Angular fluctuations of a multi-component order describe the pseudogap regime of the cuprate superconductors

Quantum Dynamics in Low Dimensional Systems
Harvard, September 21, 2013

Subir Sachdev

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Talk online: sachdev.physics.harvard.edu

Lauren Hayward

Roger Melko

David Hawthorn

University of Waterloo

Jay Deep Sau

Max Metlitski

Rolando La Placa

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Outline

1. Review of recent experiments

2. Theory of pseudogap

3. Microscopic basis

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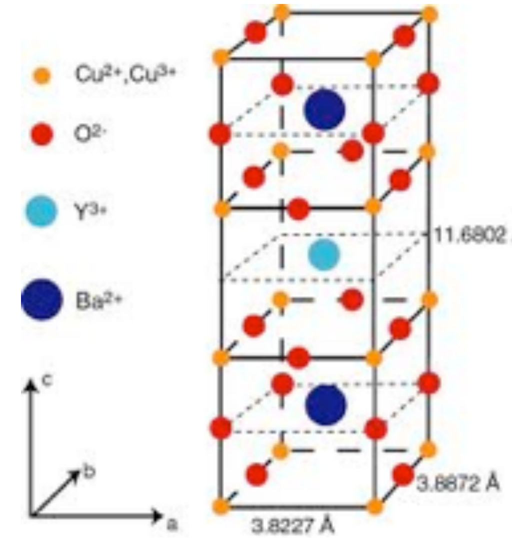

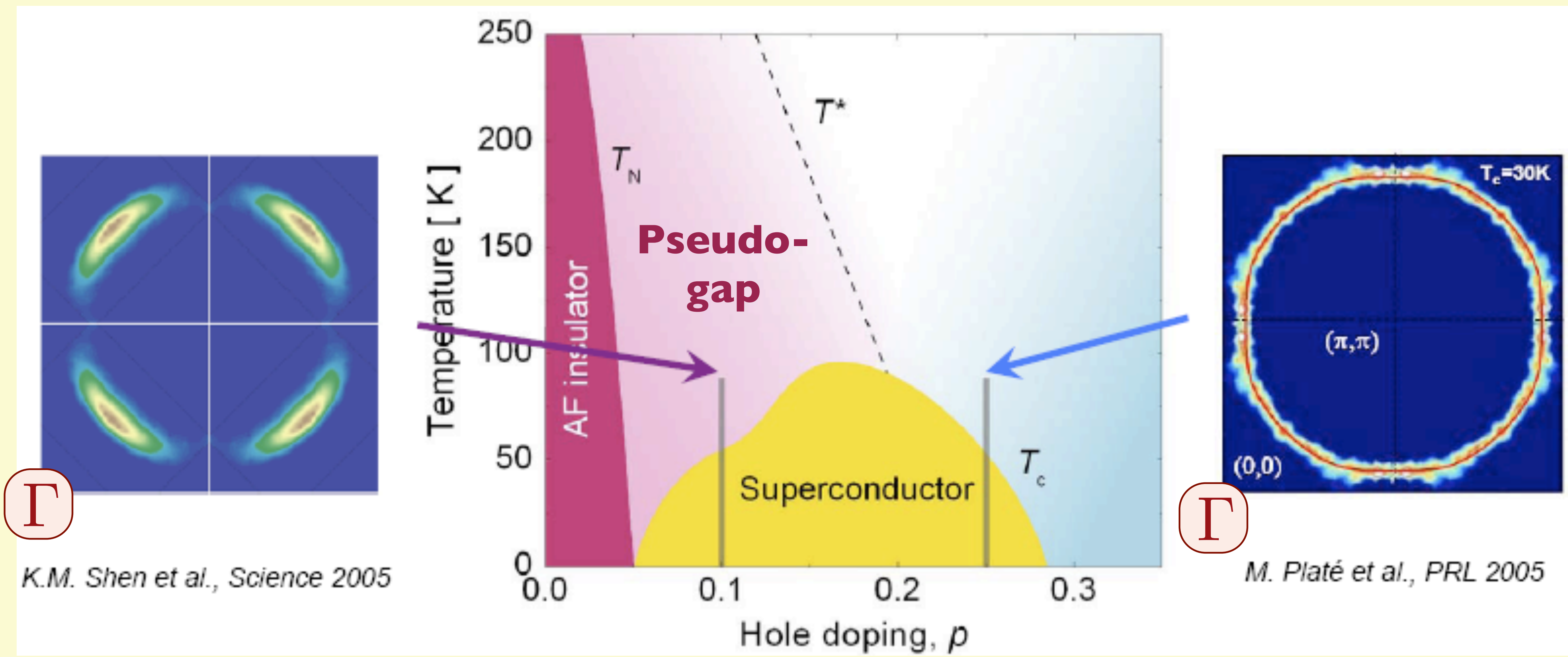

Fermi arcs ?

Large hole Fermi surface

Quantum oscillations and the Fermi surface in an underdoped high-T_c superconductor

Nicolas Doiron-Leyraud[1], Cyril Proust[2], David LeBoeuf[1], Julien Levallois[2], Jean-Baptiste Bonnemaison[1], Ruixing Liang[3,4], D. A. Bonn[3,4], W. N. Hardy[3,4] & Louis Taillefer[1,4]

Nature **447**, 565 (2007)

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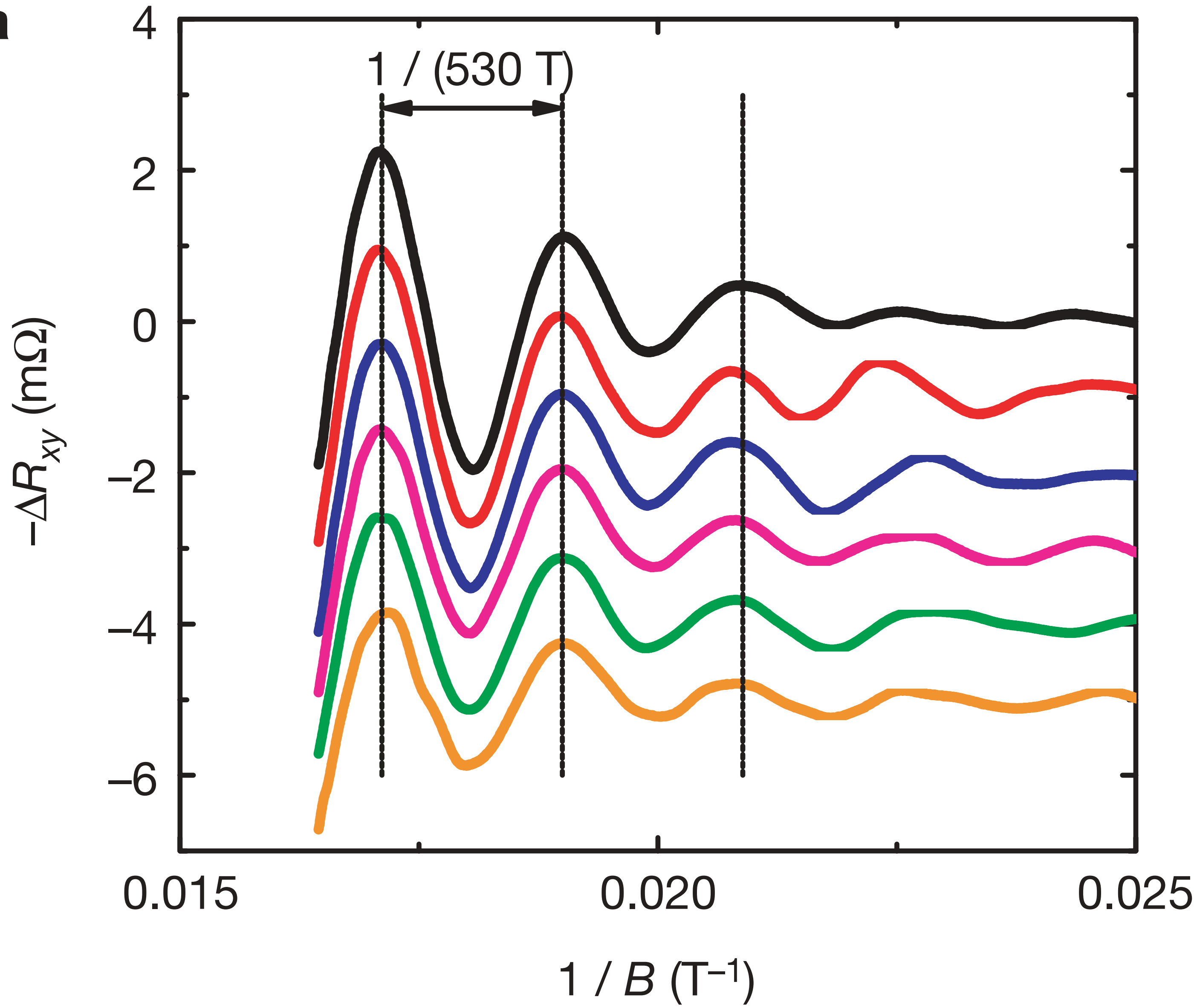
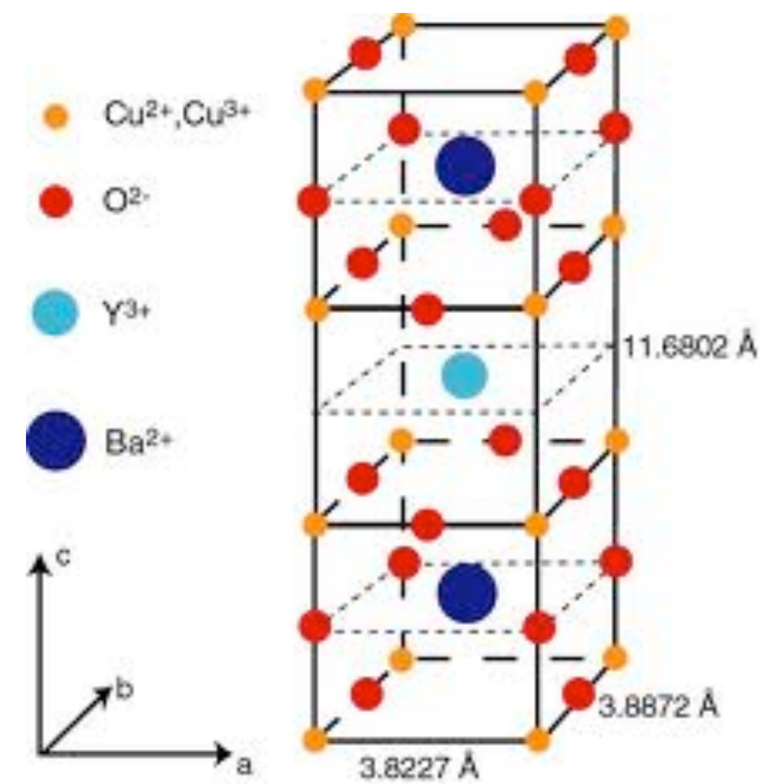
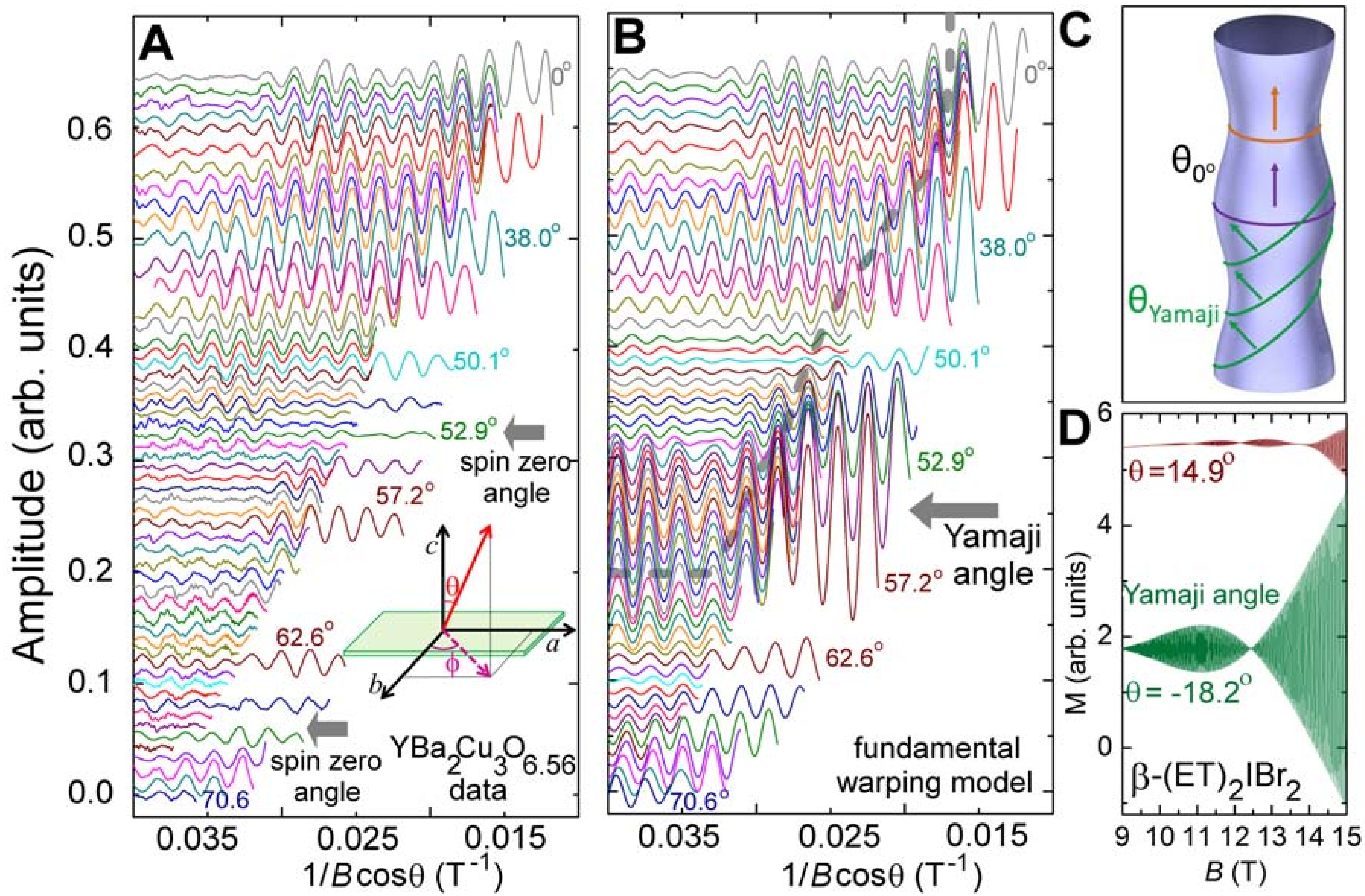






Twofold twisted Fermi surface from staggered order in an underdoped high T_c superconductor

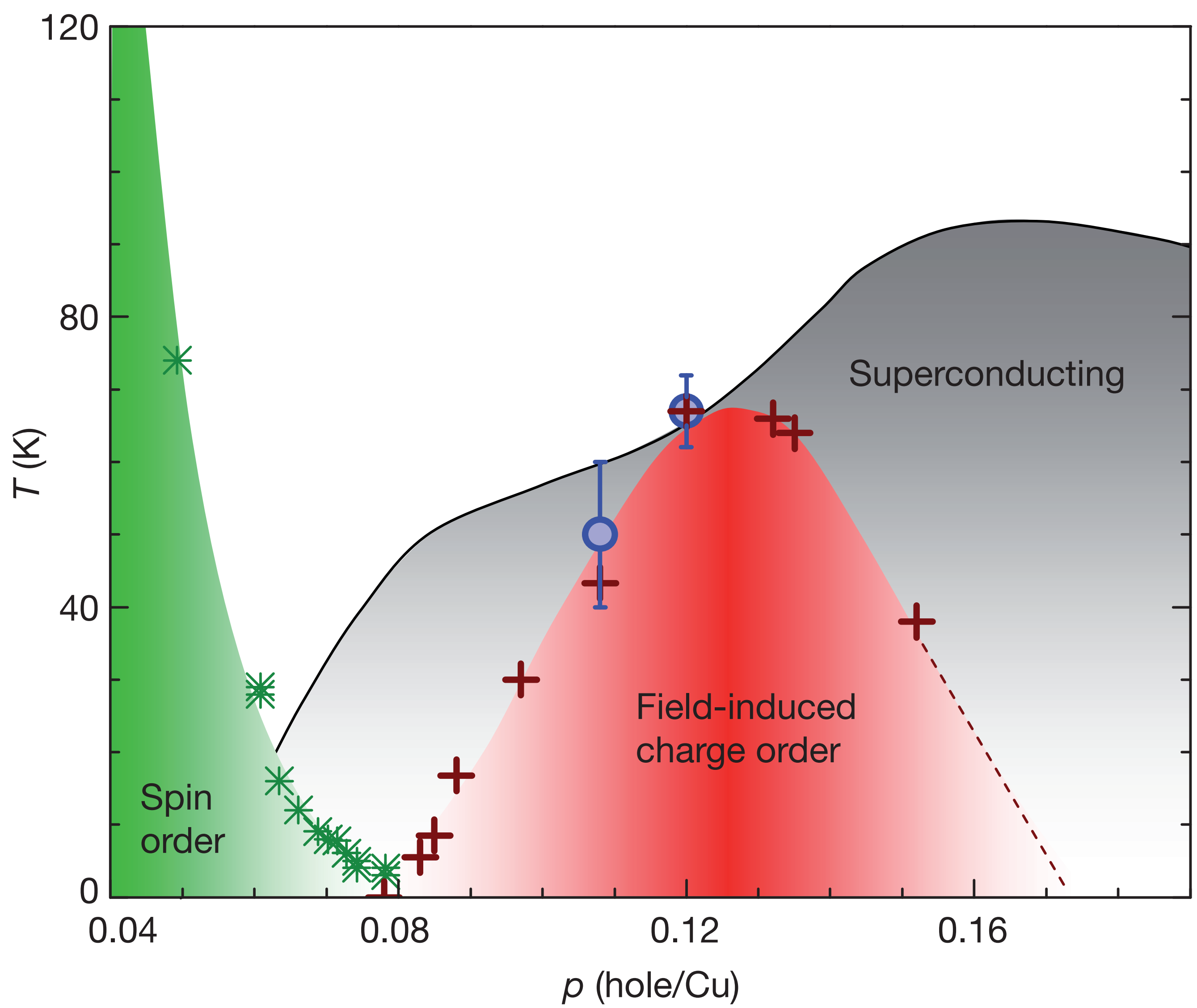
Suchitra E. Sebastian,[1*] N. Harrison,[2] F. F. Balakirev,[2] M. M. Altarawneh,[2,3]
Ruixing Liang,[4,5] D. A. Bonn,[4,5] W. N. Hardy,[4,5] G. G. Lonzarich,[1]

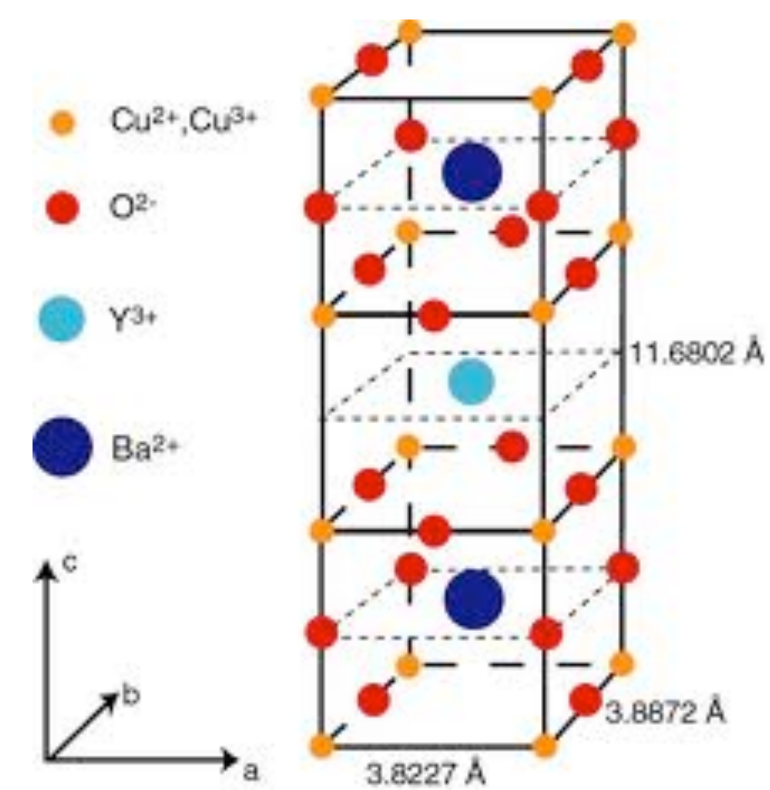
APS March meeting 2013 B2.00004

Magnetic-field-induced charge-stripe order in the high-temperature superconductor $YBa_2Cu_3O_y$

Tao Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Mladen Horvatić[1], Claude Berthier[1], W. N. Hardy[2,3], Ruixing Liang[2,3], D. A. Bonn[2,3] & Marc-Henri Julien[1]

8 SEPTEMBER 2011 | VOL 477 | NATURE | 191





Distinct Charge Orders in the Planes and Chains of Ortho-III-Ordered $YBa_2Cu_3O_{6+\delta}$ Superconductors Identified by Resonant Elastic X-ray Scattering

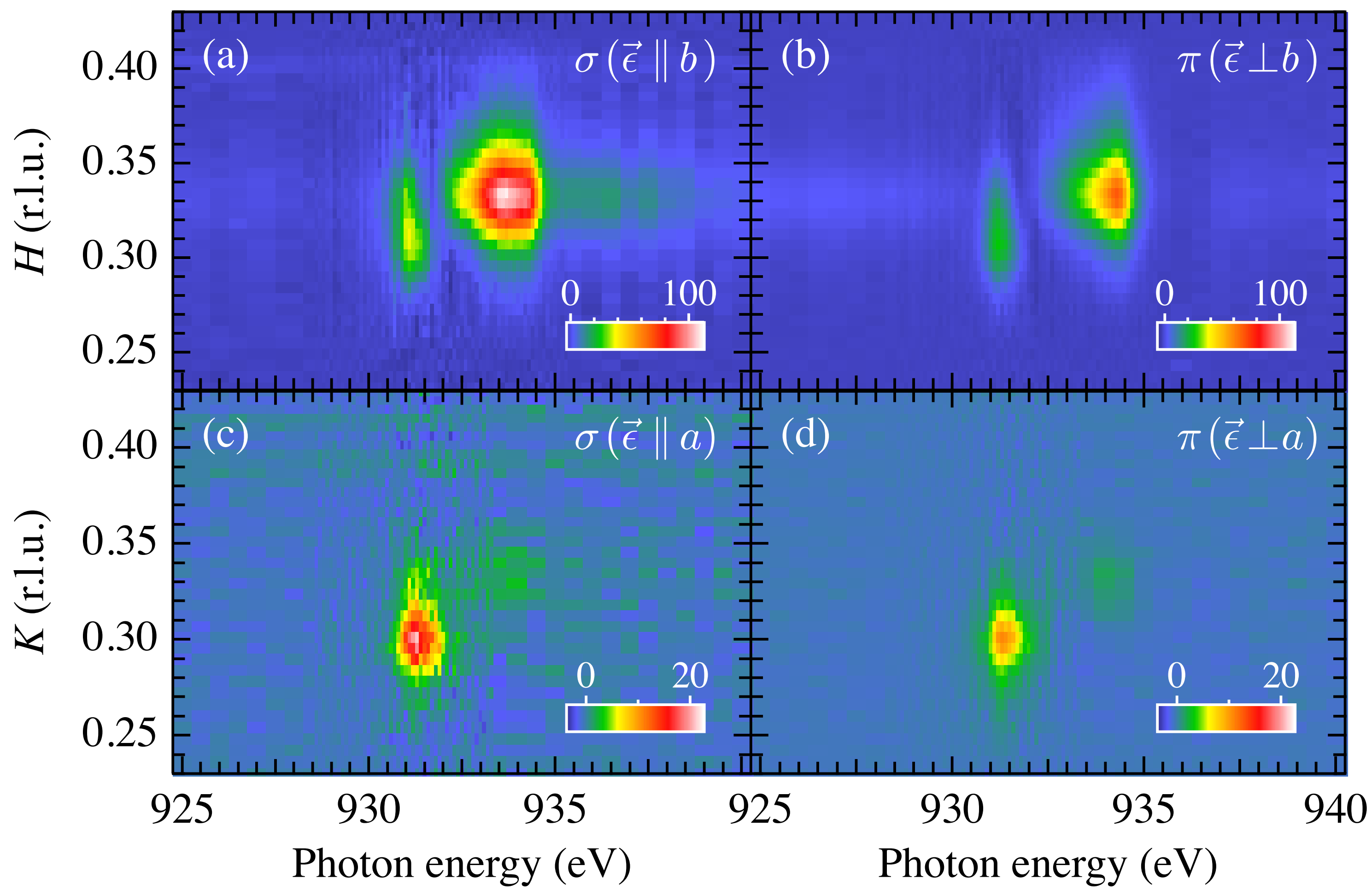
A. J. Achkar,[1] R. Sutarto,[2,3] X. Mao,[1] F. He,[3] A. Frano,[4,5] S. Blanco-Canosa,[4] M. Le Tacon,[4] G. Ghiringhelli,[6] L. Braicovich,[6] M. Minola,[6] M. Moretti Sala,[7] C. Mazzoli,[6] Ruixing Liang,[2] D. A. Bonn,[2] W. N. Hardy,[2] B. Keimer,[4] G. A. Sawatzky,[2] and D. G. Hawthorn[1,*]

PRL **109,** 167001 (2012)

Resonant X-Ray Scattering Measurements of a Spatial Modulation of the Cu 3*d* and O 2*p* Energies in Stripe-Ordered Cuprate Superconductors

A. J. Achkar,[1] F. He,[2] R. Sutarto,[3] J. Geck,[4] H. Zhang,[5] Y.-J. Kim,[5] and D. G. Hawthorn[1]

PRL **110,** 017001 (2013)



may point to a valence-bond-solid interpretation of the stripe phase.

