ in the infrared.

- The CFT3 describing this fixed point resembles "critical spin liquid" theories.

- This CFT3 is the low energy limit of string theory on an M2 brane. The AdS/CFT correspondence provides a dual description using 11-dimensional supergravity on $\mathrm{AdS}_4 \times S_7$.

- The CFT3 has a global SO(8) R symmetry, and correlators of the SO(8) charge density can be computed exactly in the large N limit, even at $T > 0$.

SU(N) SYM3 with $\mathcal{N} = 8$ supersymmetry

- The SO(8) charge correlators of the CFT3 are given by the usual AdS/CFT prescription applied to the following gauge theory on AdS4:

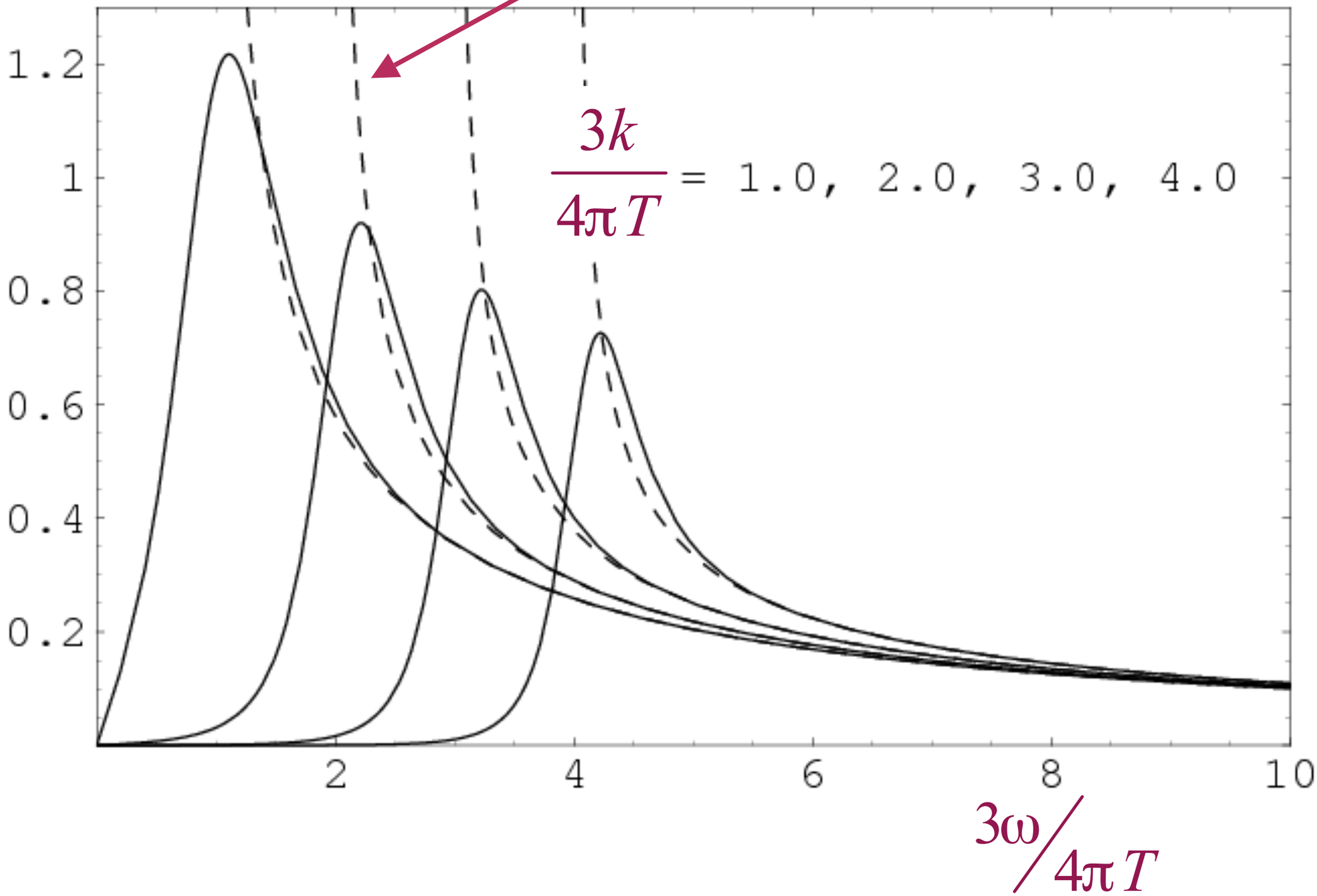
$$\mathcal{S} = -\frac{1}{4g_{4D}^2}\int d^4x\sqrt{-g}g^{MA}g^{NB}F^a_{MN}F^a_{AB}$$

where $a = 1\ldots 28$ labels the generators of SO(8). Note that in large N theory, this looks like 28 copies of an Abelian gauge theory.