Long-Range Incommensurate Charge Fluctuations in (Y,Nd)$Ba_2Cu_3O_{6+x}$

G. Ghiringhelli,[1*] M. Le Tacon,[2] M. Minola,[1] S. Blanco-Canosa,[2] C. Mazzoli,[1] N. B. Brookes,[3] G. M. De Luca,[4] A. Frano,[2,5] D. G. Hawthorn,[6] F. He,[7] T. Loew,[2] M. Moretti Sala,[3] D. C. Peets,[2] M. Salluzzo,[4] E. Schierle,[5] R. Sutarto,[7,8] G. A. Sawatzky,[8] E. Weschke,[5] B. Keimer,[2*] L. Braicovich[1]

SCIENCE VOL 337 17 AUGUST 2012

resonant soft x-ray scattering

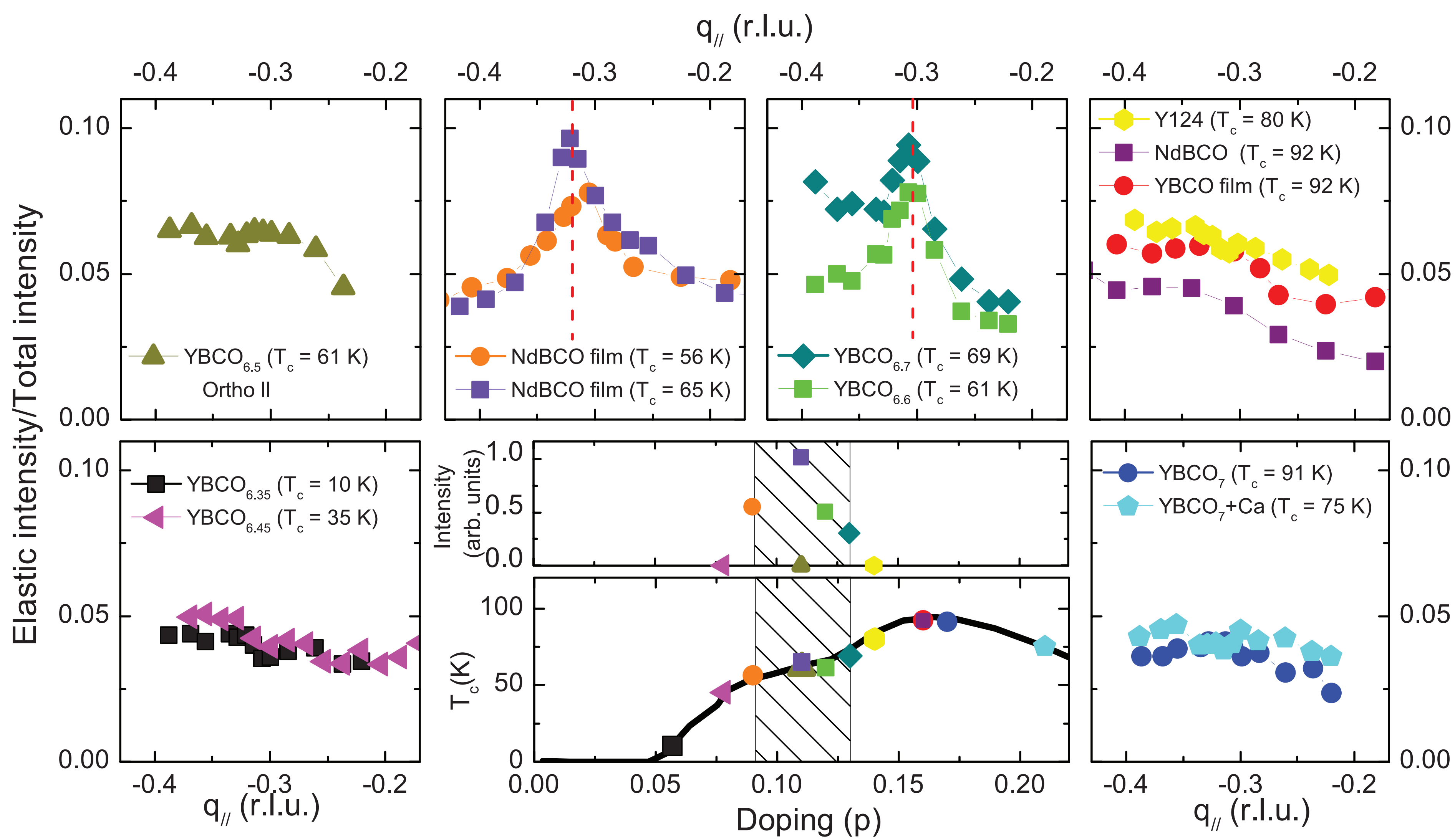


Fig. 3. Dependence of the CDW signal at 15 K on the hole doping level p. The CDW signal is present in several $YBa_2Cu_3O_{6+x}$ and $Nd_{1+y}Ba_{2-y}Cu_3O_7$ samples, but only for $0.09 \leq p \leq 0.13$. In this doping range (shaded in the central panel), the T_c-versus-p relation exhibits a plateau. The CDW peak position does not change with p outside of the experimental error, but its intensity is maximum at $p \simeq 0.11$.

Direct observation of competition between superconductivity and charge density wave order in $YBa_2Cu_3O_{6.67}$

NATURE PHYSICS | VOL 8 | DECEMBER 2012 |

J. Chang[1,2]★, E. Blackburn[3], A. T. Holmes[3], N. B. Christensen[4], J. Larsen[4,5], J. Mesot[1,2], Ruixing Liang[6,7], D. A. Bonn[6,7], W. N. Hardy[6,7], A. Watenphul[8], M. v. Zimmermann[8], E. M. Forgan[3] and S. M. Hayden[9]

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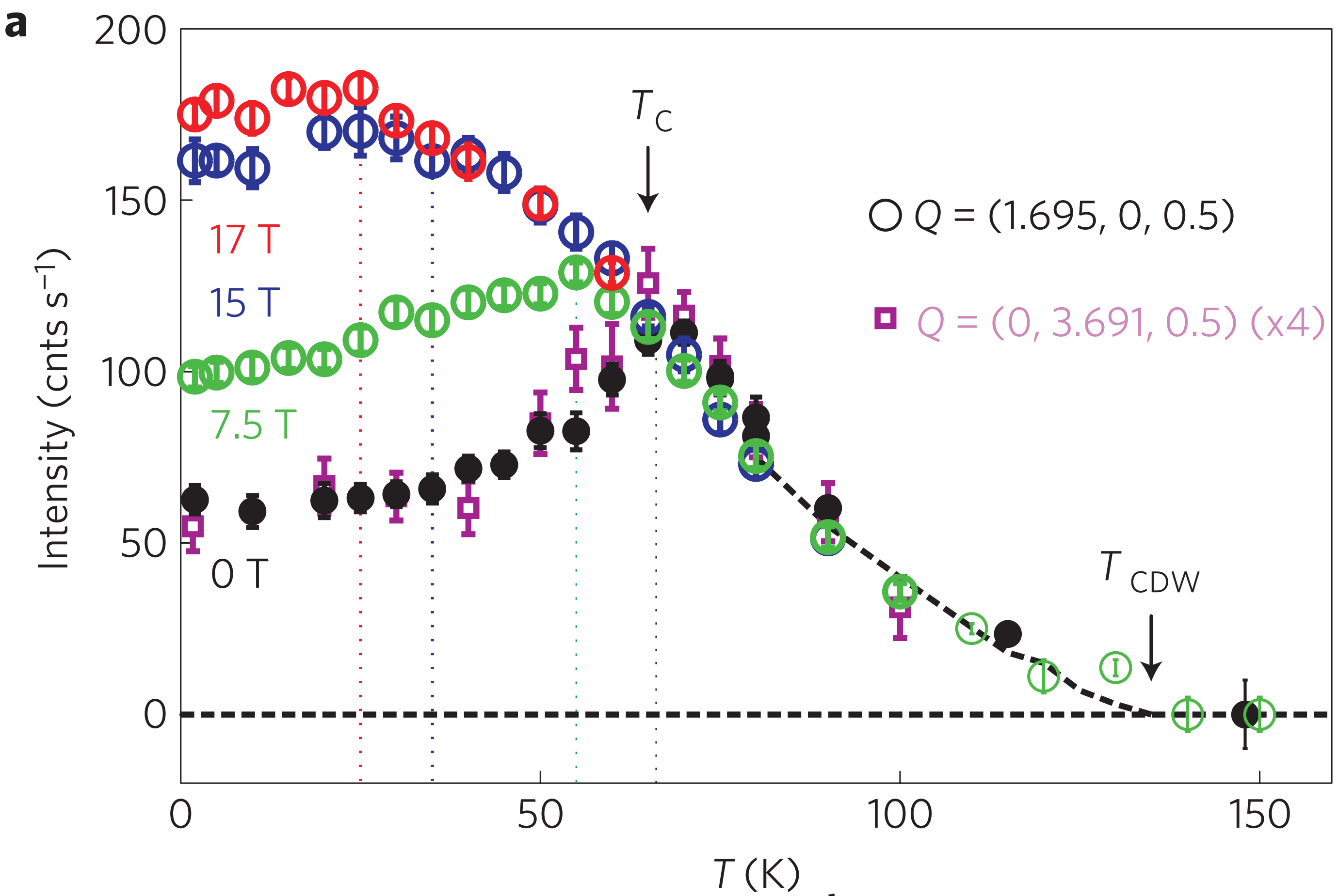

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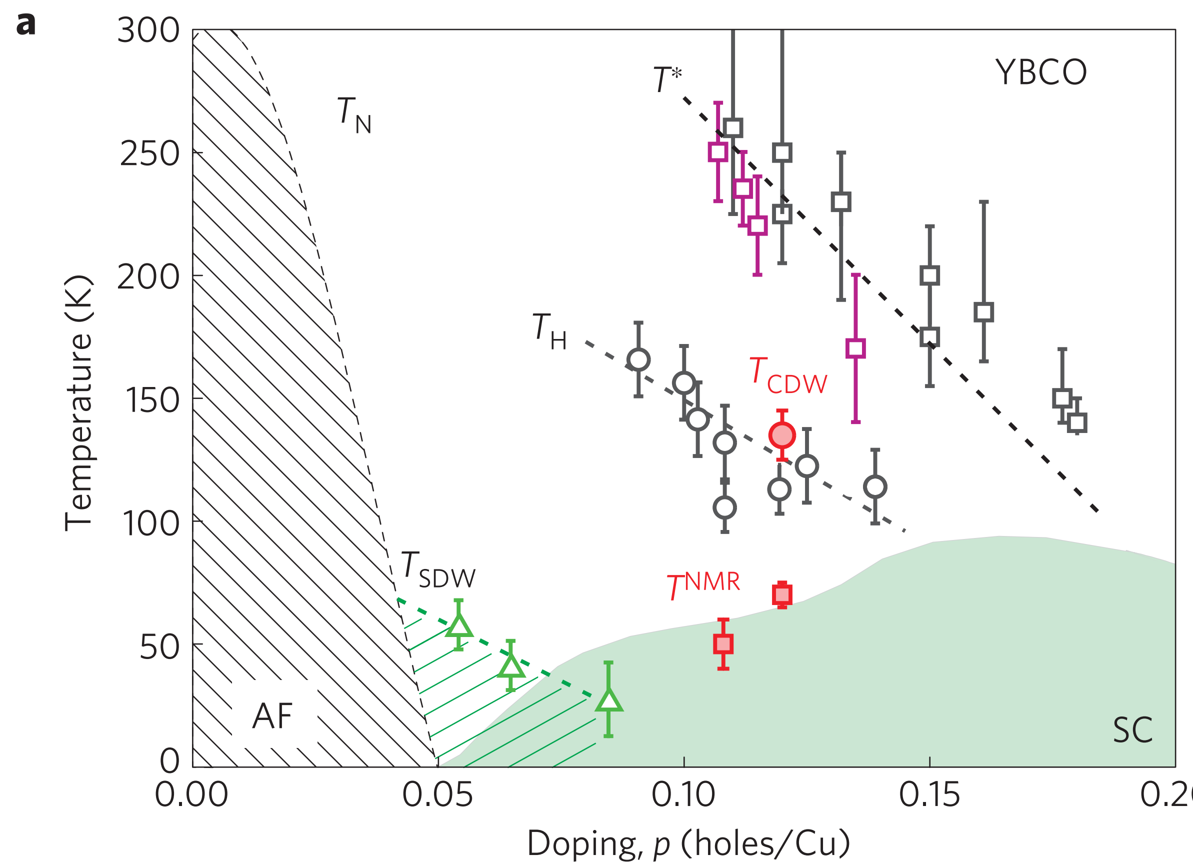
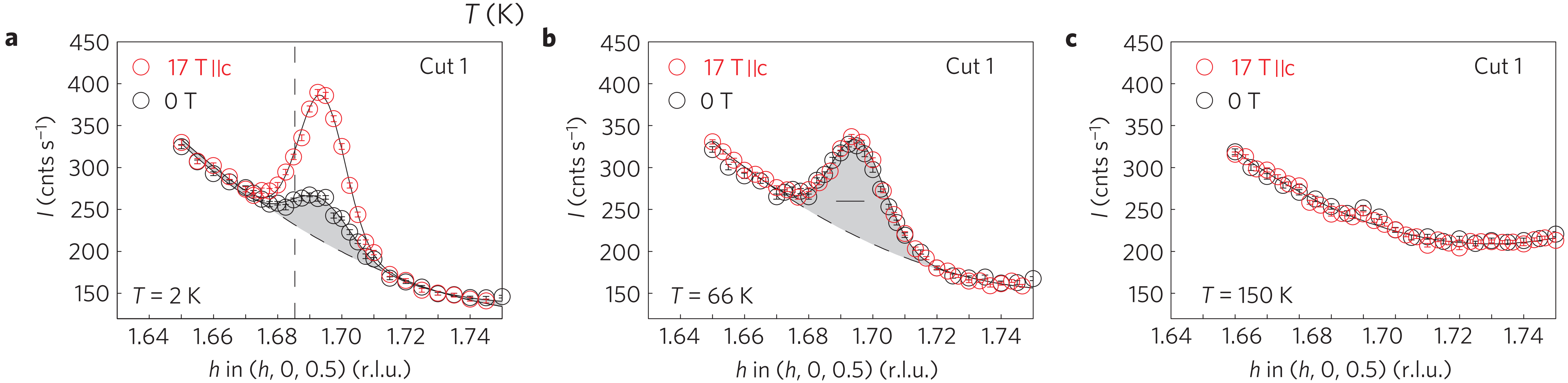


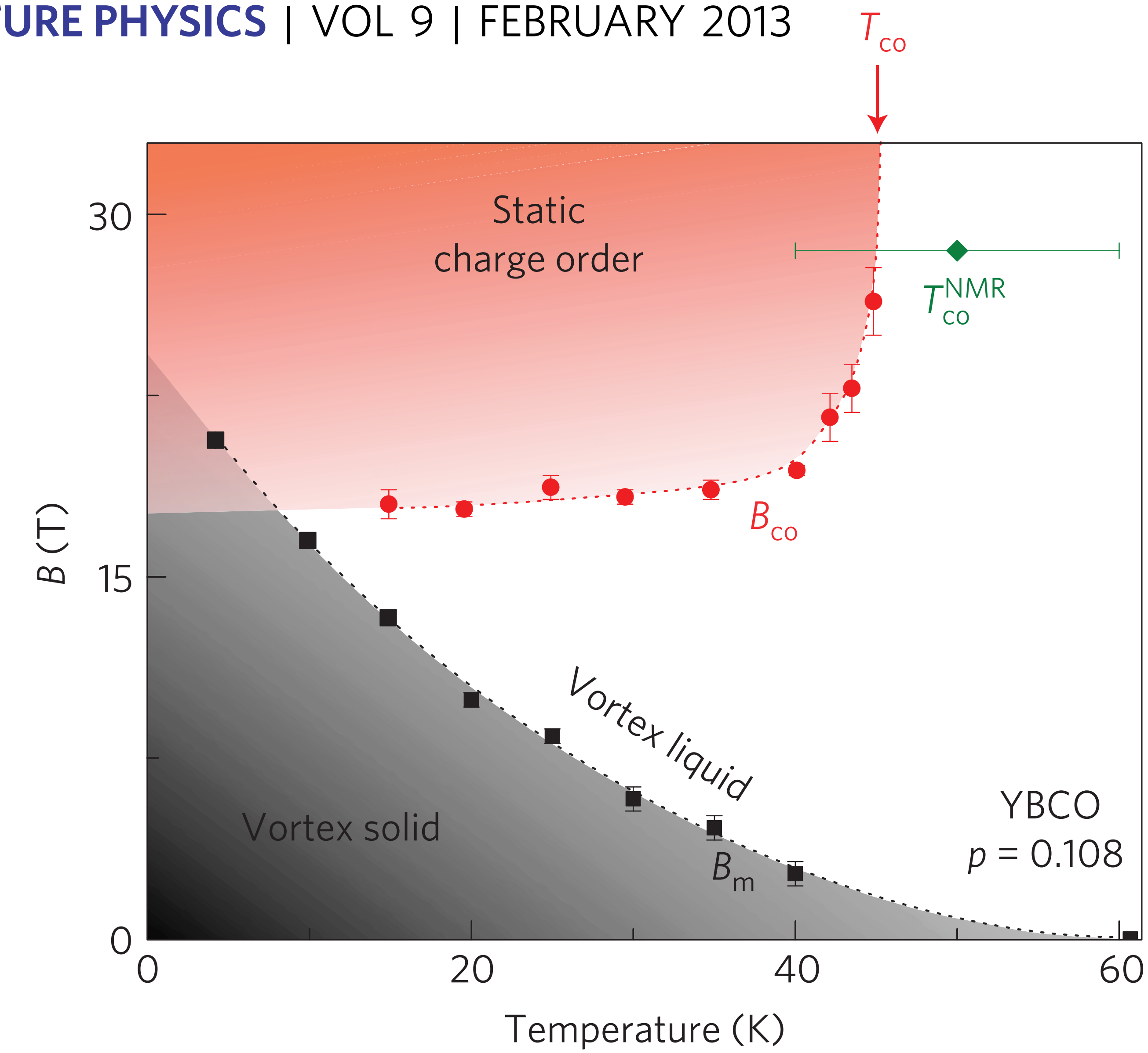


Sunday, September 22, 13

Thermodynamic phase diagram of static charge order in underdoped $YBa_2Cu_3O_y$

David LeBoeuf[1]★, S. Krämer[2], W. N. Hardy[3,4], Ruixing Liang[3,4], D. A. Bonn[3,4] and Cyril Proust[1,4]★

NATURE PHYSICS | VOL 9 | FEBRUARY 2013



The comparison of different acoustic modes indicates that the charge modulation is biaxial, which differs from a uniaxial stripe charge order.

A.J. Achkar et al., to appear

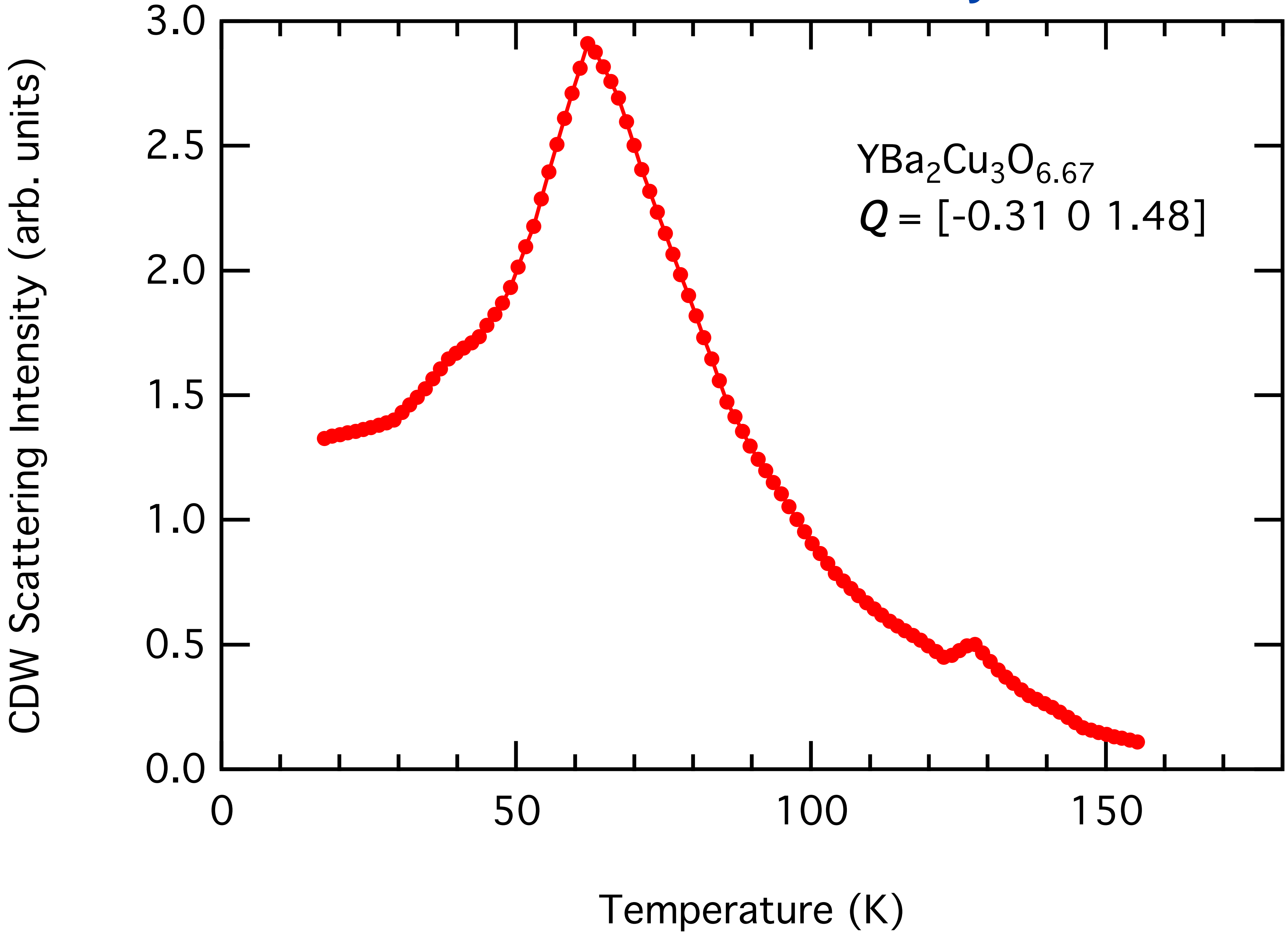


FIG. 1: The temperature dependence of the CDW scattering intensity at **Q** = [-0.31 0 1.48] in $YBa_2Cu_3O_{6.67}$ measured by resonant x-ray scattering in Ref. [4]. This sample has $T_c \approx 65.5$K.

PHYSICAL REVIEW B **69**, 144504 (2004)

Competing orders in thermally fluctuating superconductors in two dimensions

Subir Sachdev
Department of Physics, Yale University, P.O. Box 208120, New Haven, Connecticut 06520-8120, USA

Eugene Demler
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
(Received 6 August 2003; revised manuscript received 24 November 2003; published 6 April 2004)

We extend recent low-temperature analyses of competing orders in the cuprate superconductors to the pseudogap regime where all orders are fluctuating. A universal continuum limit of a classical Ginzburg-Landau functional is used to characterize fluctuations of the superconducting order: this describes the crossover from Gaussian fluctuations at high temperatures to the vortex-binding physics near the onset of global phase coherence. These fluctuations induce affiliated corrections in the correlations of other orders, and in particular, in the different realizations of charge order. Implications for scanning tunneling spectroscopy and neutron-scattering experiments are noted: there may be a regime of temperatures near the onset of superconductivity where the charge order is enhanced with increasing temperatures.

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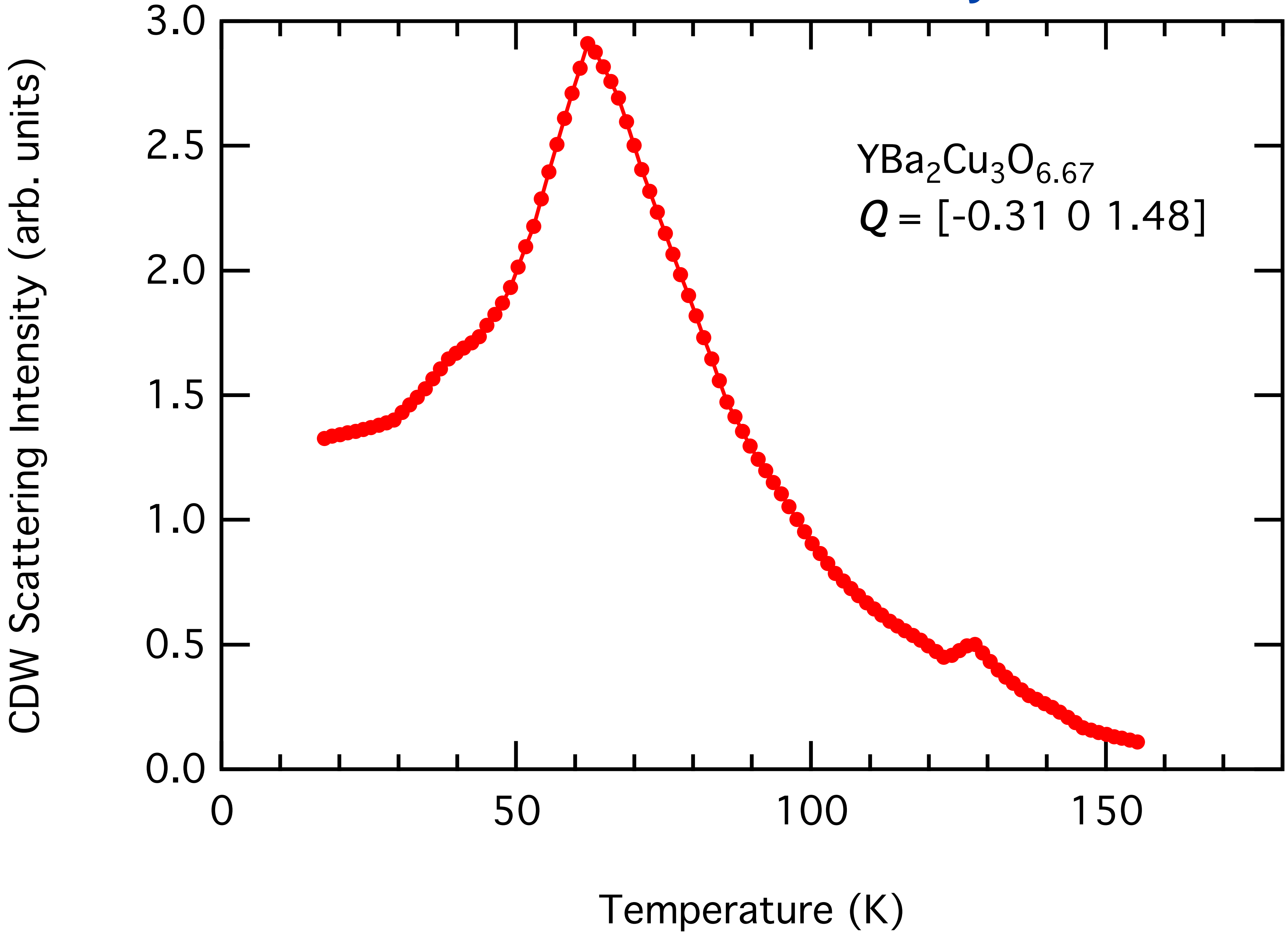
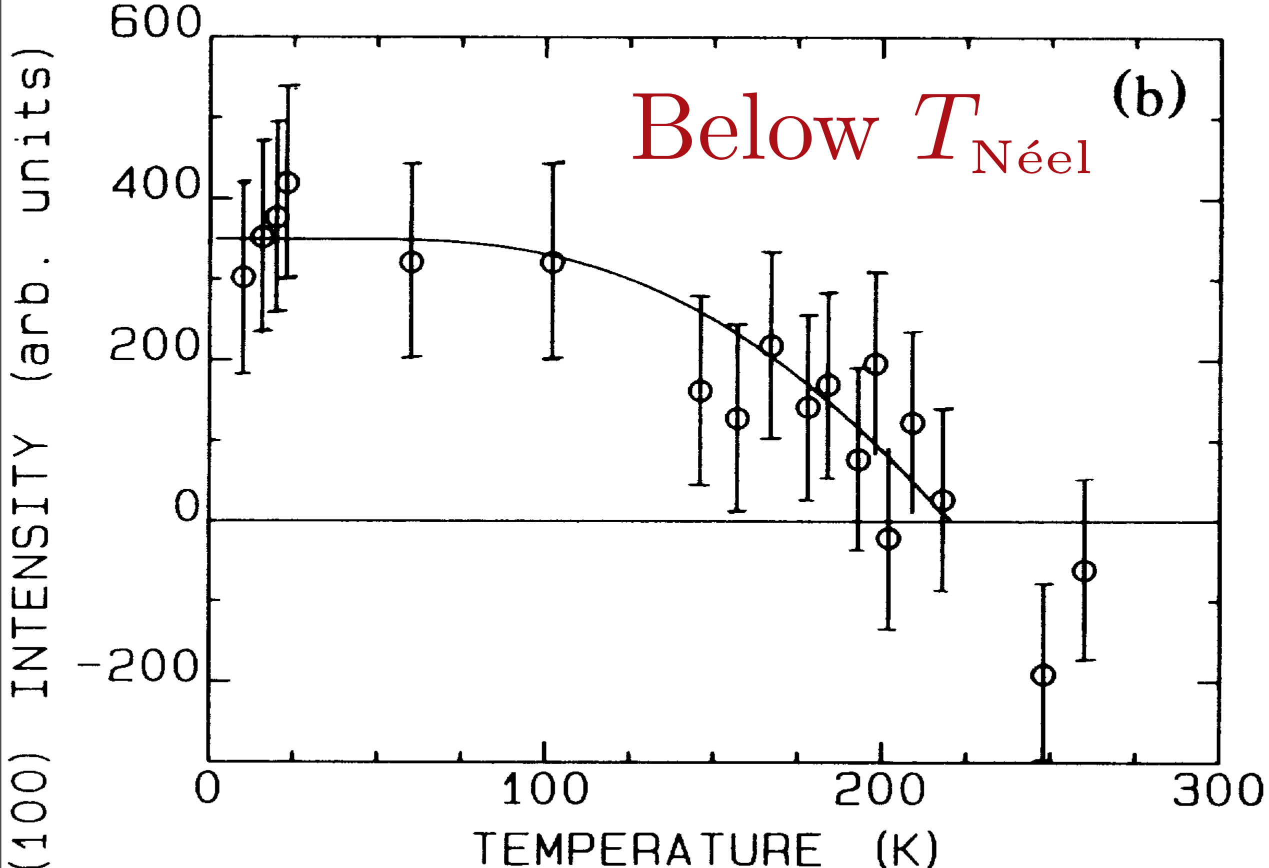


Onset is unlike an arrested ordering transition,
or precursor critical fluctuations

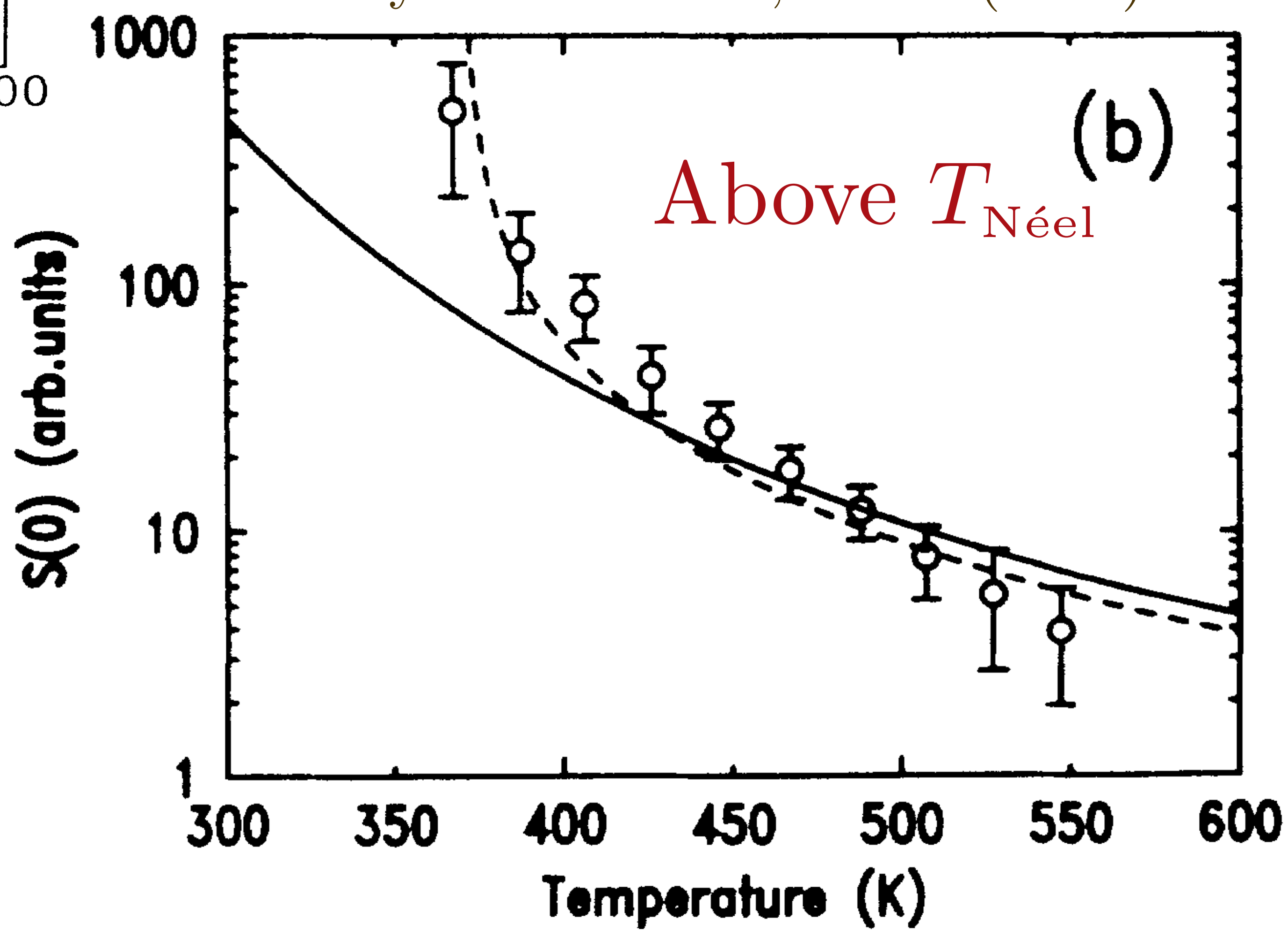
Key idea: analogy with the onset of antiferromagnetism in the *insulator* La_2CuO_4



D. Vaknin *et al.*,
Phys. Rev. Lett. **58**, 2802 (1987).

$$T_{\text{Néel}} = 325\text{K}$$

B. Keimer *et al.*,
Phys. Rev. B **46**, 14034 (1992).



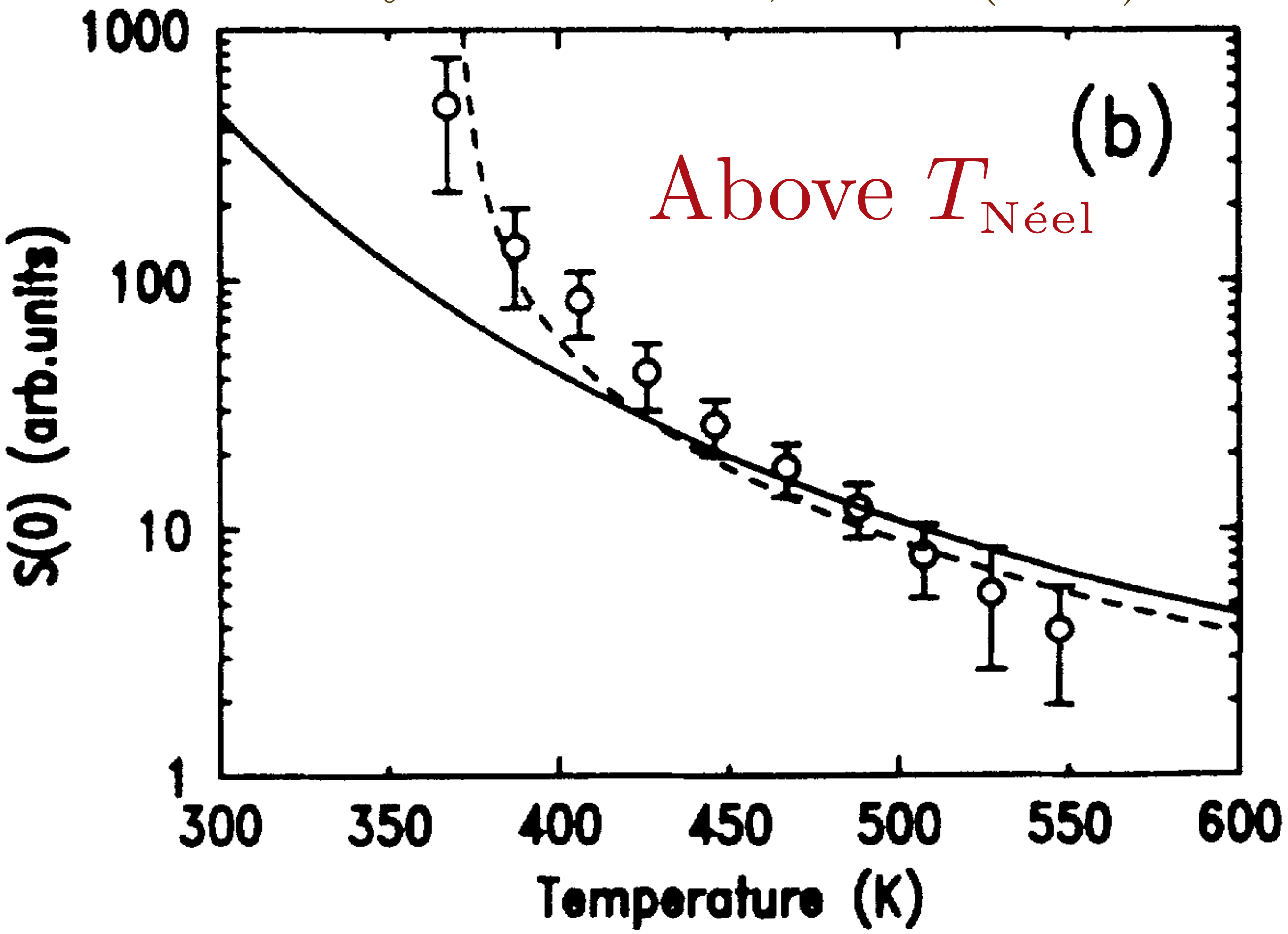
Key idea: analogy with the onset of antiferromagnetism in the *insulator* La_2CuO_4

$$T_{\text{Néel}} = 325\text{K}$$

Gradual onset of intensity over a wide range of T is a consequence of *angular thermal* fluctuations of an order parameter with 3 or more components in 2 spatial dimensions

Polyakov, 1975
Chakravarty, Halperin, Nelson 1989

B. Keimer *et al.*,
Phys. Rev. B **46**, 14034 (1992).



Multi-component order parameter

Superconducting order $\Psi(\mathbf{r})$:

$$\left\langle c_{i\alpha}^{\dagger} c_{j\beta}^{\dagger} \right\rangle = \varepsilon_{\alpha\beta} \left[\sum_{\mathbf{k}} \Delta_S(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] \Psi\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right)$$

Charge/bond order $\Phi_{x,y}(\mathbf{r})$ at wavevectors $\mathbf{Q}_{x,y}$:

$$\begin{aligned} \left\langle c_{i\alpha}^{\dagger} c_{j\beta} \right\rangle = & \ \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_{\mathbf{Q}_x}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_x \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_x\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right) \\ & + \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_{\mathbf{Q}_y}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_y \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_y\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right) \end{aligned}$$

Multi-component order parameter

Symmetries:
Charge conservation (O(2)), x translations (O(2)), y translations (O(2)), $x \leftrightarrow y$ ($\mathbb{Z}_2$), inversion, time-reversal.
Landau-Ginzburg free energy:

$$
\begin{aligned}
F = \int d^2r \Big[& |\nabla \Psi|^2 + s_1 |\Psi|^2 + u_1 |\Psi|^4 \\
& + |\nabla \Phi_x|^2 + |\nabla \Phi_y|^2 + s_2 \left(|\Phi_x|^2 + |\Phi_y|^2 \right) \\
& + u_2 \left(|\Phi_x|^2 + |\Phi_y|^2 \right)^2 + w \left(|\Phi_x|^4 + |\Phi_y|^4 \right) \\
& + v |\Psi|^2 \left(|\Phi_x|^2 + |\Phi_y|^2 \right) \Big]
\end{aligned}
$$

S. Sachdev and E. Demler, Phys. Rev. B 69, 144504 (2004)

Competing orders: v is positive (and large).

Multi-component order parameter

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Charge conservation (O(2)), x translations (O(2)), y translations (O(2)), $x \leftrightarrow y$ ($\mathbb{Z}_2$), inversion, time-reversal.
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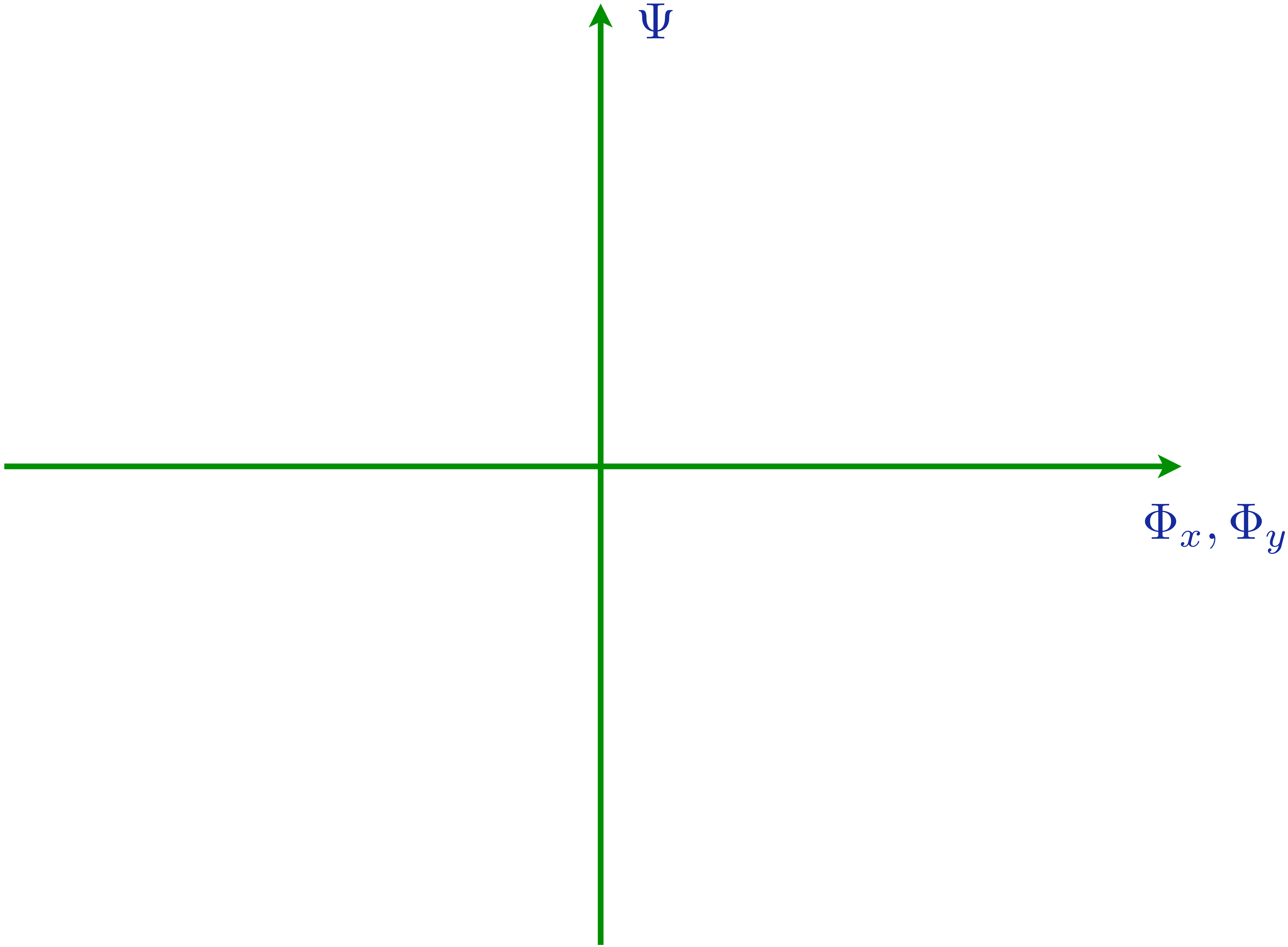
\end{aligned}
$$

S. Sachdev and E. Demler, Phys. Rev. B 69, 144504 (2004)

Competing orders: **Needed: a theory for large v**

Multi-component order parameter



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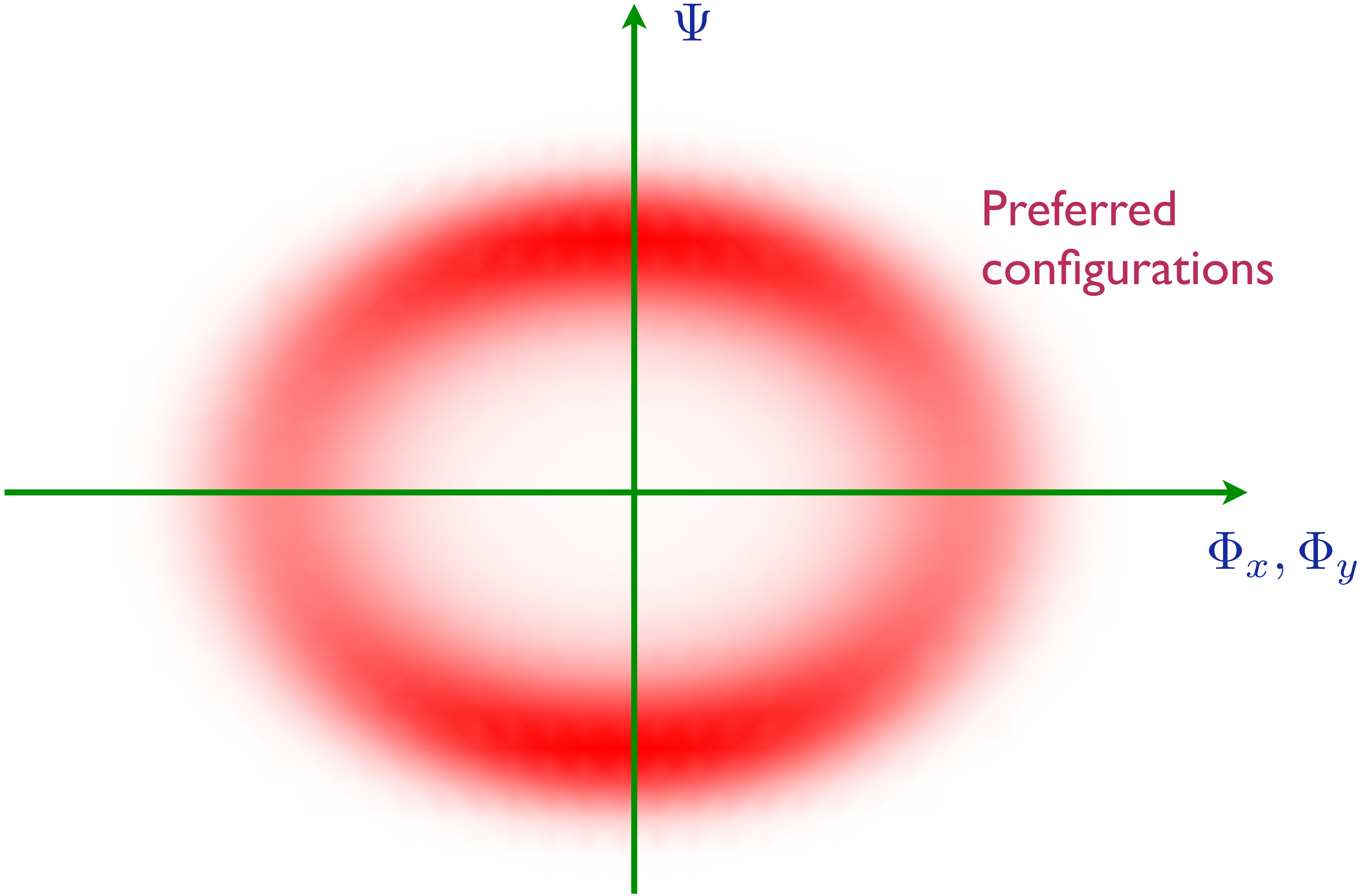
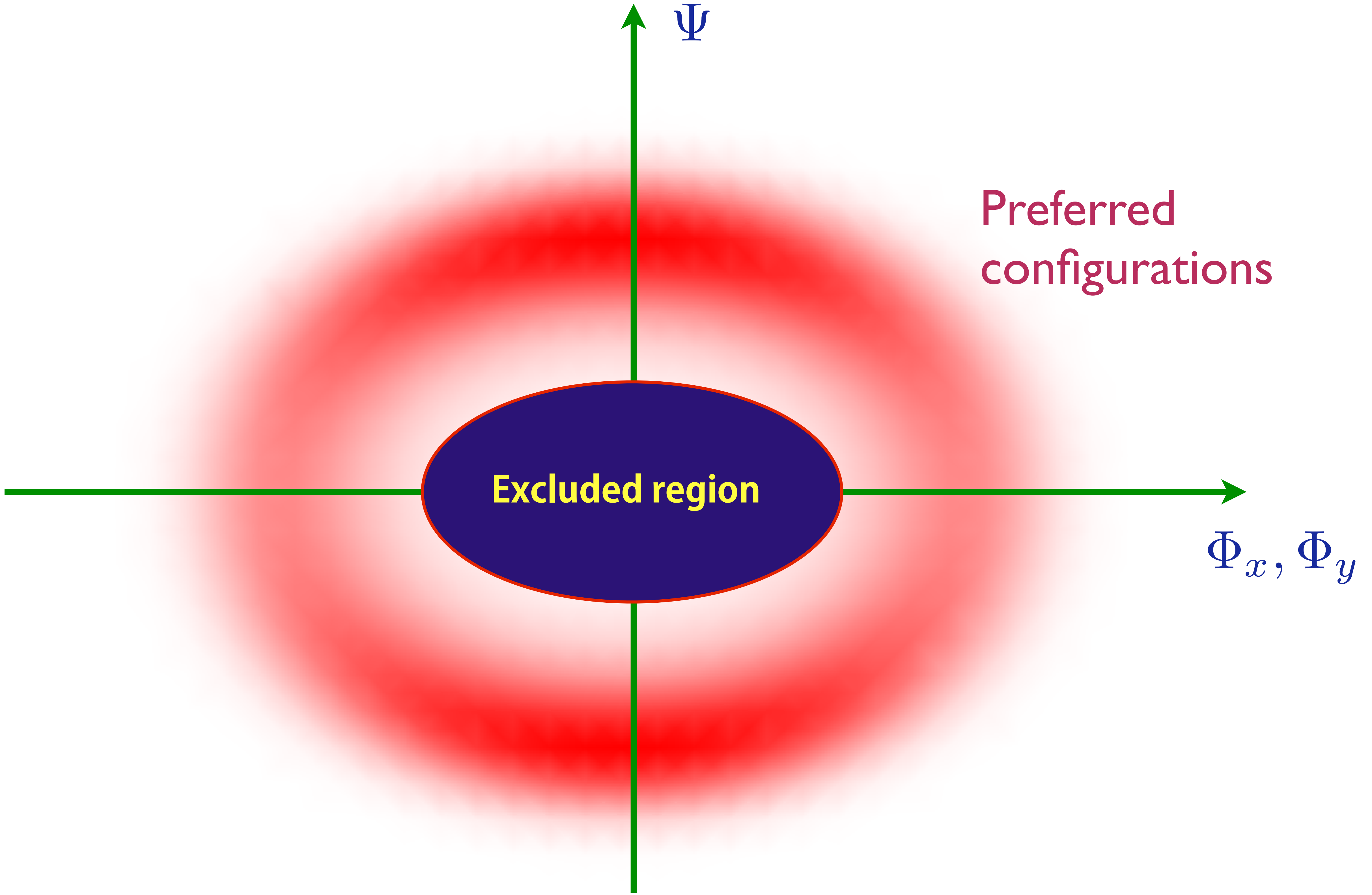