Collisionless to hydrodynamic crossover of SYM3

$\mathrm{Im}\chi(k,\omega)/k^2$

$$\mathrm{Im}\,\frac{K}{\sqrt{k^2-\omega^2}}$$



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

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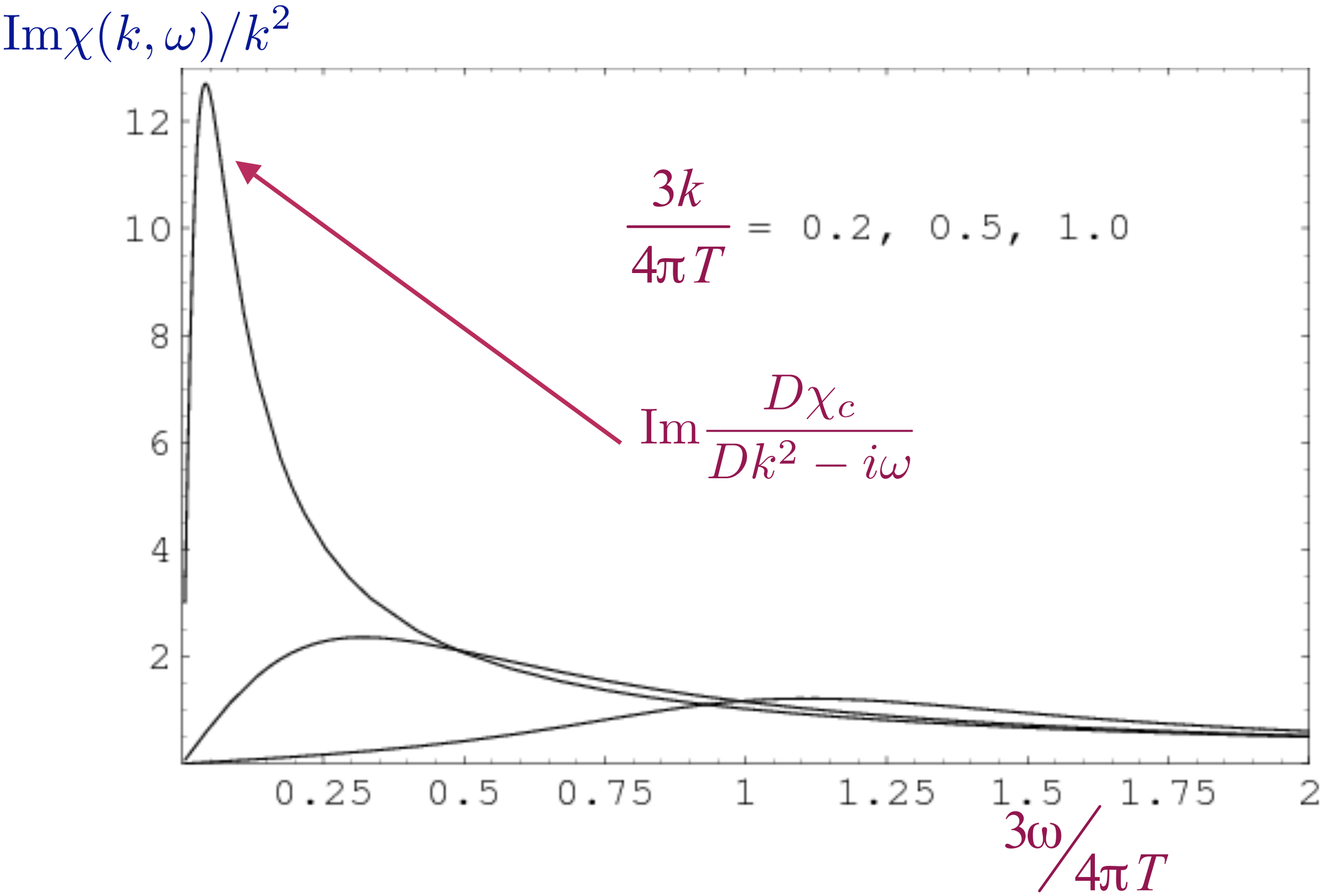

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Universal constants of SYM3

$$\chi_c = \frac{k_B T}{(hv)^2}\Theta_1 \qquad D = \frac{hv^2}{k_B T}\Theta_2$$

$$\sigma(\omega) = \begin{cases} \dfrac{4e^2}{h}K & , \quad \hbar\omega \gg k_B T \\ \dfrac{4e^2}{h}\Theta_1\Theta_2 & , \quad \hbar\omega \ll k_B T \end{cases}$$

$$K = \frac{\sqrt{2}N^{3/2}}{3}$$

$$\Theta_1 = \frac{8\pi^2\sqrt{2}N^{3/2}}{9}$$

$$\Theta_2 = \frac{3}{8\pi^2}$$

C. Herzog, JHEP **0212**, 026 (2002)

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Electromagnetic self-duality

- Unexpected result, $K = \Theta_1 \Theta_2$. Actually, a stronger result holds: $\sigma(\omega)$ is independent of ω for all $\hbar\omega/(k_B T)$.