Support from electron theory:

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

K. B. Efetov, H. Meier, and C. Pepin, *Nature Physics* **9**, 442 (2013)

Multi-component order parameter



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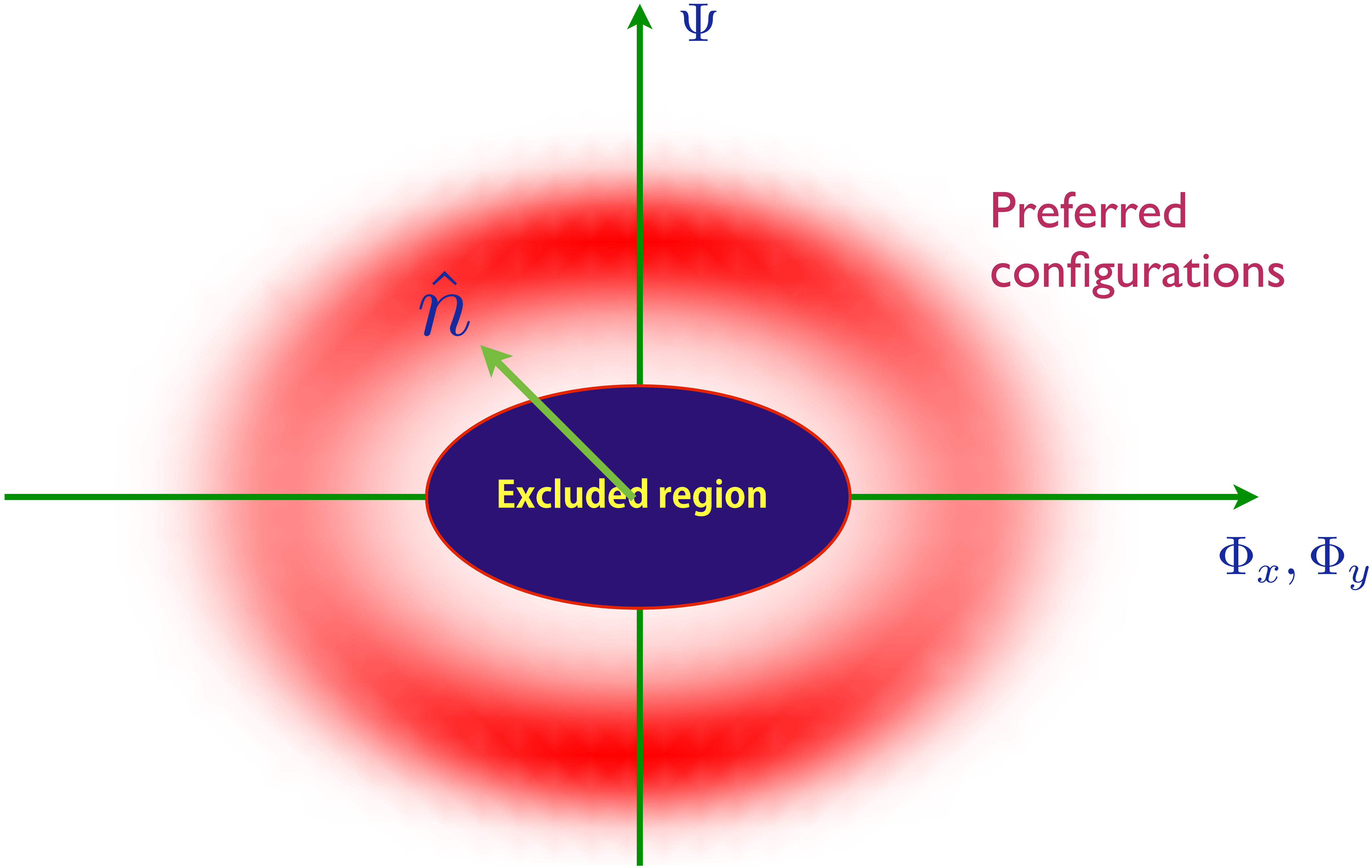

Label order parameter by a
6-component unit vector n_α with $\sum_\alpha n_\alpha^2 = 1$

O(6) non-linear sigma model

$$\mathcal{Z} = \int \mathcal{D}n_\alpha(\mathbf{r})\,\delta\left(\sum_{\alpha=1}^{6} n_\alpha^2(\mathbf{r}) - 1\right)\exp\left(- \frac{\rho_s}{2T}\int d^2r \left[\sum_{\alpha=1}^{2}(\nabla n_\alpha)^2 + \lambda\sum_{\alpha=3}^{6}(\nabla n_\alpha)^2 + g\sum_{\alpha=3}^{6} n_\alpha^2 + w\left[\left(n_3^2+n_4^2\right)^2 + \left(n_5^2+n_6^2\right)^2\right]\right]\right).$$

where $\Psi \propto n_1 + in_2$, $\Phi_x \propto n_3 + in_4$, $\Phi_y \propto n_5 + in_6$.

Describes $\mathrm{O}(6) \Rightarrow \mathrm{O}(2)\times\mathrm{O}(2)\times\mathrm{O}(2)\times\mathbb{Z}_2$.

Solve by cluster Monte Carlo and $1/N$ expansion.

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, arXiv:1309.xxxx

Comparison of Monte Carlo with 1/N expansion

$$S_{\Phi_x} = \int d^2r \, \langle \Phi_x(\mathbf{r}) \Phi_x(0) \rangle$$

Charge order structure factor S_{Φ_x}

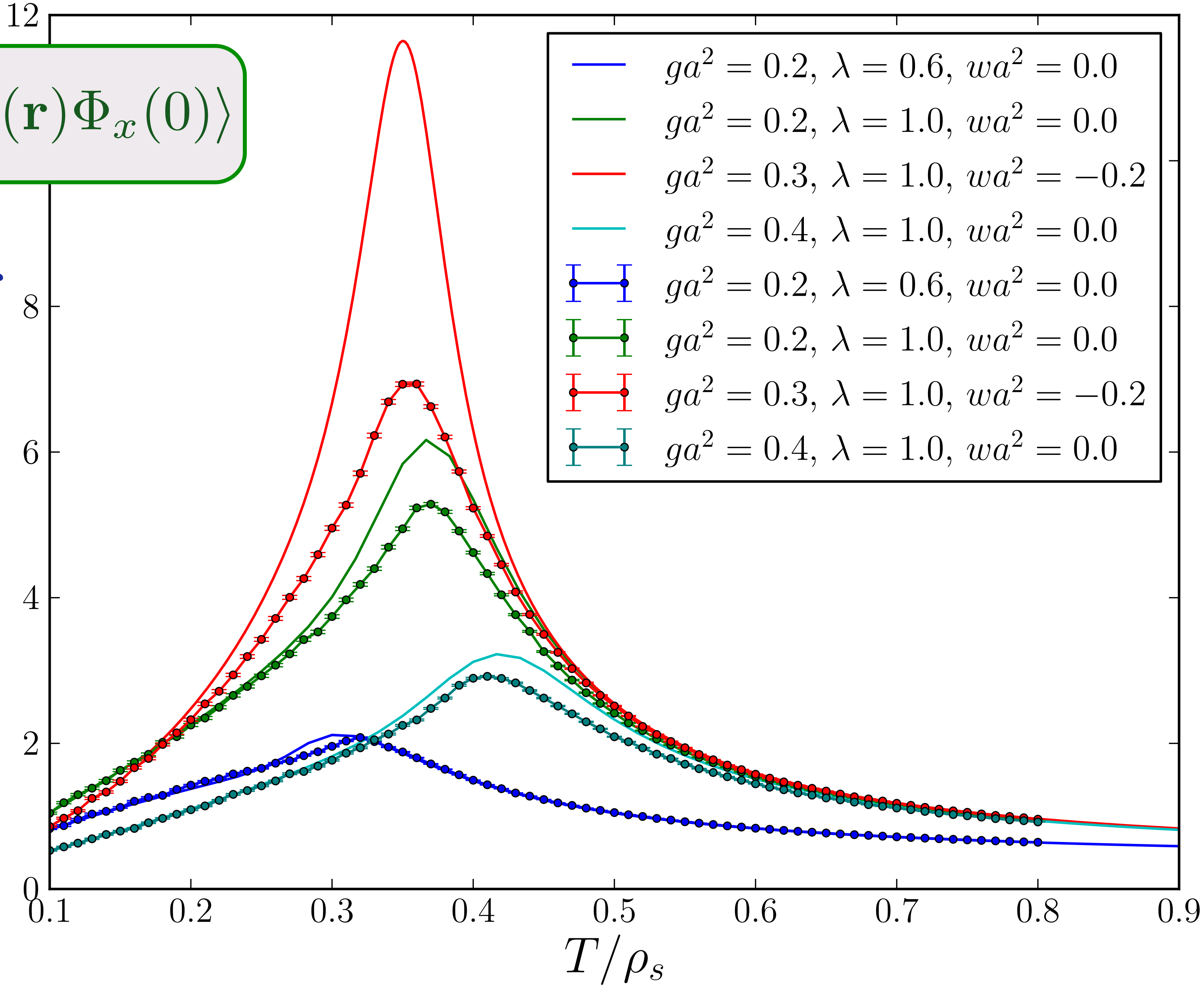

FIG. 6: Comparison of the charge order structure factor as obtained from the large N expansion at order $1/N$, with the computations of the Monte Carlo for the same parameters, and size $L = 32$. Large N calculations are solid lines, and Monte Carlo data is plotted as circles with statistical error bars.

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, arXiv:1309.xxxx

Comparison of Monte Carlo with experiments

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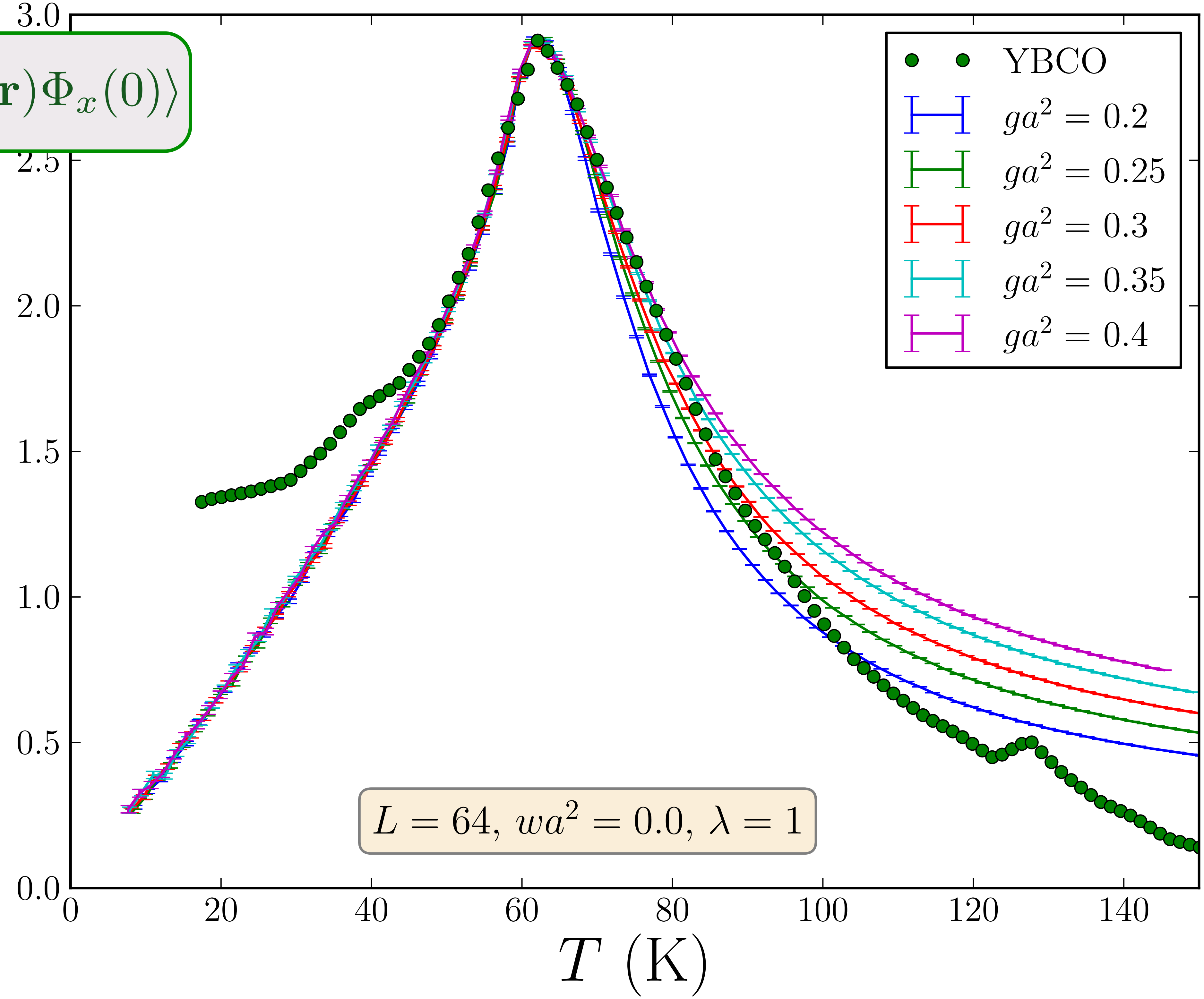

For $ga^2 = 0.30$ and $wa^2 = 0.0$ we have $\rho_s = 160\text{K}$.
The height was also rescaled to make the peak heights match.

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, arXiv:1309.xxxx

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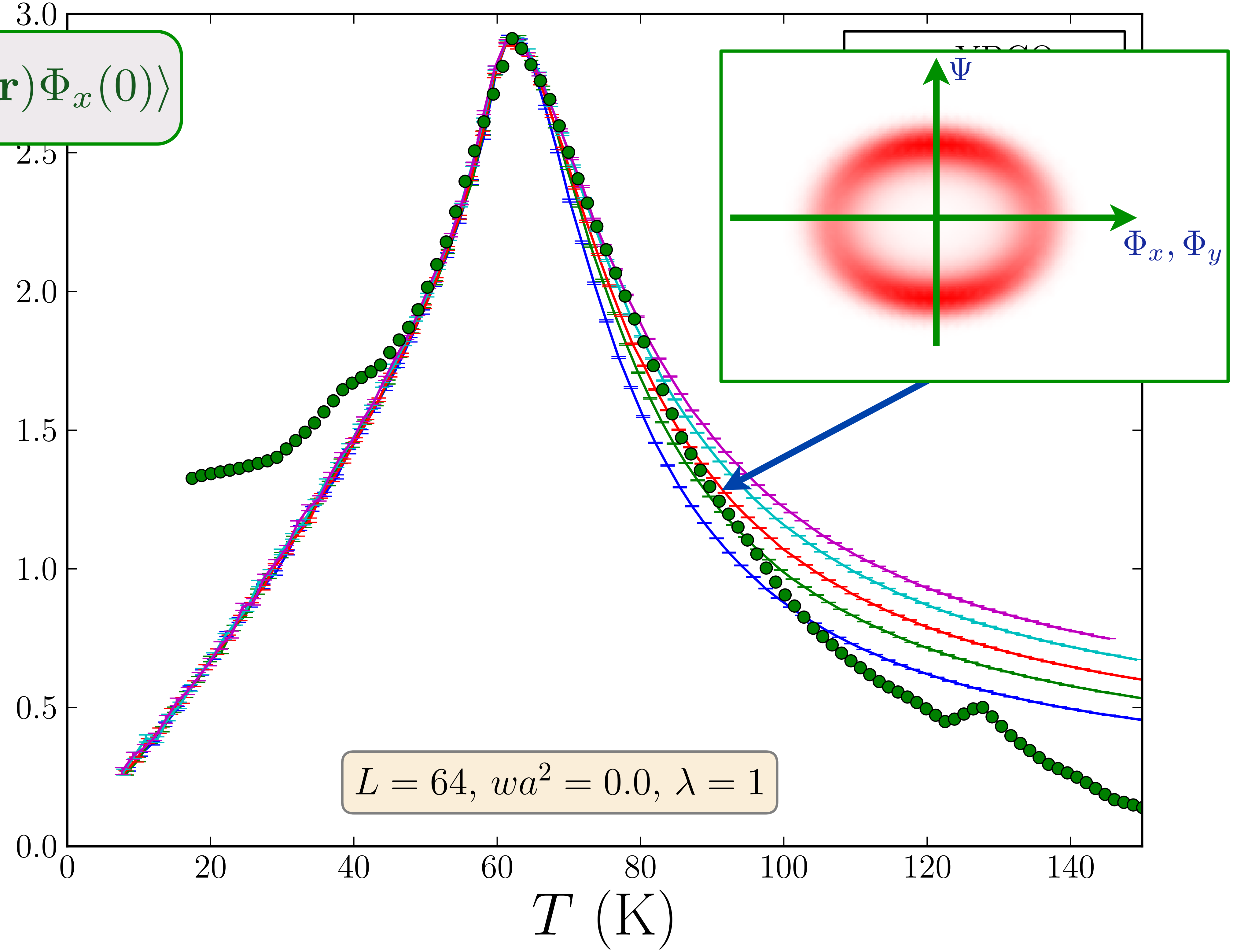

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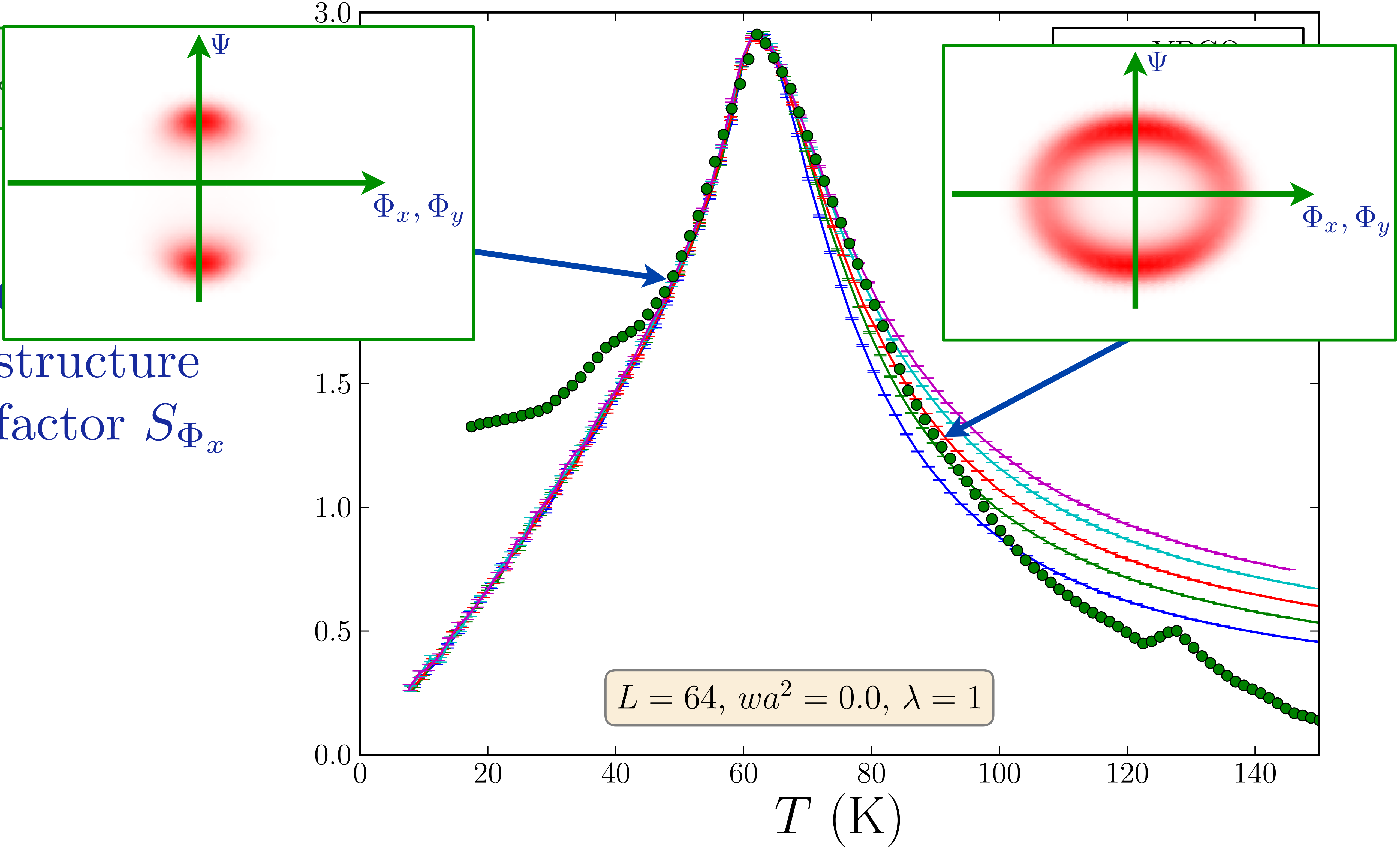

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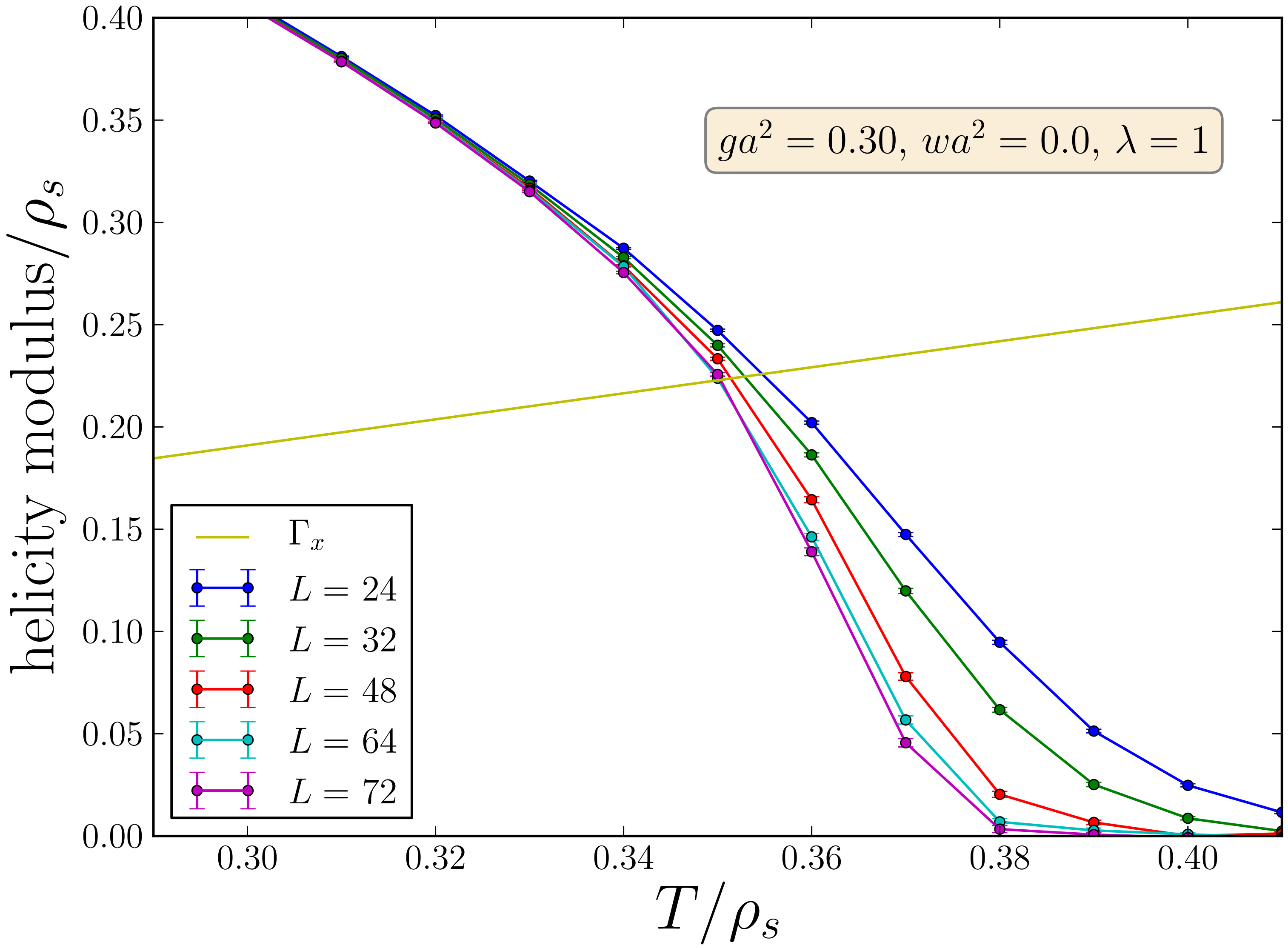
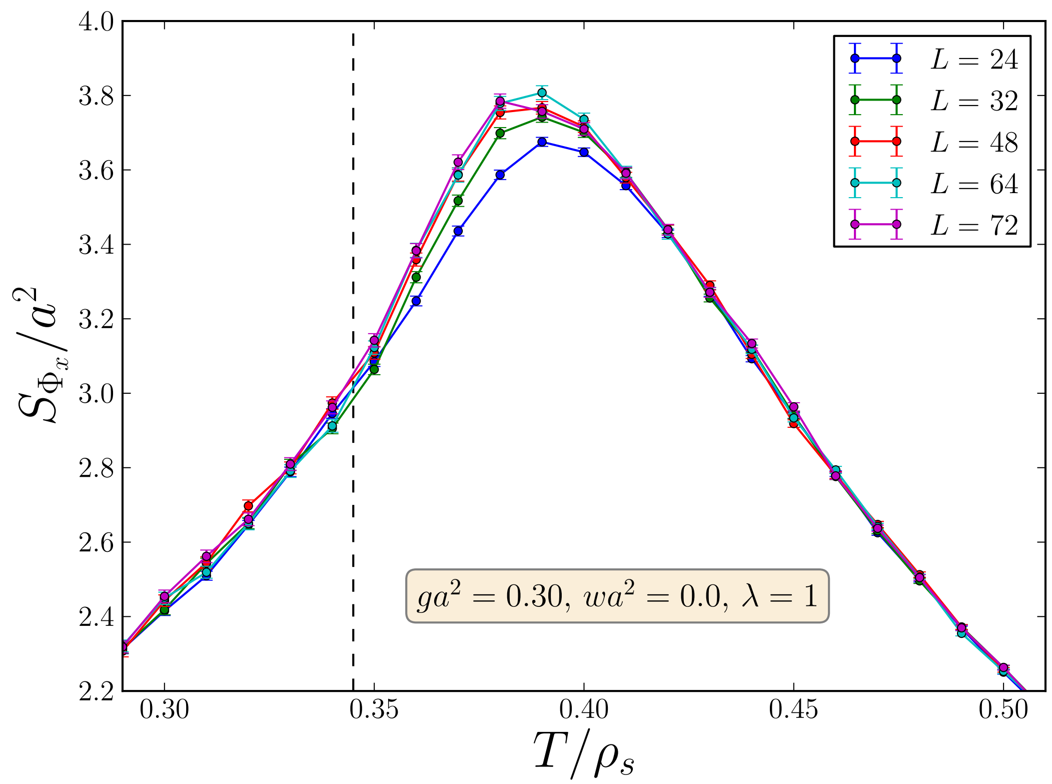
Onset of superconductivity in Monte Carlo

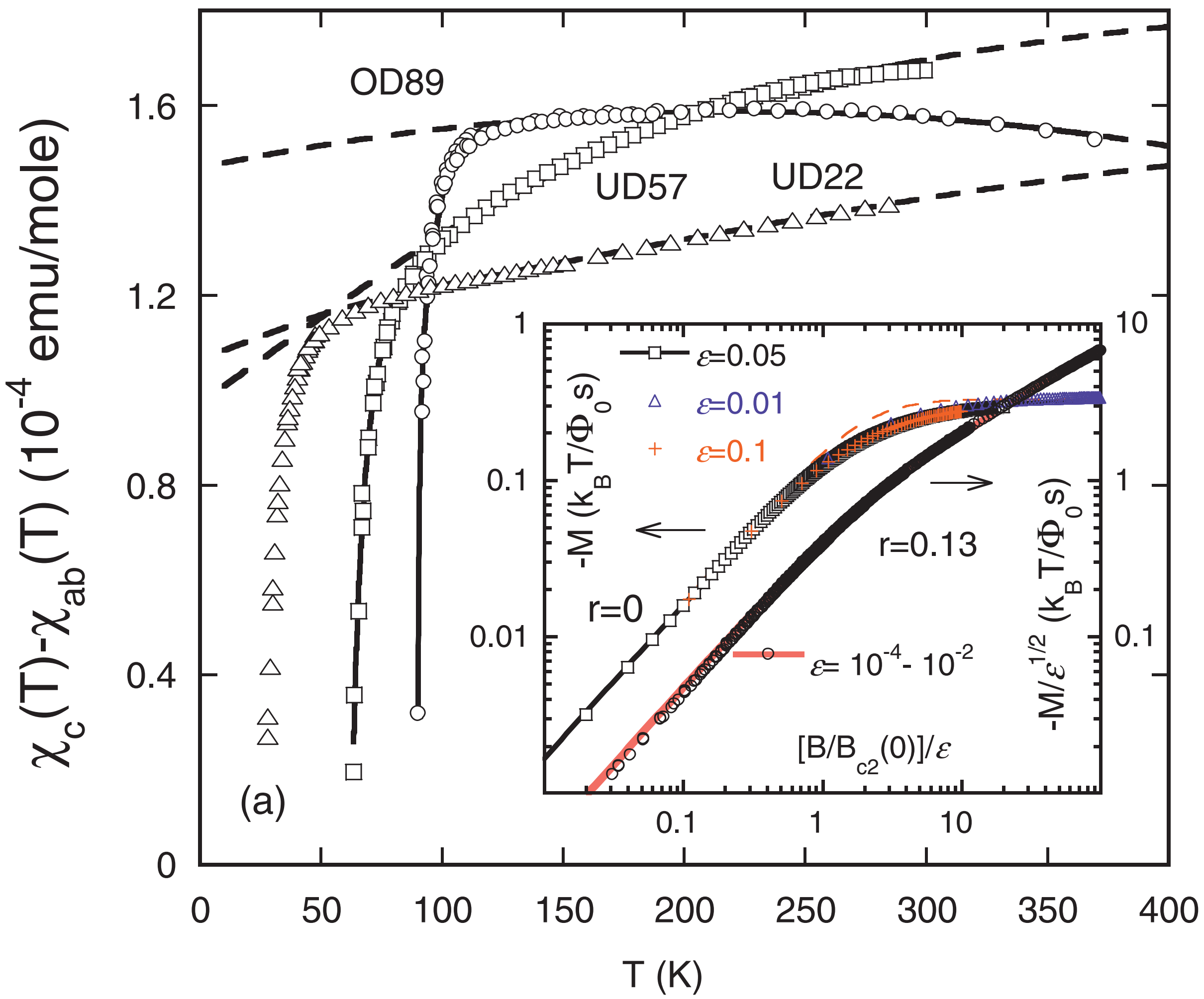




L. E. Hayward, D. G. Hawthorn,
R. G. Melko, and S. Sachdev, arXiv:1309.xxxx

Other experiments in the pseudogap

- The *same* set of parameters used to describe X-ray scattering, also predict the strength of superconducting fluctuations above T_c. Indeed $YBa_2Cu_3O_{6+x}$ shows significant fluctuation diamagnetism over the same range of temperatures. (S. Chatterjee et al, in progress).



PHYSICAL REVIEW B **88**, 060505(R) (2013)

I. Kokanović,[1,2,*] D. J. Hills,[1] M. L. Sutherland,[1] R. Liang,[3] and J. R. Cooper[1]

Diamagnetism of $YBa_2Cu_3O_{6+x}$ crystals above T_c: Evidence for Gaussian fluctuations

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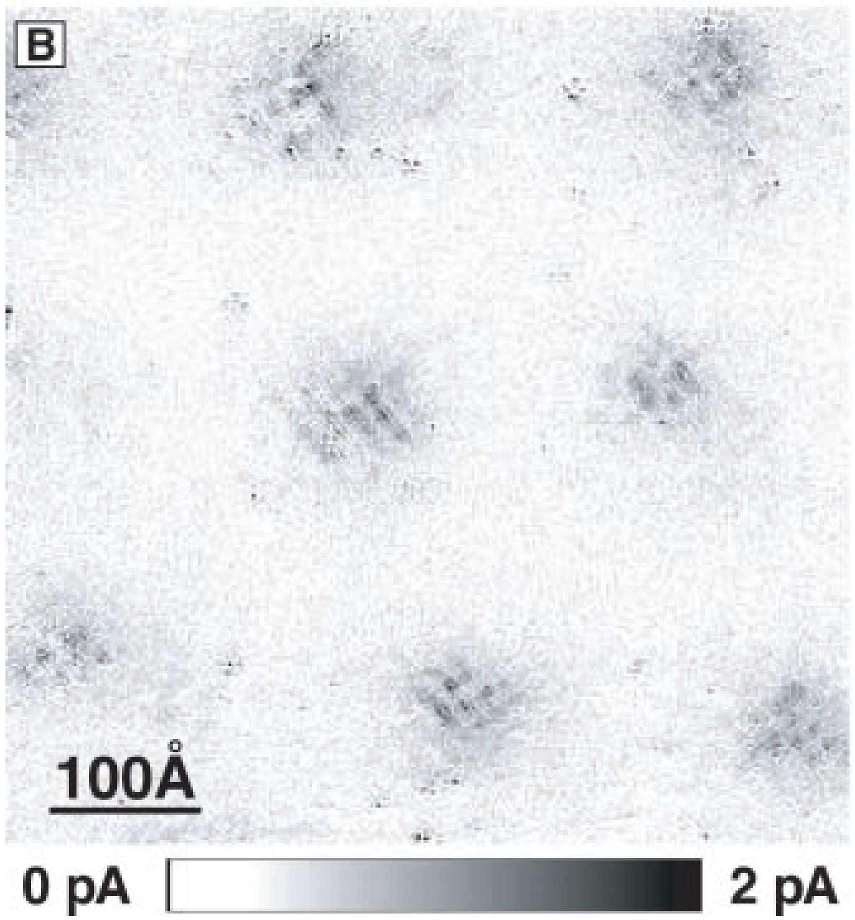
- Charge order was originally observed around vortex cores, indicating its competition with superconductivity.



A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $Bi_2Sr_2CaCu_2O_{8+\delta}$

J. E. Hoffman,[1] E. W. Hudson,[1,2*] K. M. Lang,[1] V. Madhavan,[1] H. Eisaki,[3†] S. Uchida,[3] J. C. Davis[1,2‡]

SCIENCE VOL 295 18 JANUARY 2002

Other experiments in the pseudogap

- The *same* set of parameters used to describe X-ray scattering, also predict the strength of superconducting fluctuations above T_c. Indeed $YBa_2Cu_3O_{6+x}$ shows significant fluctuation diamagnetism over the same range of temperatures. (S. Chatterjee et al, in progress).

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- Fluctuating 6-component order can explain "Fermi arc" photoemission spectra (Randeria; D. Chowdhury et al, in progress)

Outline

1. Review of recent experiments

2. Theory of pseudogap

3. Microscopic basis

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Pseudospin symmetry of the exchange interaction

$$H_J = \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

with $\vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$D_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c^\dagger_{i\downarrow} \end{pmatrix} \quad , \quad D_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c^\dagger_{i\uparrow} \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left(D^\dagger_{i\alpha a} \vec{\sigma}_{\alpha\beta} D_{i\beta a} \right) \cdot \left(D^\dagger_{j\gamma b} \vec{\sigma}_{\gamma\delta} D_{j\delta b} \right)$$

where a, b are the Nambu indices. This form makes explicit the symmetry under *independent* SU(2) pseudospin transformations on each site

$$D_{i\alpha a} \to U_{i,ab} D_{i\alpha b}$$

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of H_J.

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We will find important consequences of the pseudospin symmetry in ordinary metals with antiferromagnetic correlations.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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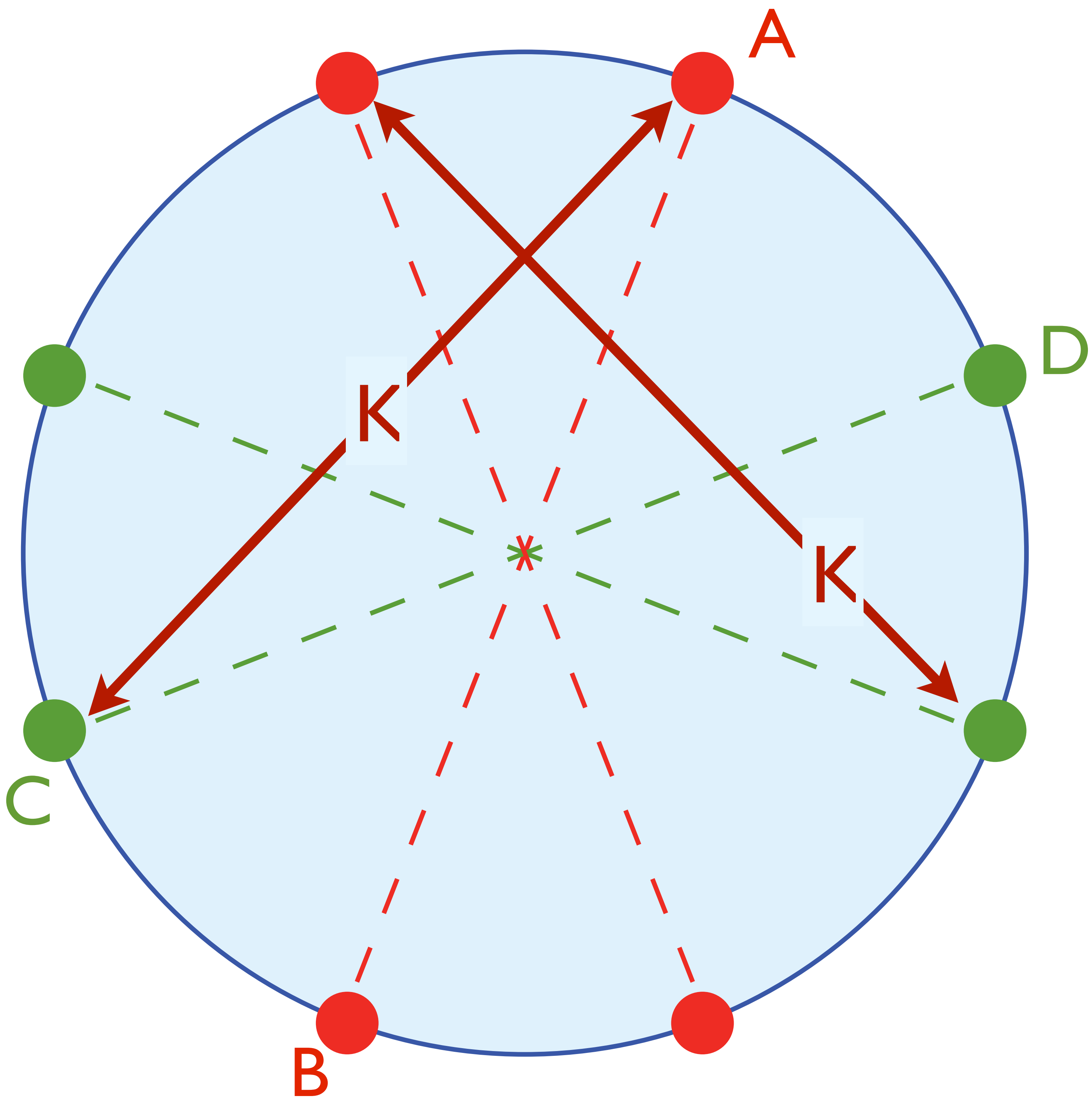
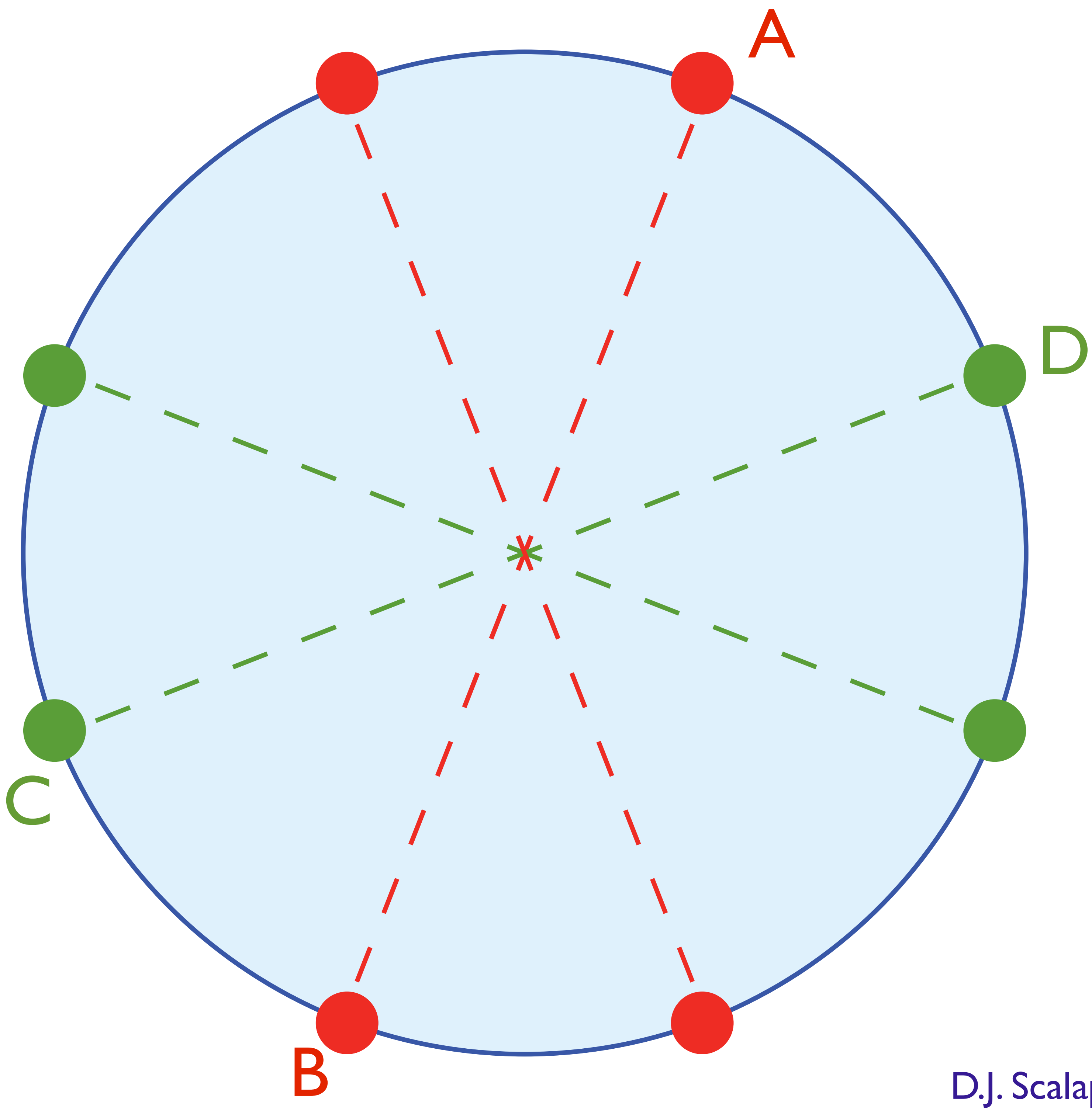
In Fourier space, the exchange interaction $J(\mathbf{q})$ is maximum near $\mathbf{q} = \mathbf{K} \equiv (\pi, \pi)$. So it is most effective near points on the Fermi surface which are separated by $\mathbf{K}$, the so-called "hot spots". Exchange interactions near the hot spots are expected to lead to d-wave superconductivity at low temperatures.

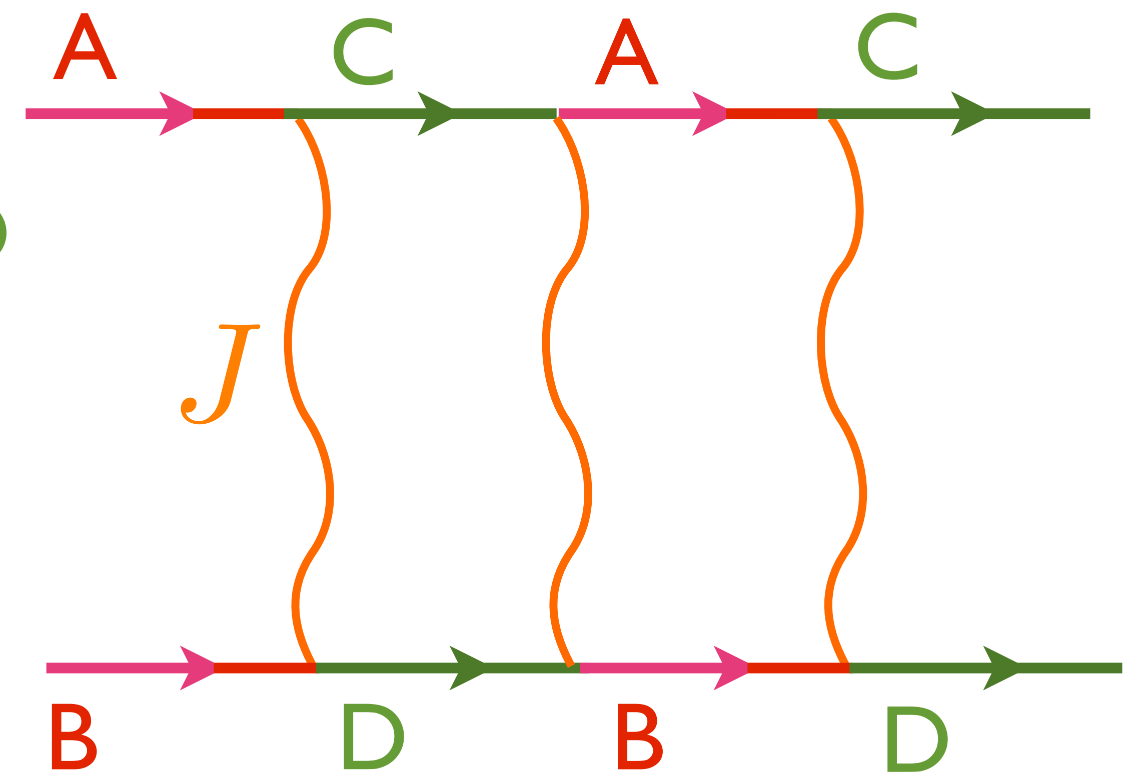
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Hot spots on the Fermi surface



Pairing "glue" from antiferromagnetic fluctuations





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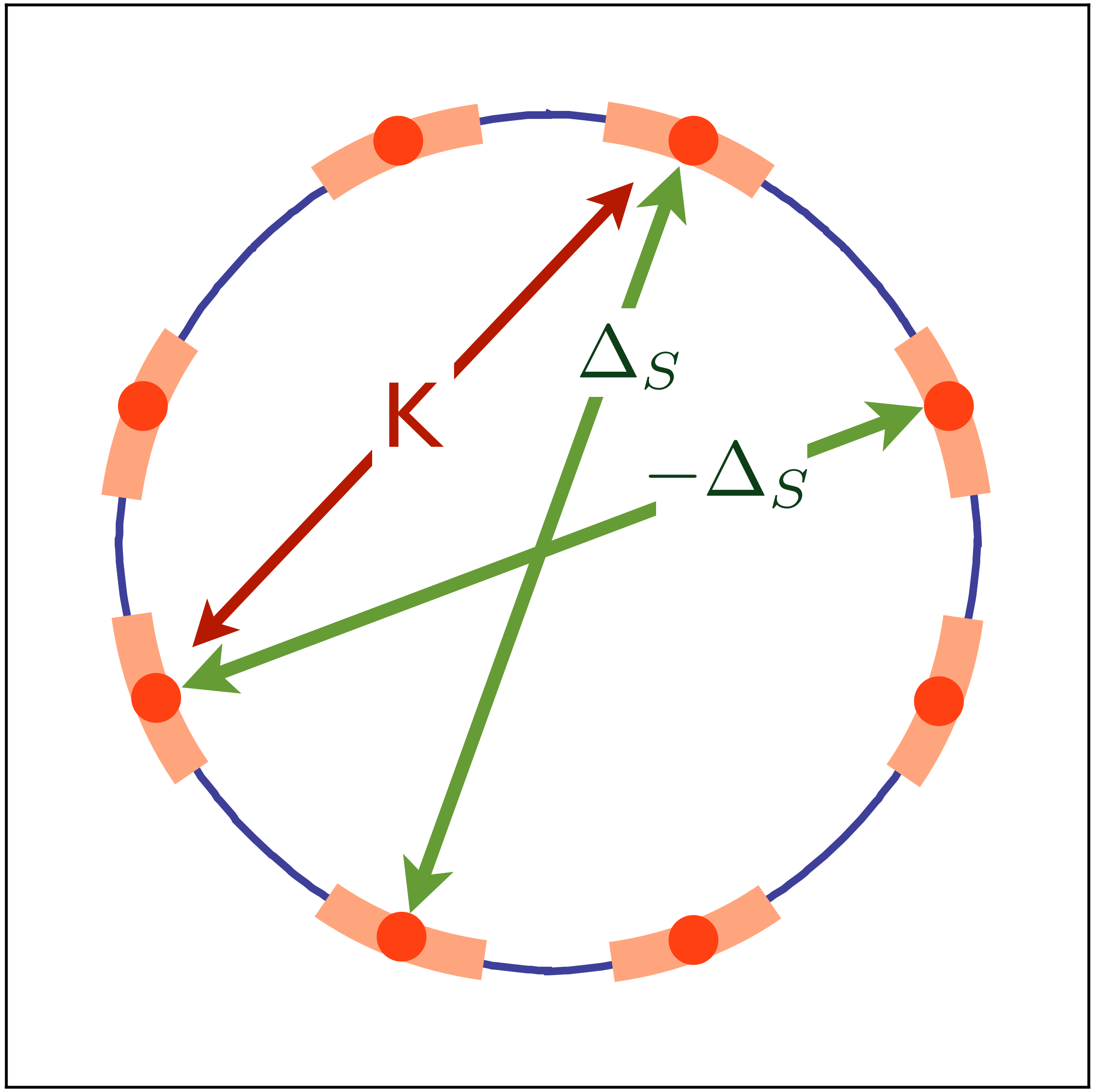
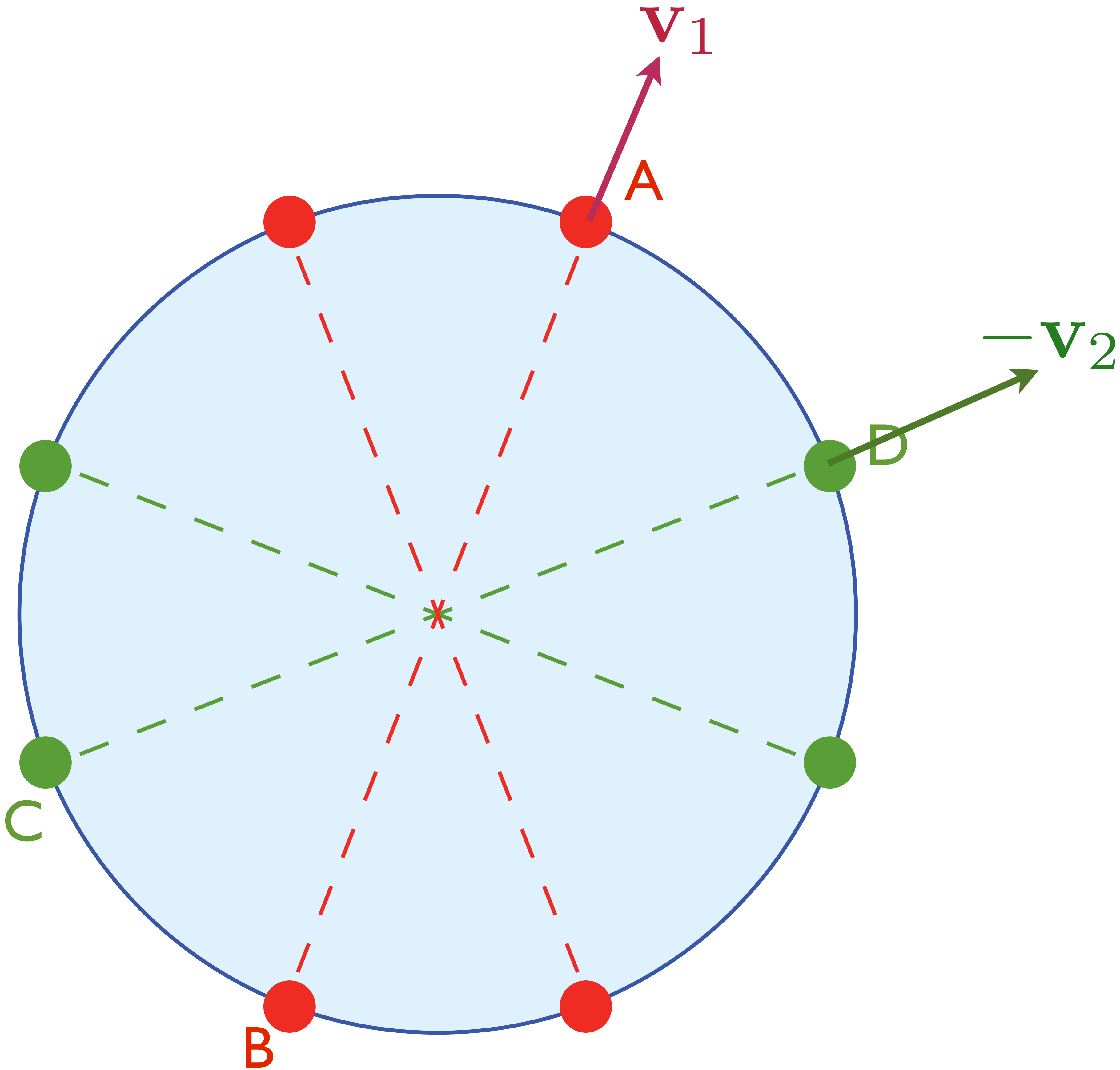


d-wave superconductor: particle-particle pairing at *and near* hot spots, with sign-changing pairing amplitude



$\mathbf{v}_1$
A
$-\mathbf{v}_2$
D
C
B

Hamiltonian at and *near* hot spots

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This Hamiltonian has an exact SU(2)xSU(2) pseudospin symmetry !