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- **Special properties of CFT3s with gravity duals**: $\sigma(\omega)$ is ω-independent and equal to the self-dual value. These results are the analog of $\eta/s = \hbar/(4k_B\pi)$.

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- **Special properties of CFT3s with gravity duals**: $\sigma(\omega)$ is ω-independent and equal to the self-dual value. These results are the analog of $\eta/s = \hbar/(4k_B\pi)$.
- **Curious fact**: Experimental studies show a quantum critical σ close to the self-dual value.

Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$

$$\sigma_{\text{Insulator}}(T \to 0) = 0$$

$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

D. B. Haviland, Y. Liu, and A. M. Goldman,
Phys. Rev. Lett. **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

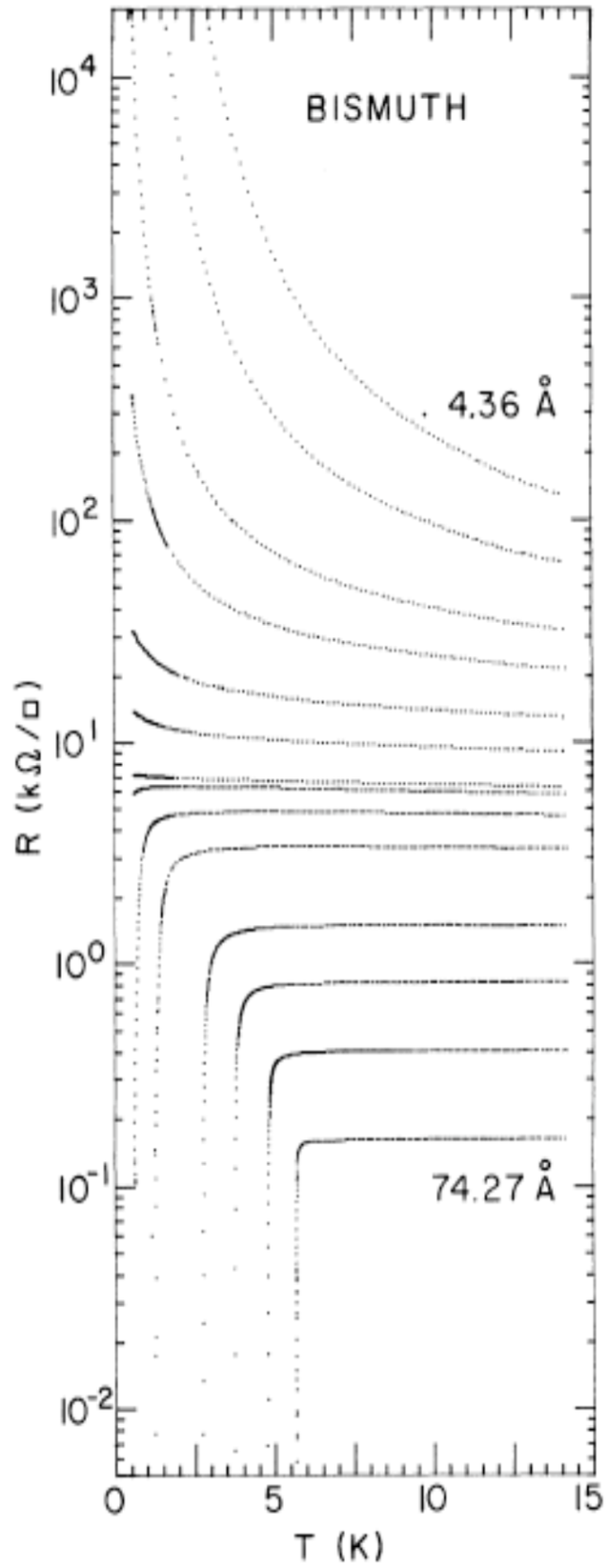

FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Quantum critical transport in graphene

$$\sigma(\omega) = \begin{cases} \dfrac{e^2}{h}\left[\dfrac{\pi}{2} + \mathcal{O}\left(\dfrac{1}{\ln(\Lambda/\omega)}\right)\right] & , \quad \hbar\omega \gg k_B T \\ \dfrac{e^2}{h\alpha^2(T)}\left[0.760 + \mathcal{O}\left(\dfrac{1}{|\ln(\alpha(T))|}\right)\right] & , \quad \hbar\omega \ll k_B T \alpha^2(T) \end{cases}$$

$$\frac{\eta}{s} = \frac{\hbar}{k_B \alpha^2(T)} \times 0.130$$

where the "fine structure constant" is

$$\alpha(T) = \frac{\alpha}{1 + (\alpha/4)\ln(\Lambda/T)} \overset{T\to 0}{\sim} \frac{4}{\ln(\Lambda/T)}$$

L. Fritz, J. Schmalian, M. Müller and S. Sachdev, *Physical Review B* **78**, 085416 (2008)
M. Müller, J. Schmalian, and L. Fritz, *Physical Review Letters* **103**, 025301 (2009)