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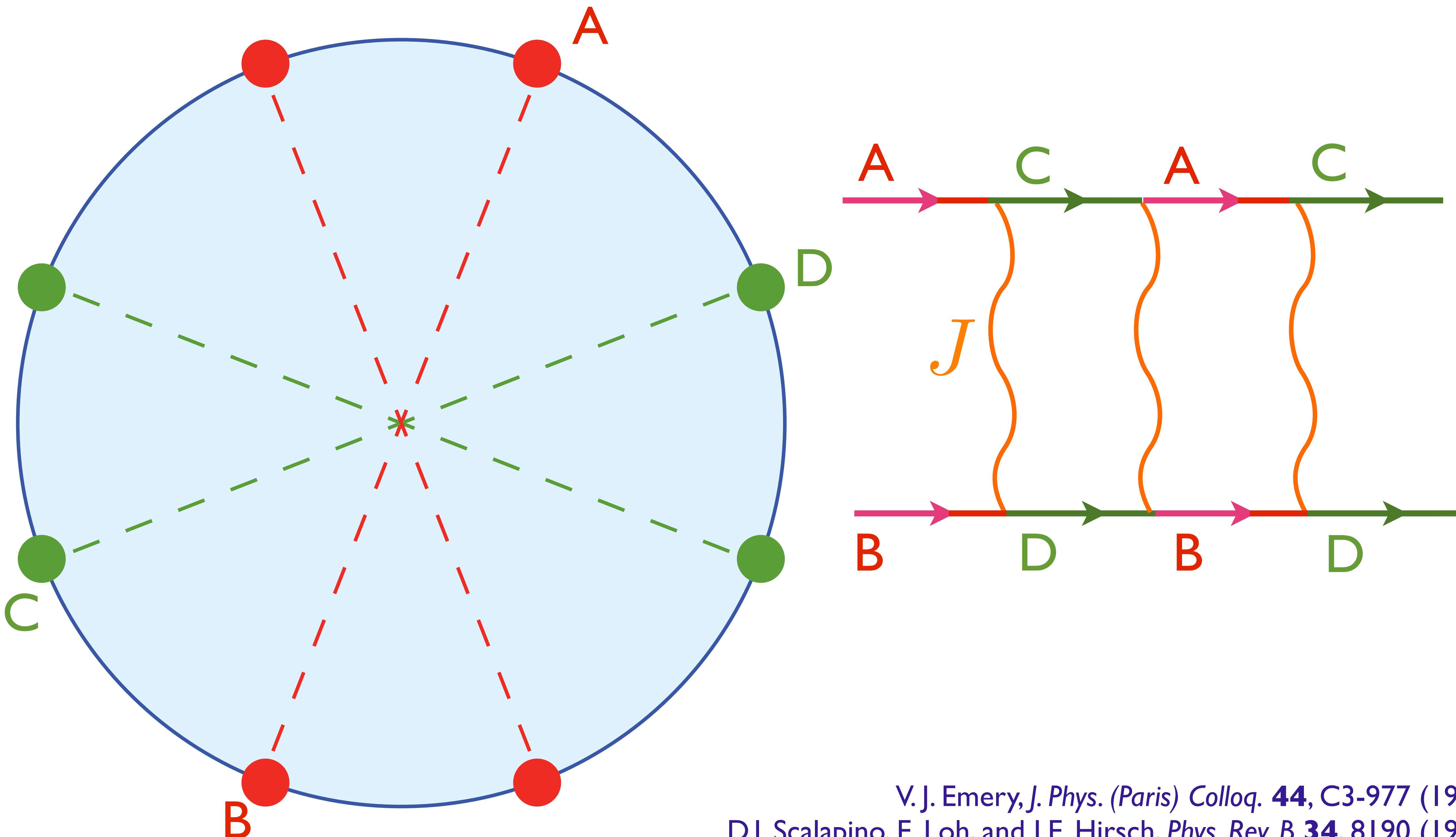

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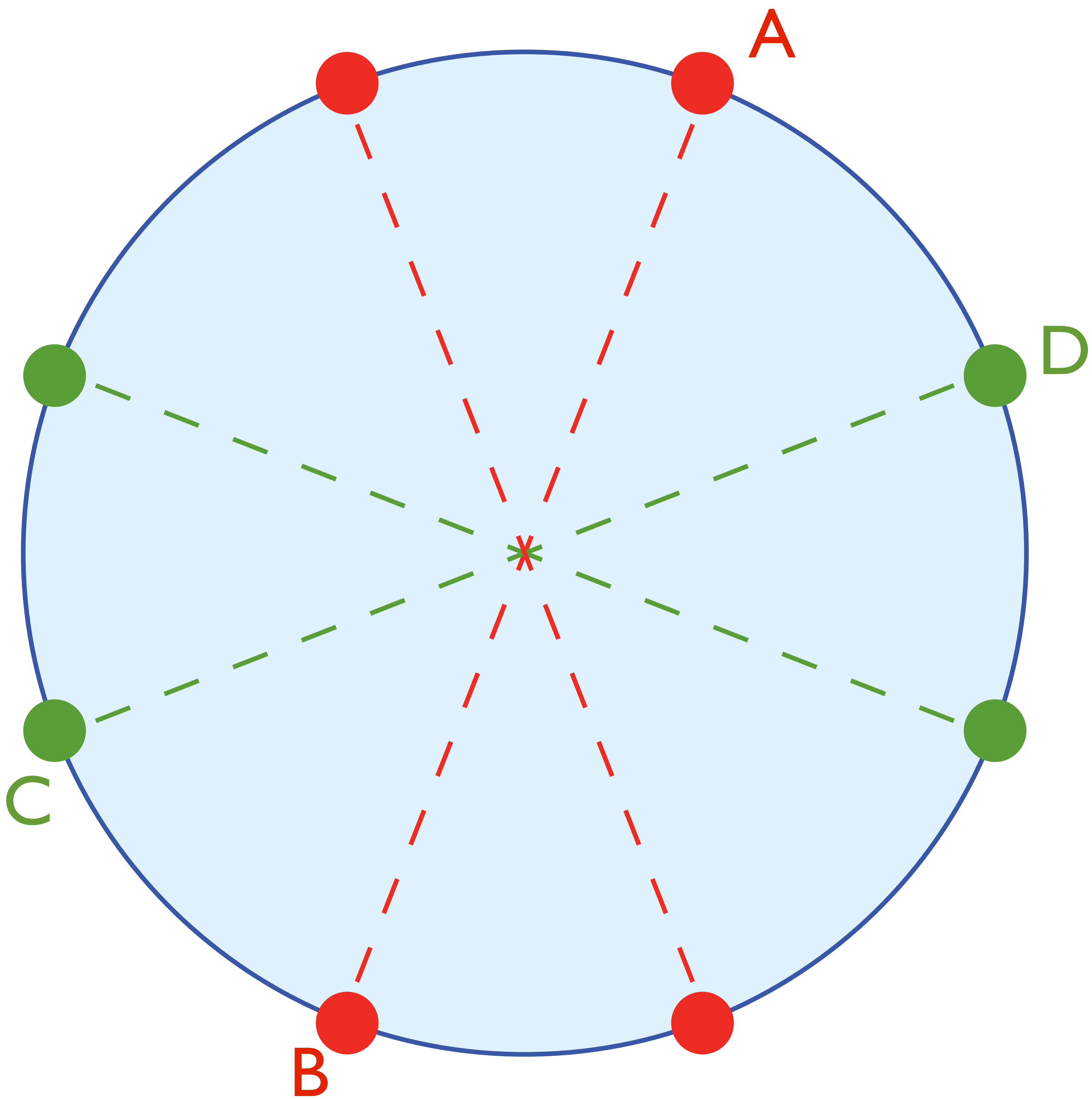
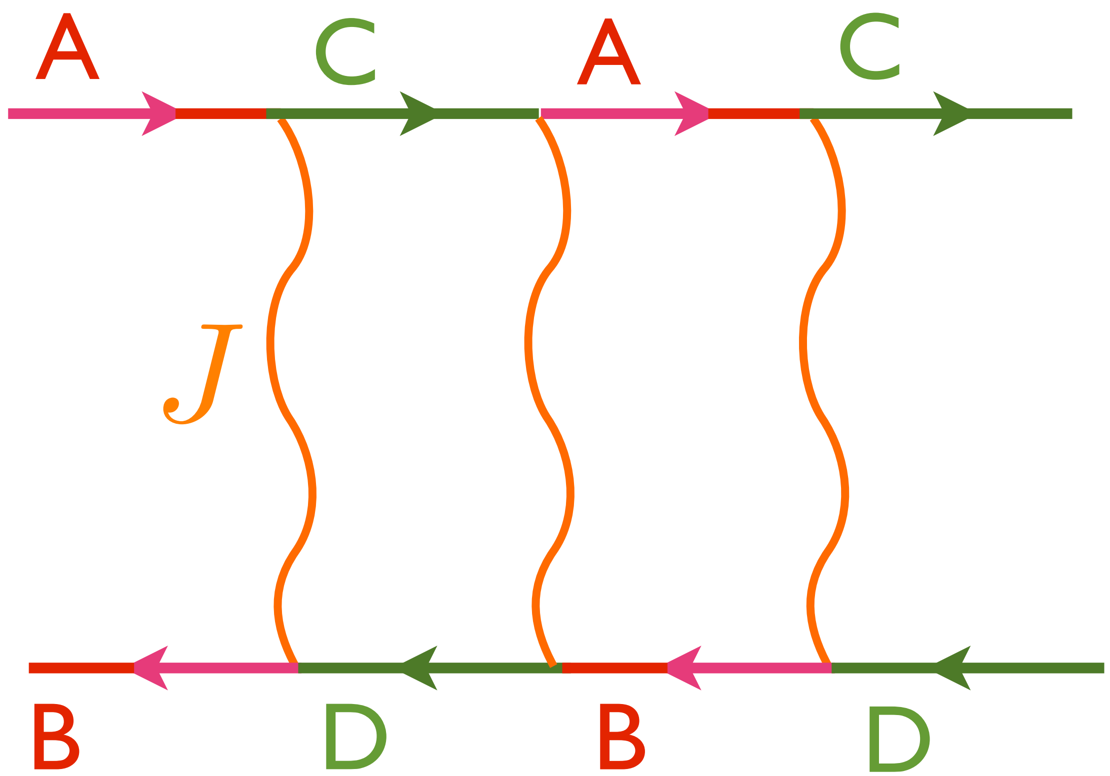
Same "glue" leads to particle-hole pairing !





M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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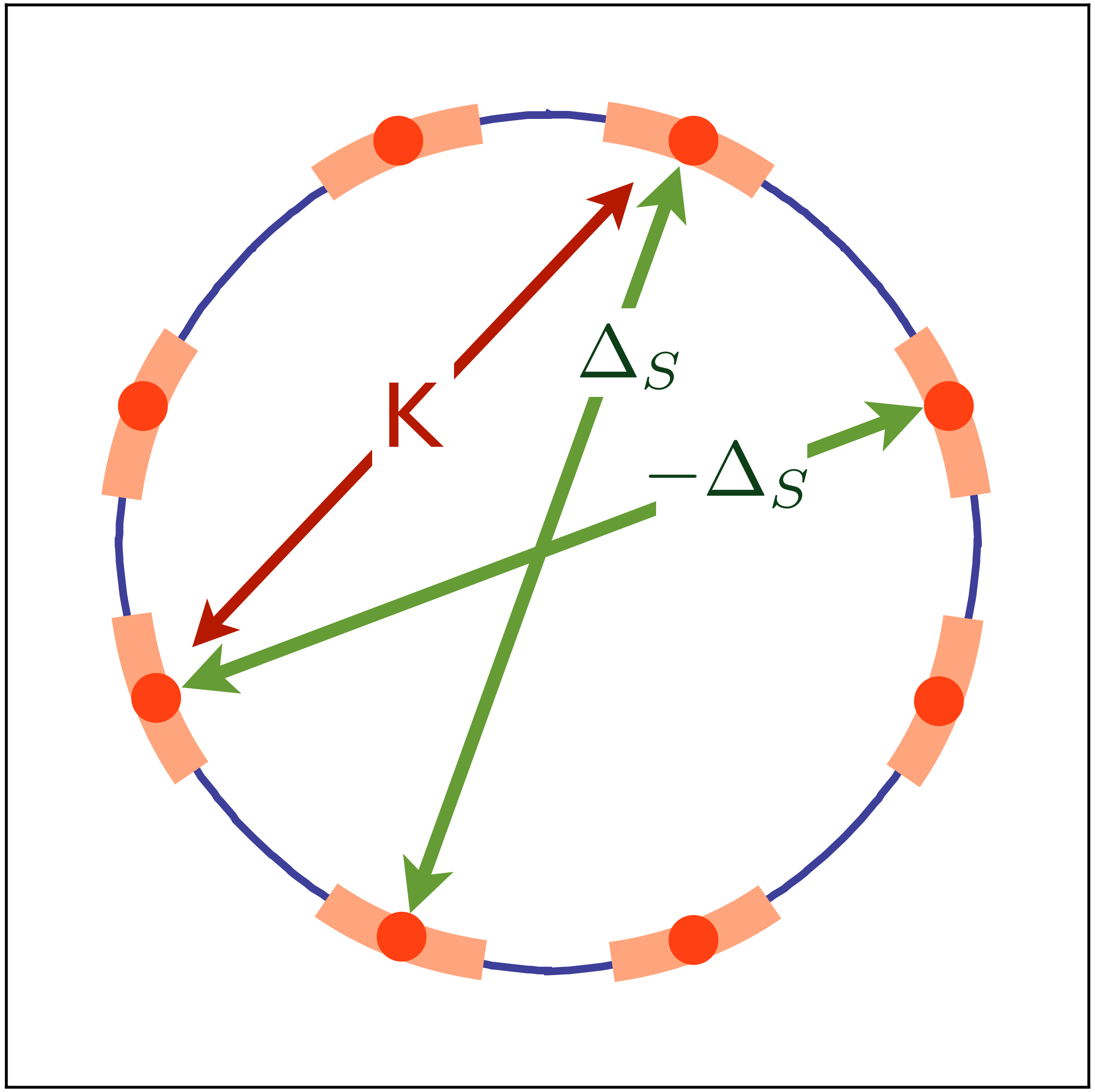
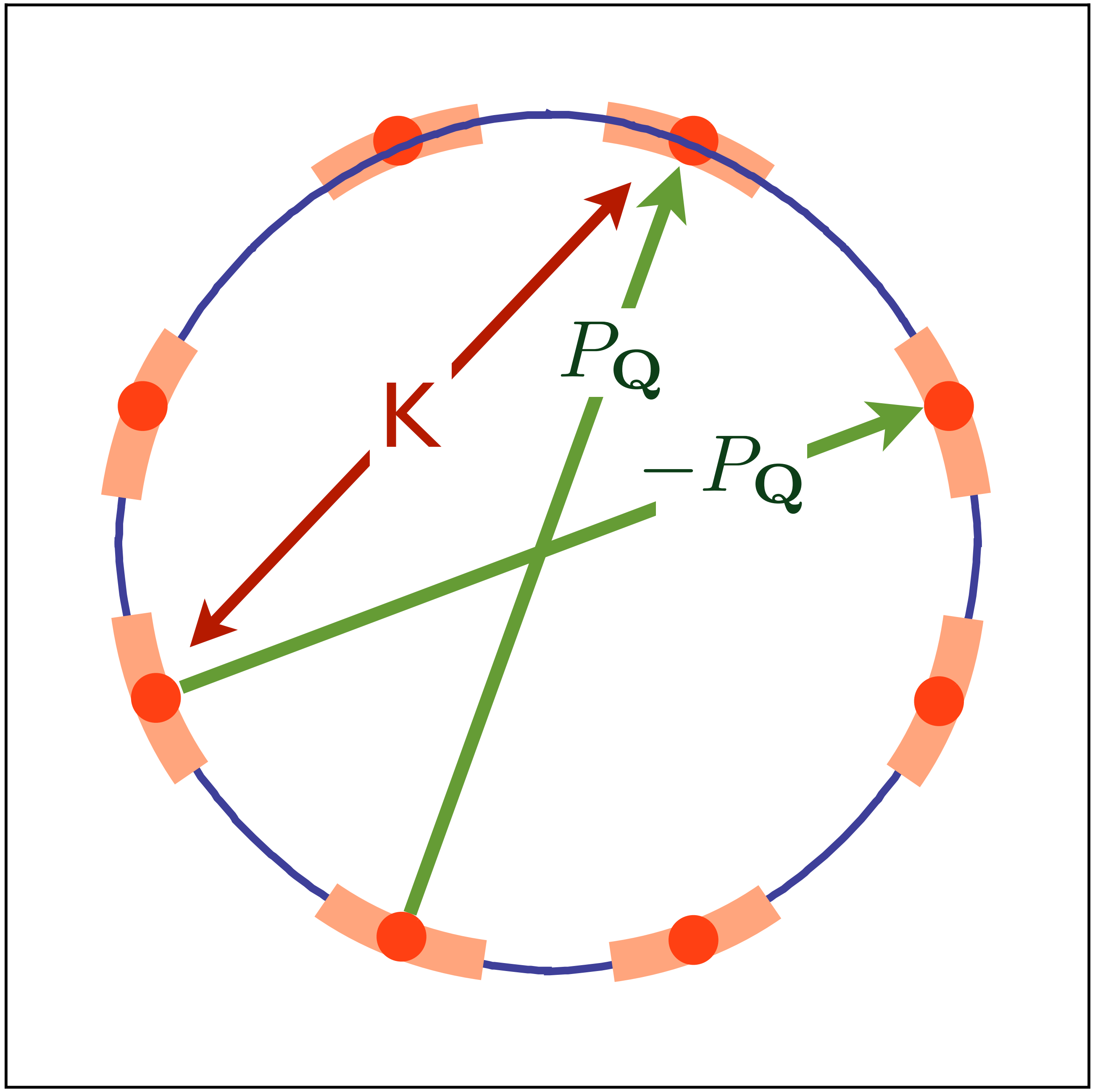


d-wave superconductor: particle-particle pairing at *and near* hot spots, with sign-changing pairing amplitude

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = P_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

After pseudospin rotation on *half* the hot-spots

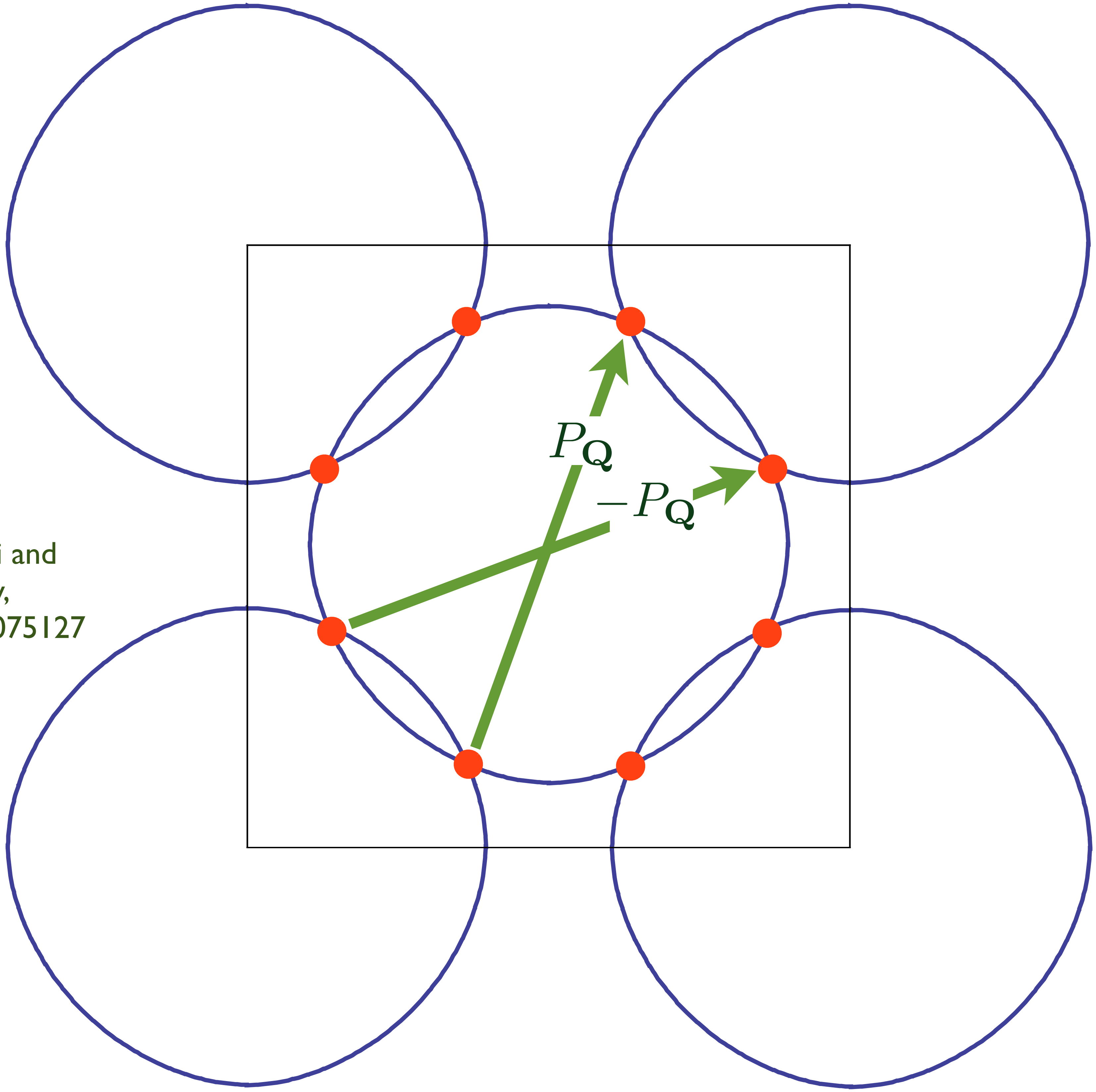
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



$\mathbf{Q}$ is '$2k_F$' wavevector

Incommensurate d-wave bond order: particle-hole pairing at ***and near*** hot spots, with sign-changing pairing amplitude

Incommensurate *d*-wave bond order



M. A. Metlitski and
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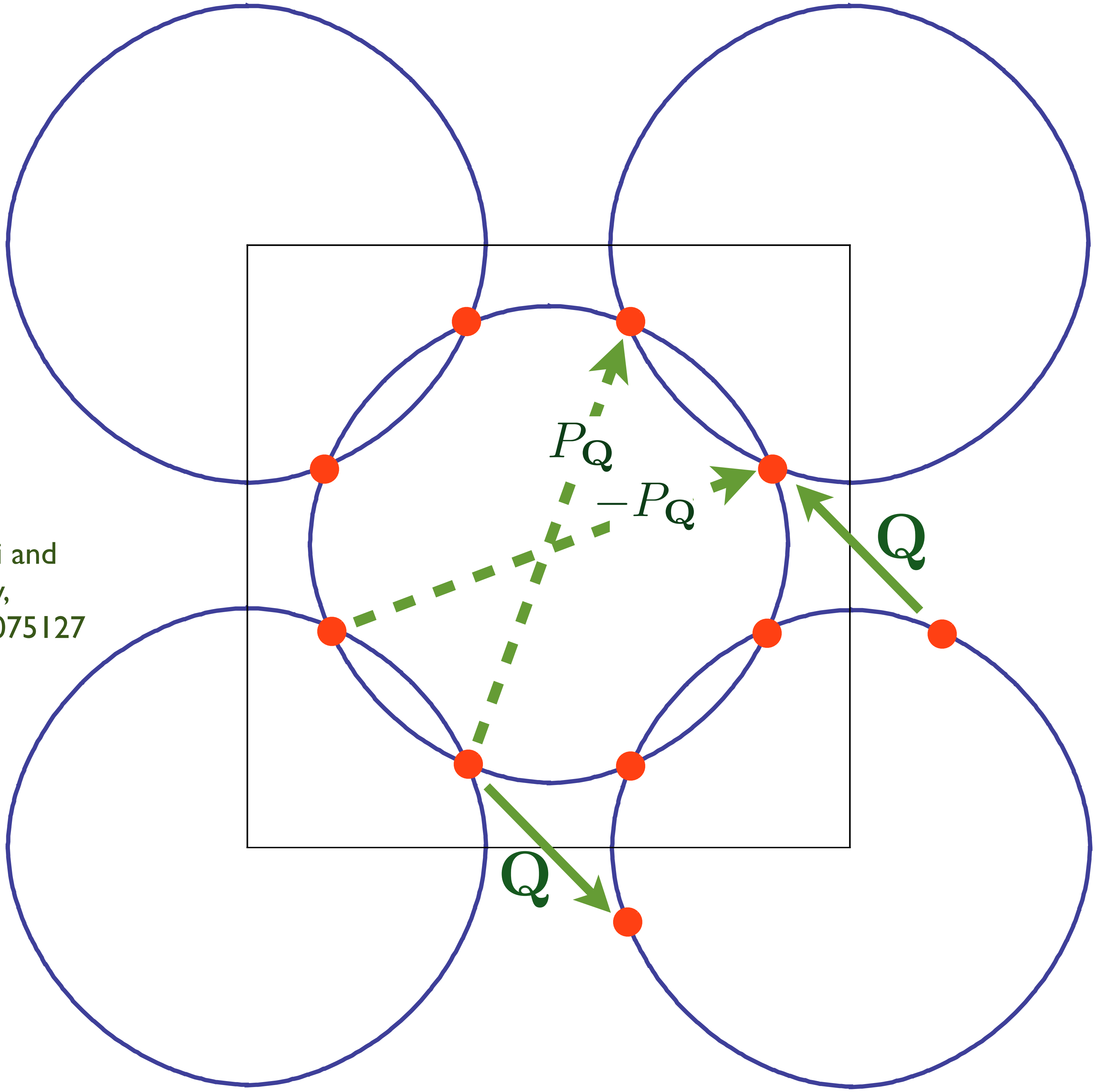

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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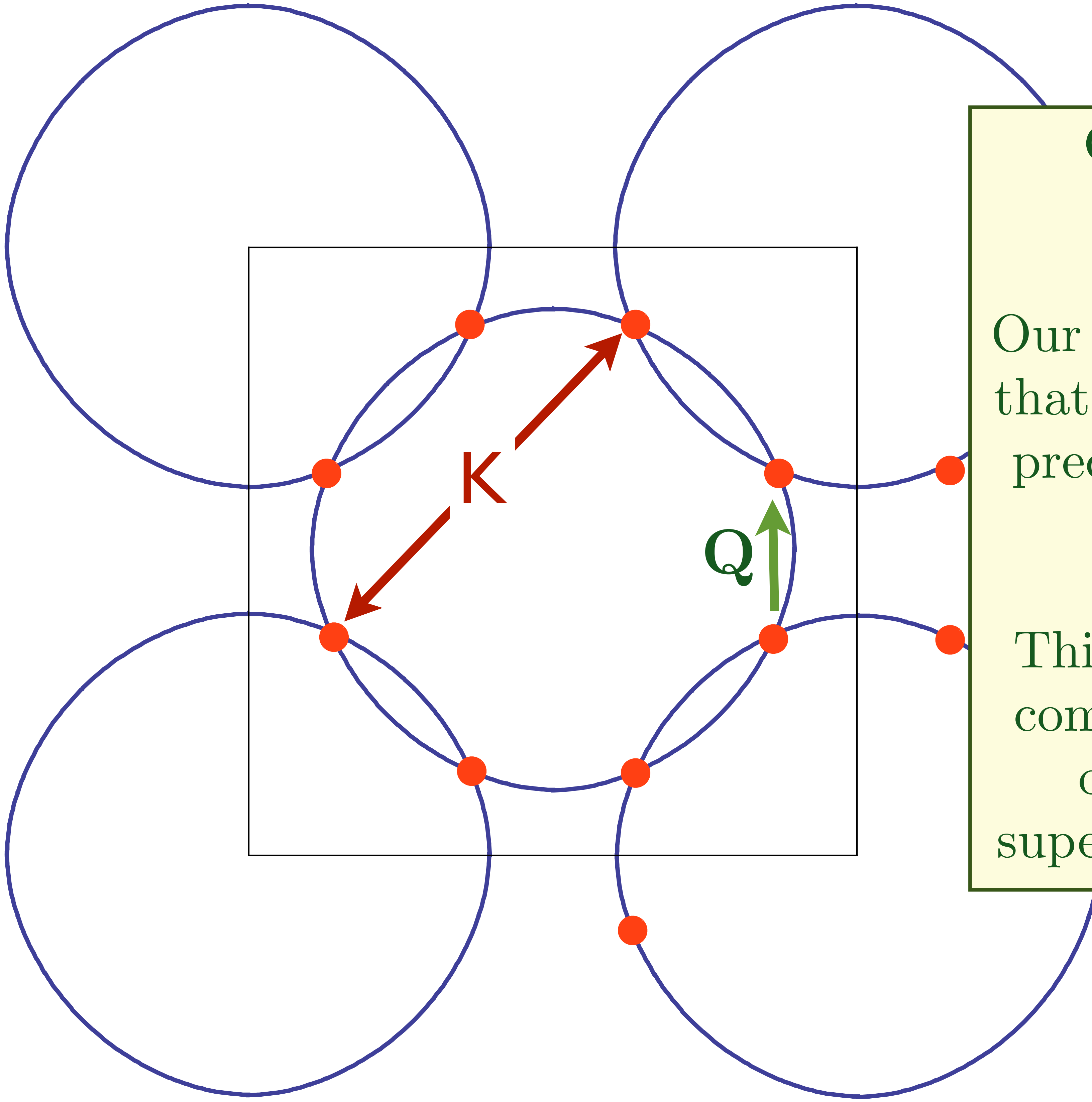

Observed low T ordering.

Our computations show that the charge order is predominantly d-wave also at this $\mathbf{Q}$.

This $\mathbf{Q}$ is preferred in computations of bond order within the superconducting phase.

S. Sachdev and R. La Placa, Physical Review Letters **111**, 027202 (2013)
M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
M. Vojta and O. Rosch, Physical Review B 77, 094504 (2008)

Multi-component order parameter

Superconducting order $\Psi(\mathbf{r})$:

$$\left\langle c_{i\alpha}^{\dagger} c_{j\beta}^{\dagger} \right\rangle = \varepsilon_{\alpha\beta} \left[\sum_{\mathbf{k}} \Delta_S(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] \Psi\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right)$$

Charge/bond order $\Phi_{x,y}(\mathbf{r})$ at wavevectors $\mathbf{Q}_{x,y}$:

$$\left\langle c_{i\alpha}^{\dagger} c_{j\beta} \right\rangle = \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_{\mathbf{Q}_x}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_x\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_x\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right)$$
$$+ \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_{\mathbf{Q}_y}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_y\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_y\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right)$$

Hamiltonian at and *near* hot spots

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Fermi surface curvature; breaks pseudospin symmetry

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Coulomb repulsion; breaks pseudospin symmetry

Perform standard Hartree-Fock-BCS factorizations into

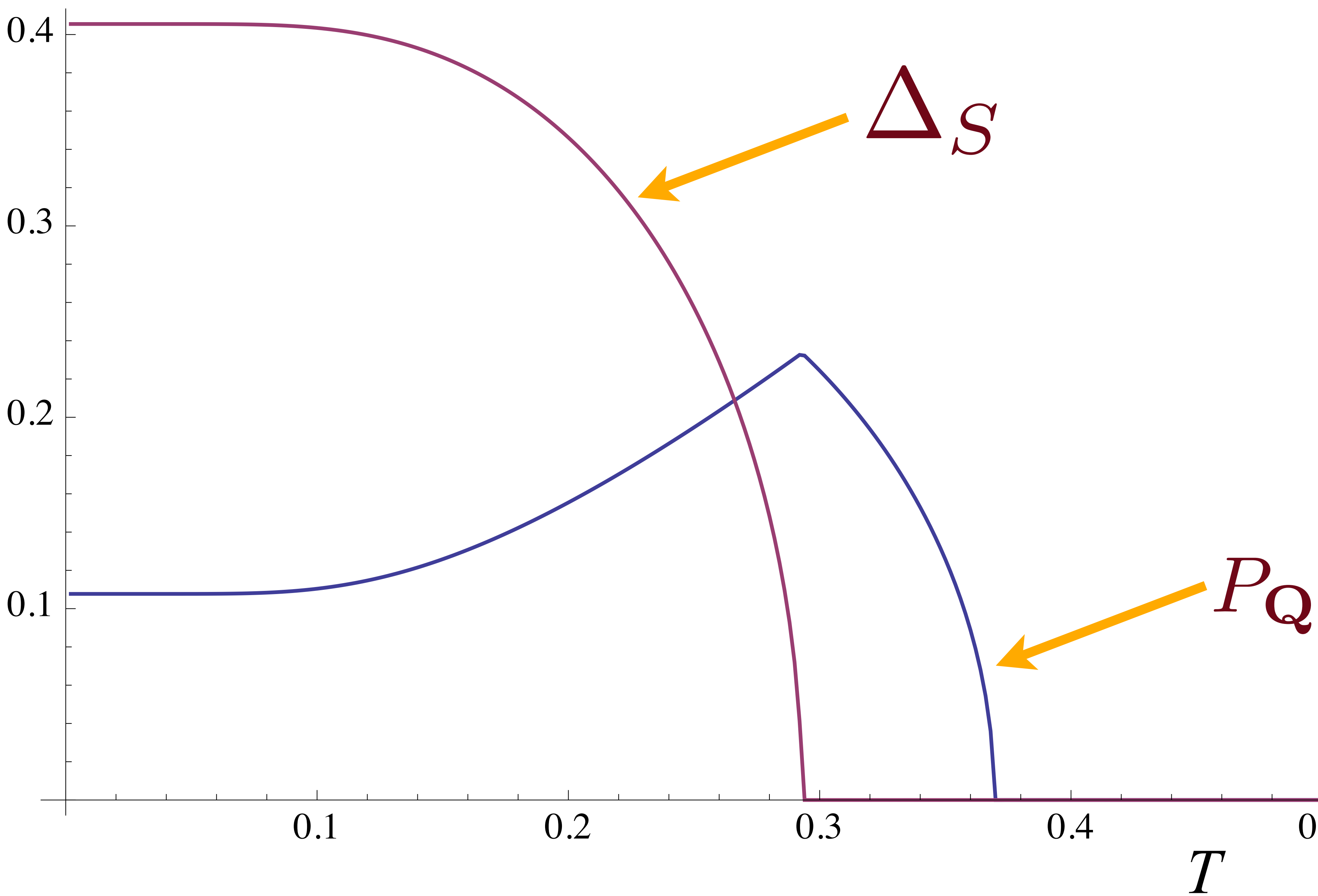
$$\Delta_S = \langle \varepsilon_{\alpha\beta} \Psi_{A\alpha} \Psi_{B\beta} \rangle = -\langle \varepsilon_{\alpha\beta} \Psi_{C\alpha} \Psi_{D\beta} \rangle$$

$$P_{\mathbf{Q}} = \left\langle \Psi^\dagger_{A\alpha} \Psi_{B\alpha} \right\rangle = -\left\langle \Psi^\dagger_{C\alpha} \Psi_{D\alpha} \right\rangle$$

With pseudospin symmetry, energy depends only on $|\Delta_S|^2 + |P_{\mathbf{Q}}|^2$.

Mean-field theory of charge/bond order and *d*-wave superconductivity

Both orders are induced by a "glue" provided by the antiferromagnetic exchange interaction



Jay Sau and S. Sachdev, to appear

Particle-hole instabilities of the full Fermi surface

$$H_{tJ} = -\sum_{i,j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Optimize the free energy w.r.t. a mean field Hamiltonian which allows for spin-singlet charge order ($P_{\mathbf{Q}}(\mathbf{k})$):

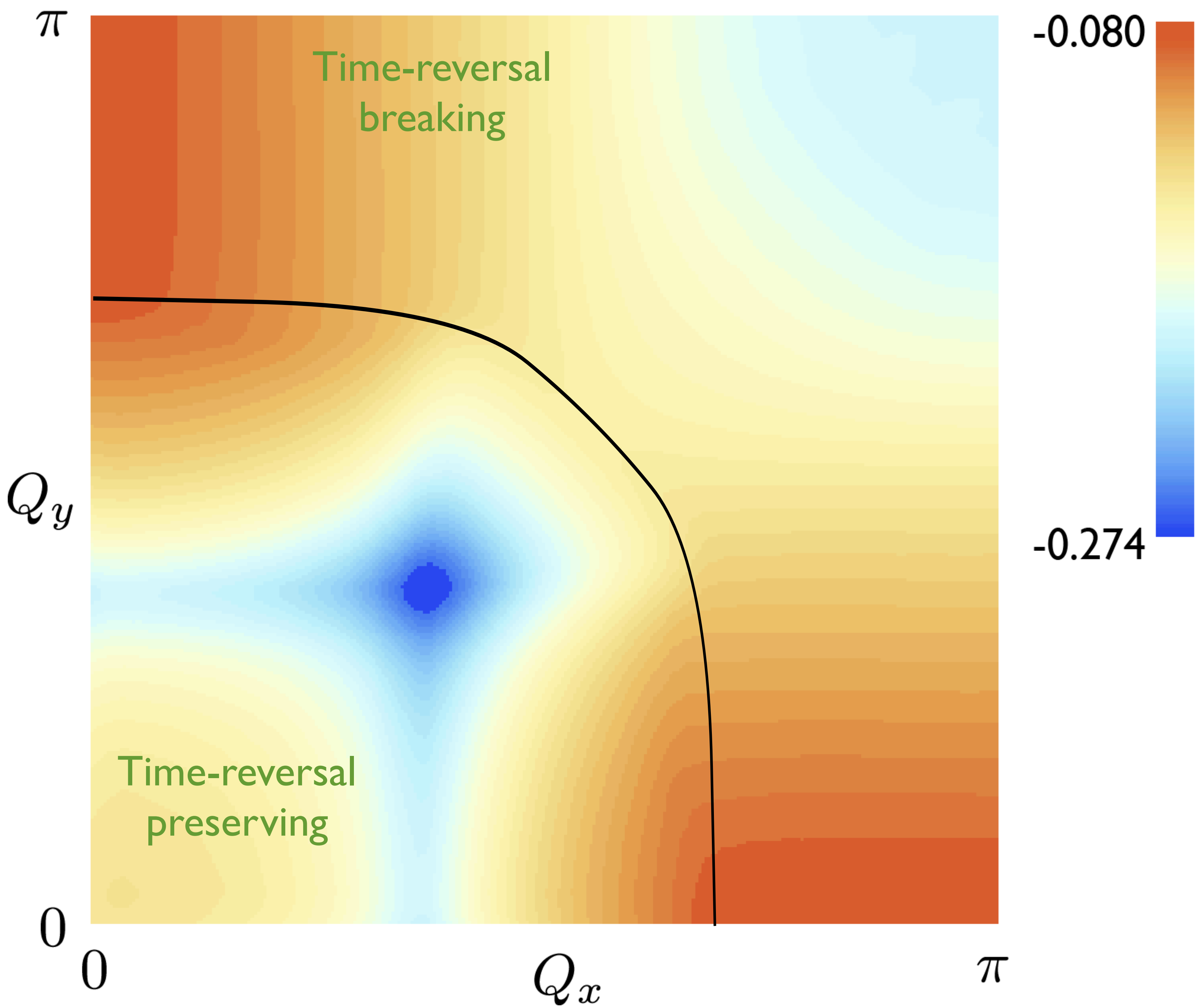
$$H_{MF} \quad = \quad -\sum_{i,j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + \sum_{\mathbf{k},\mathbf{Q}} P_{\mathbf{Q}}(\mathbf{k}) c_{\mathbf{k}+\mathbf{Q}/2,\alpha}^{\dagger} c_{\mathbf{k}-\mathbf{Q}/2,\alpha}$$

Expanding the free energy in powers of the order parameters we obtain

$$F = F_0 + \sum_{\mathbf{k},\mathbf{Q}} P_{\mathbf{Q}}^{*}(\mathbf{k}) \mathcal{M}_{\mathbf{Q}}(\mathbf{k},\mathbf{k}') P_{\mathbf{Q}}(\mathbf{k}')$$

We compute the eigenvalues, $1+\lambda_{\mathbf{Q}}$, and eigenfunctions, $P_{\mathbf{Q}}(\mathbf{k})$ of the kernel $\mathcal{M}_{\mathbf{Q}}(\mathbf{k},\mathbf{k}')$

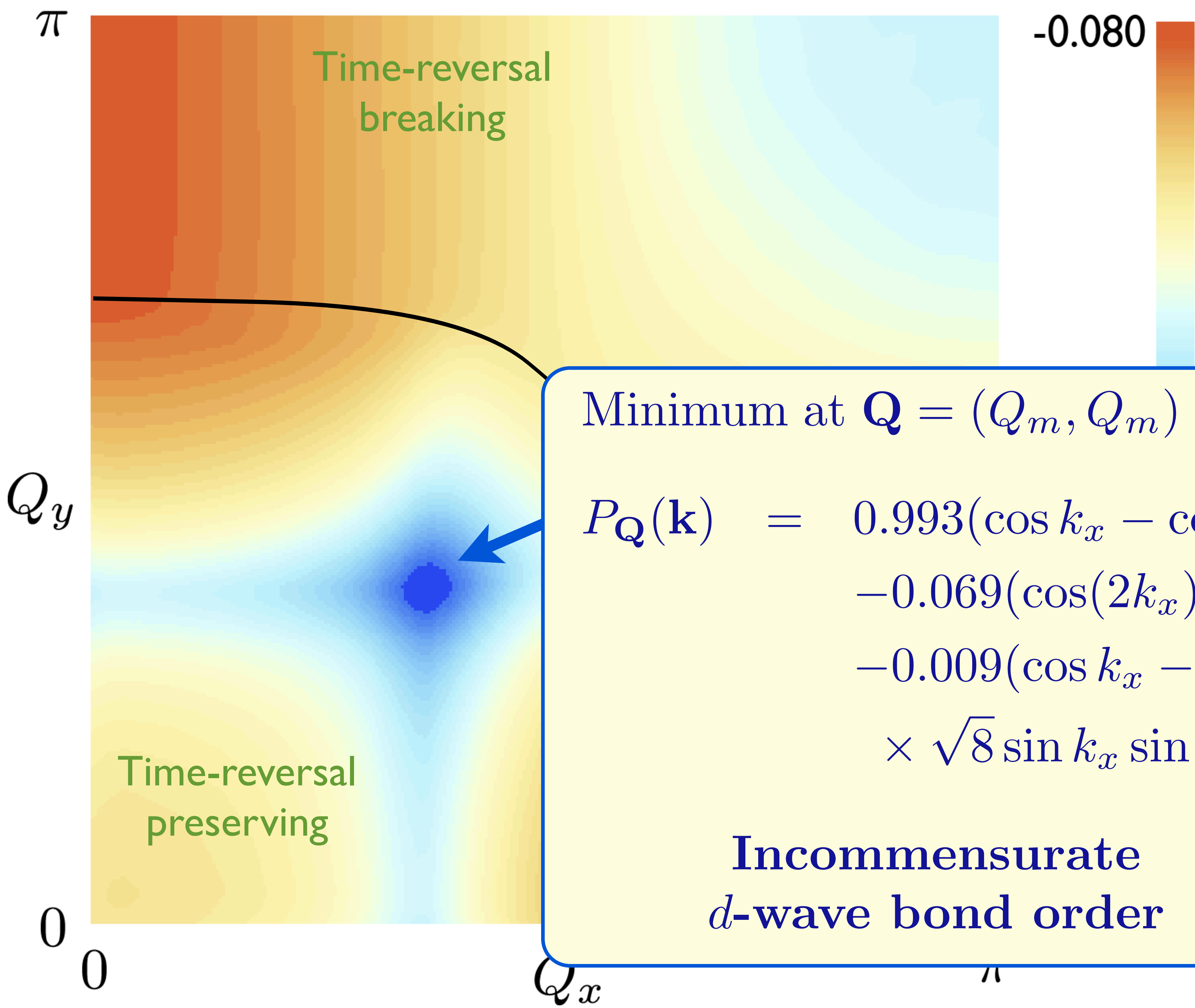
S. Sachdev and R. La Placa, Phys. Rev. Lett. 111, 027202 (2013)



Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$ of particle-hole propagator. The corresponding eigenvector is $P_{\mathbf{Q}}(\mathbf{k})$ and this leads to the order

$$\left\langle c_{i\alpha}^\dagger c_{j\alpha} \right\rangle = \left[\sum_{\mathbf{k}} P_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}\right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)



Minimum at $\mathbf{Q} = (Q_m, Q_m)$ with

$$
\begin{aligned}
P_{\mathbf{Q}}(\mathbf{k}) \;=\; & 0.993(\cos k_x - \cos k_y) \\
& -0.069(\cos(2k_x) - \cos(2k_y)) \\
& -0.009(\cos k_x - \cos k_y) \\
& \times \sqrt{8} \sin k_x \sin k_y
\end{aligned}
$$

Incommensurate
***d*-wave bond order**

Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$ of particle-hole propagator. The corresponding eigenvector is $P_{\mathbf{Q}}(\mathbf{k})$ and this leads to the order

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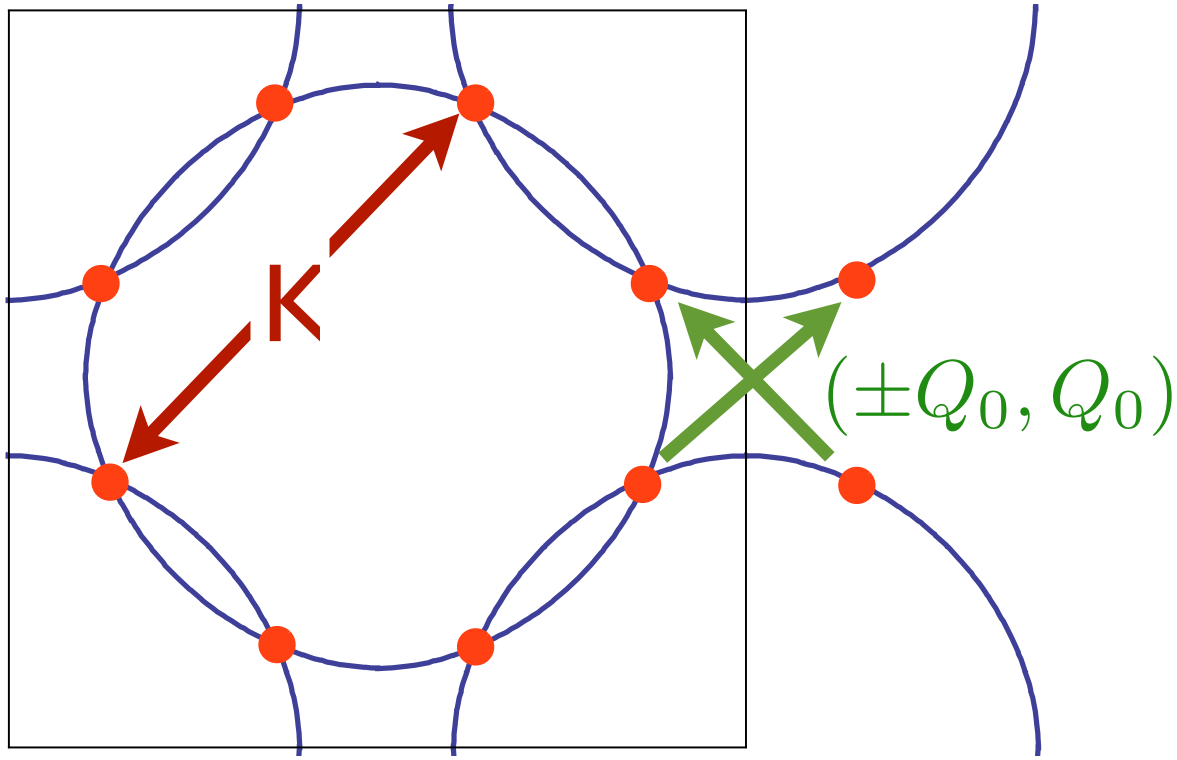
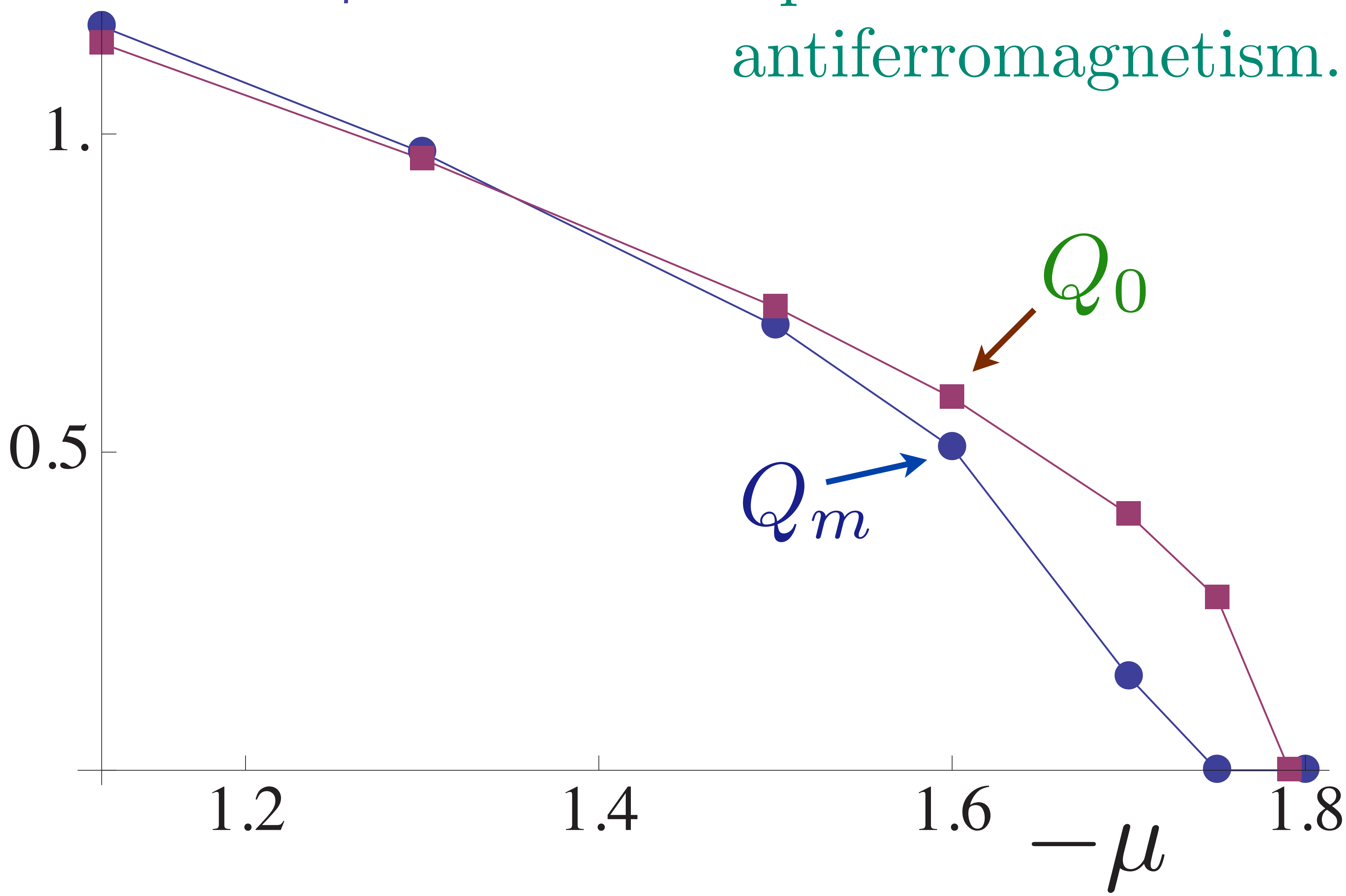
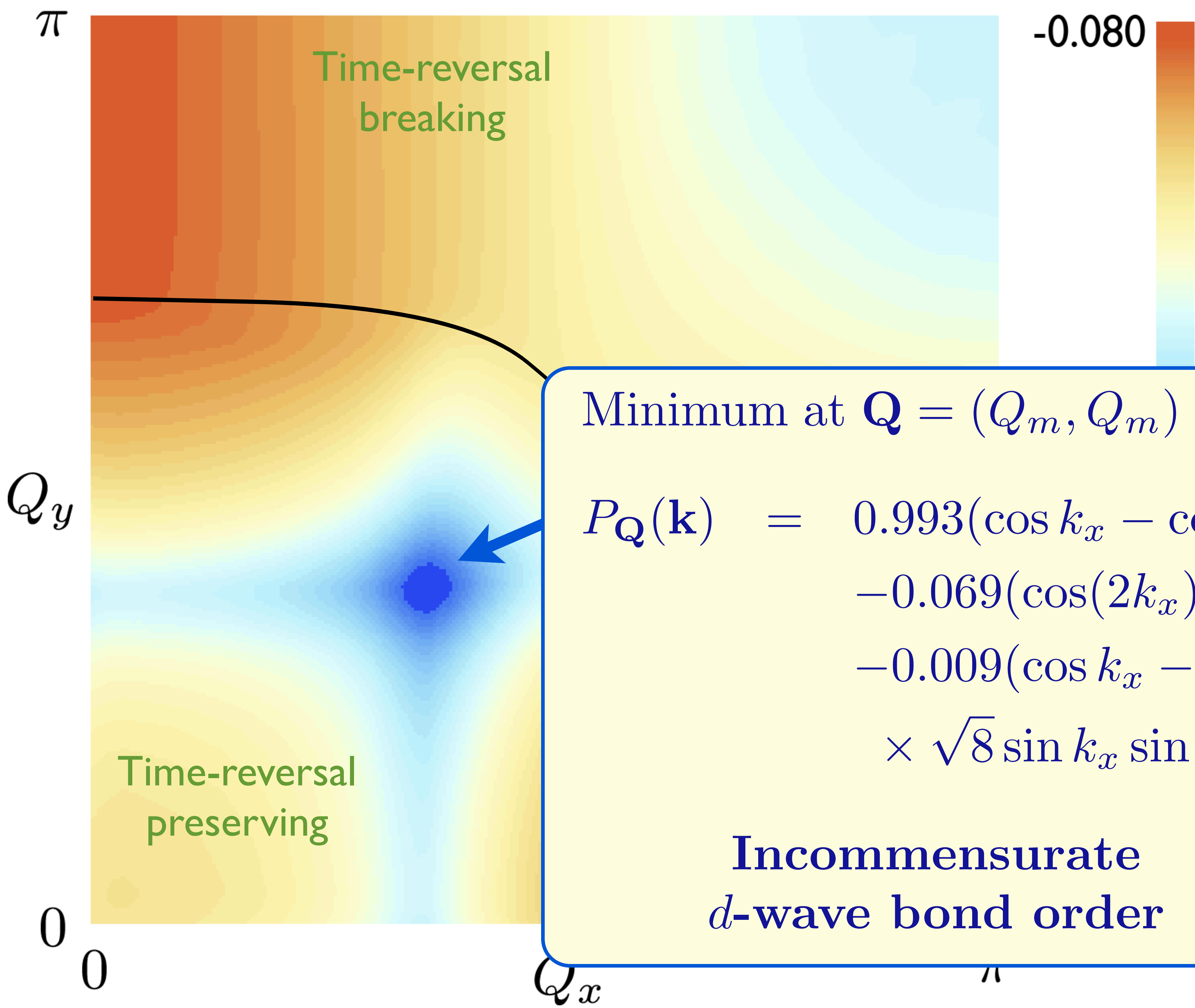


Remarkable agreement between the value of Q_m from Hartree-Fock in a metal with short-range *incommensurate* spin correlations, and the value of Q_0 from hot spots of *commensurate* antiferromagnetism.



S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)



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Incommensurate
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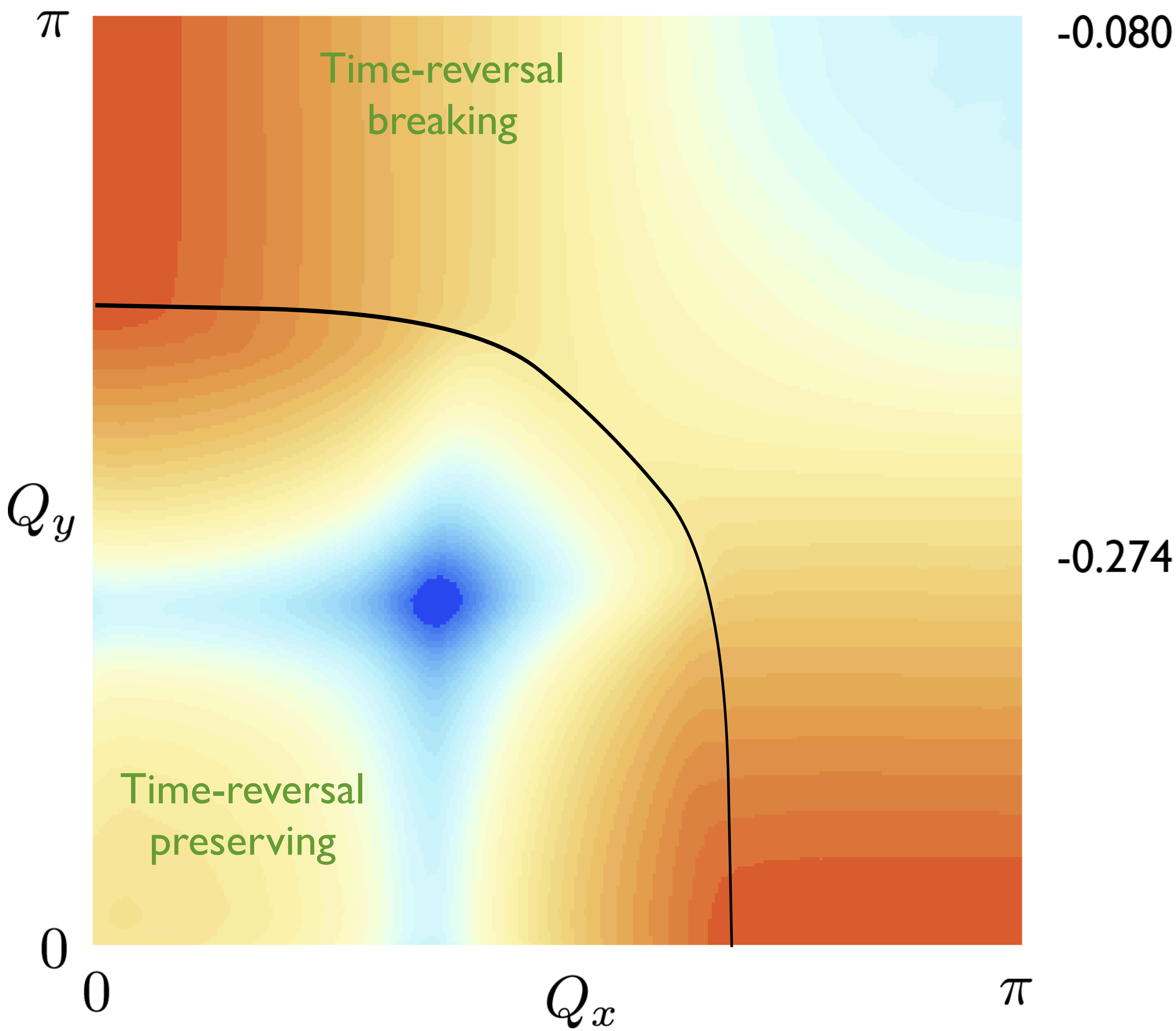

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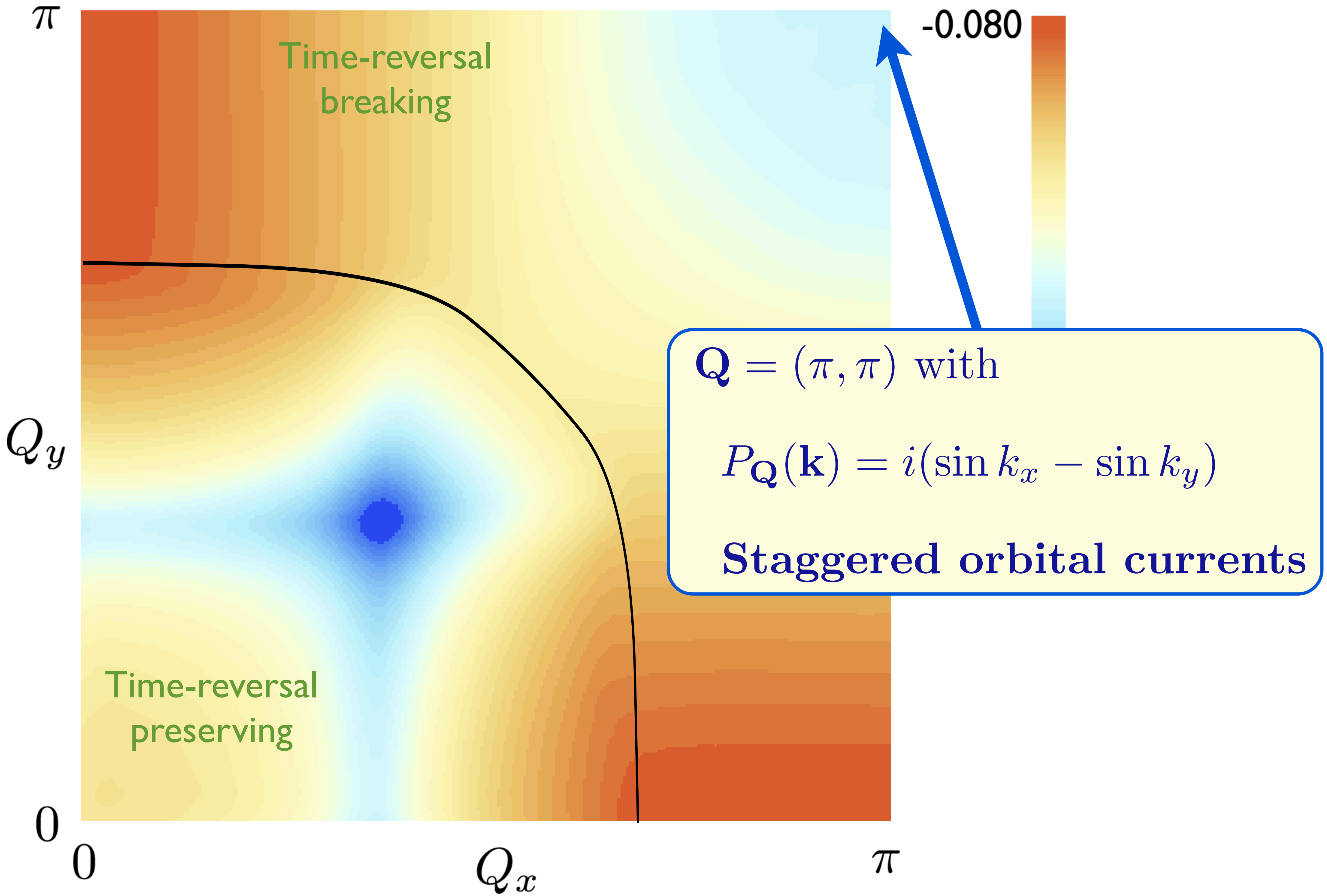

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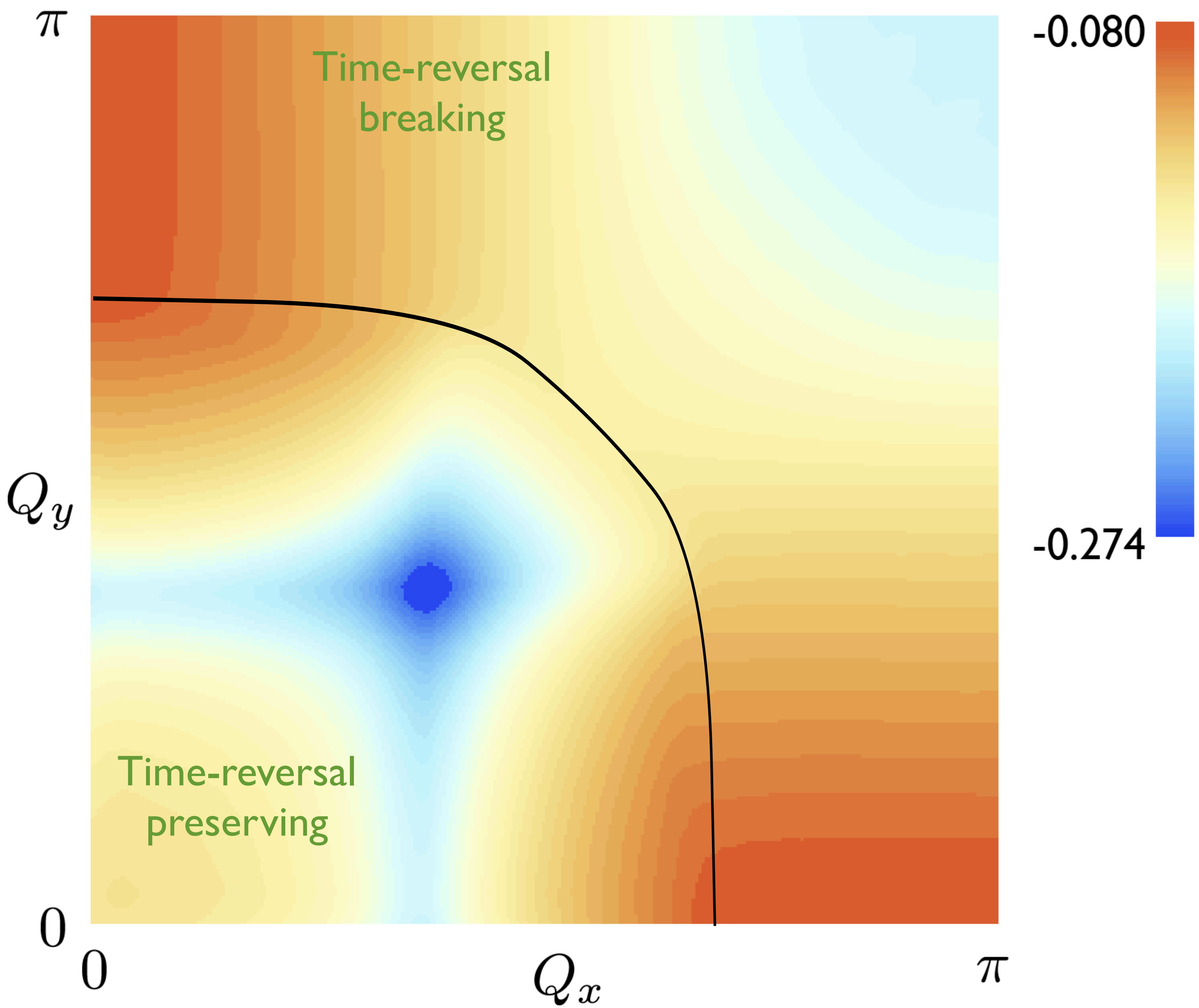

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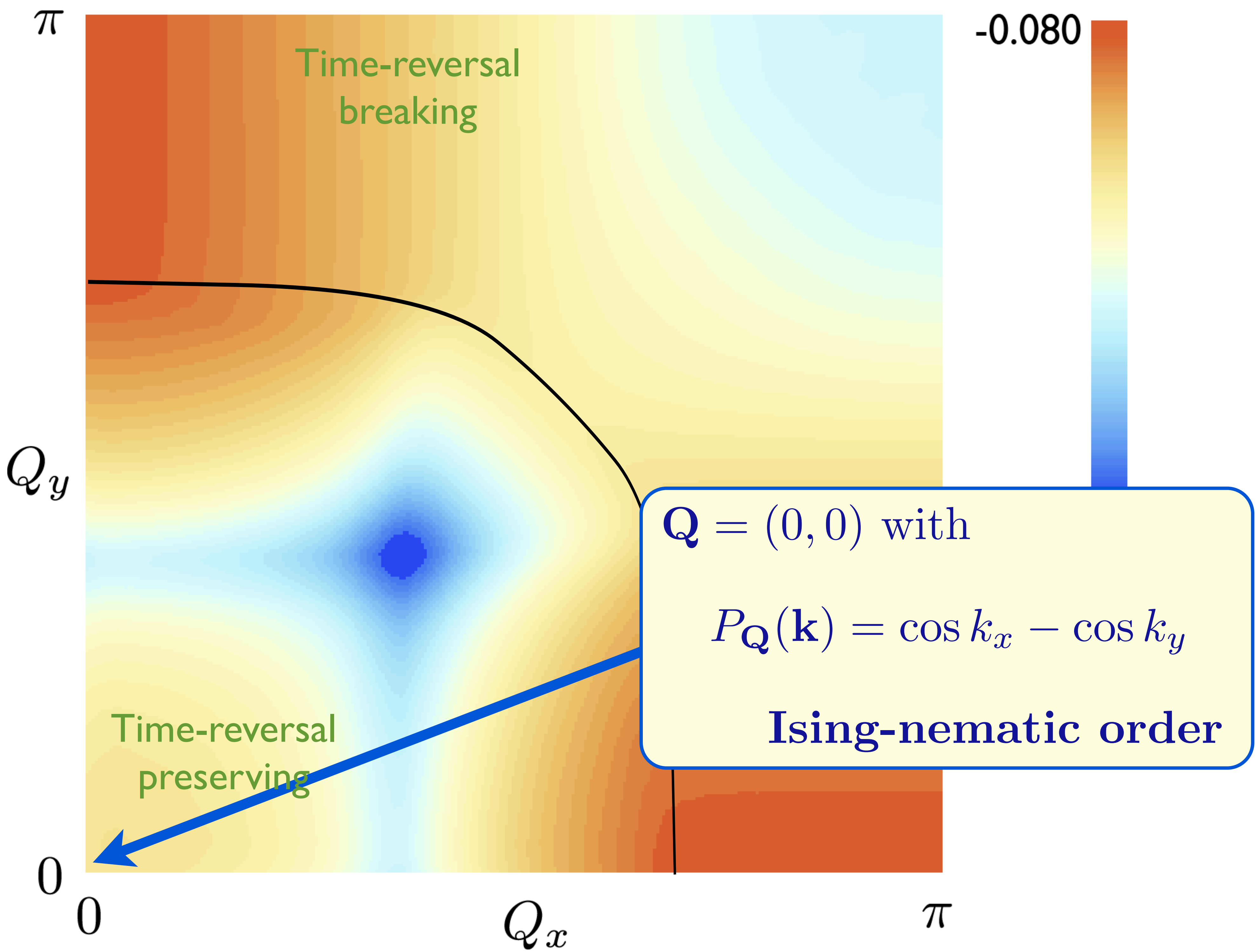

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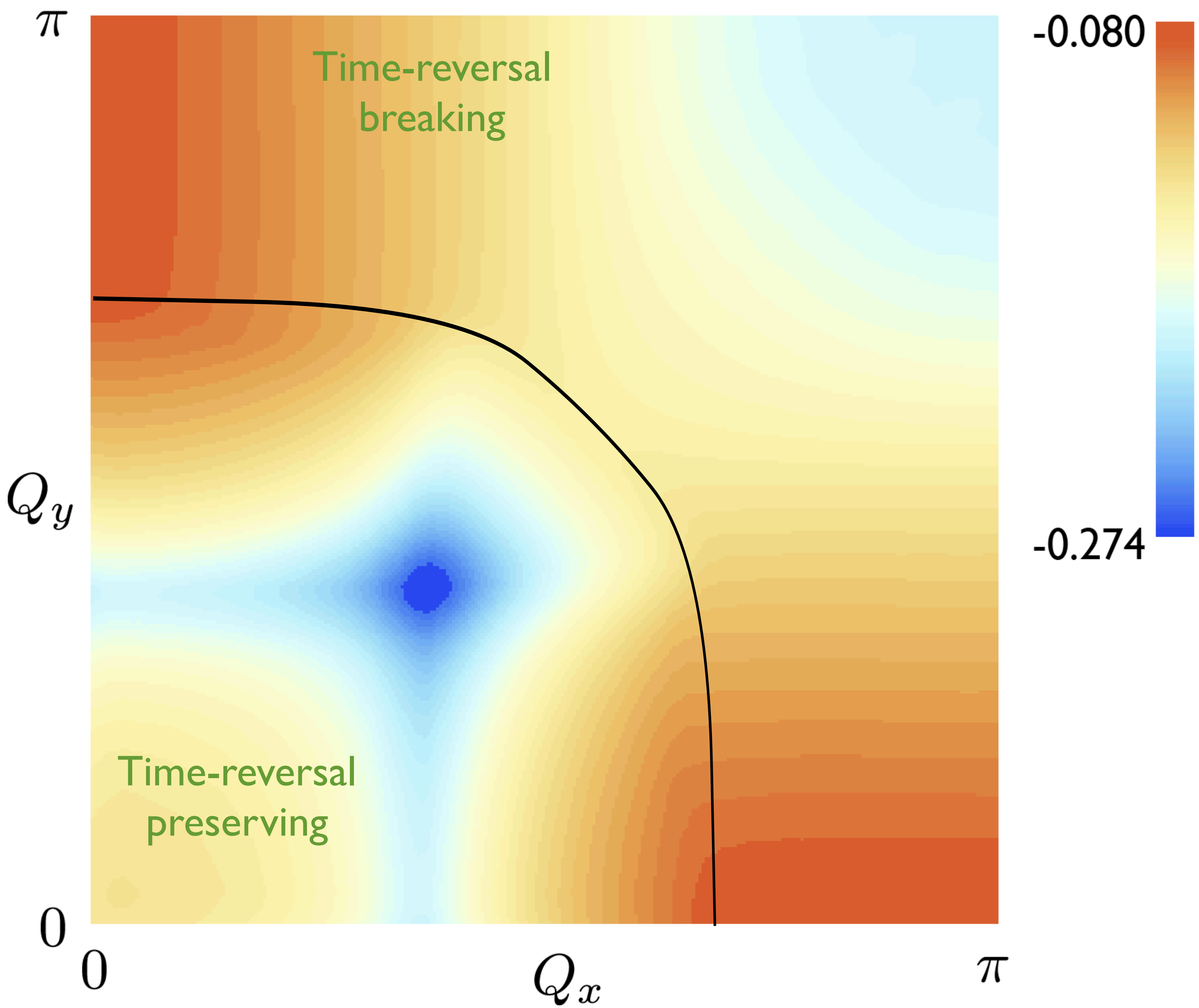

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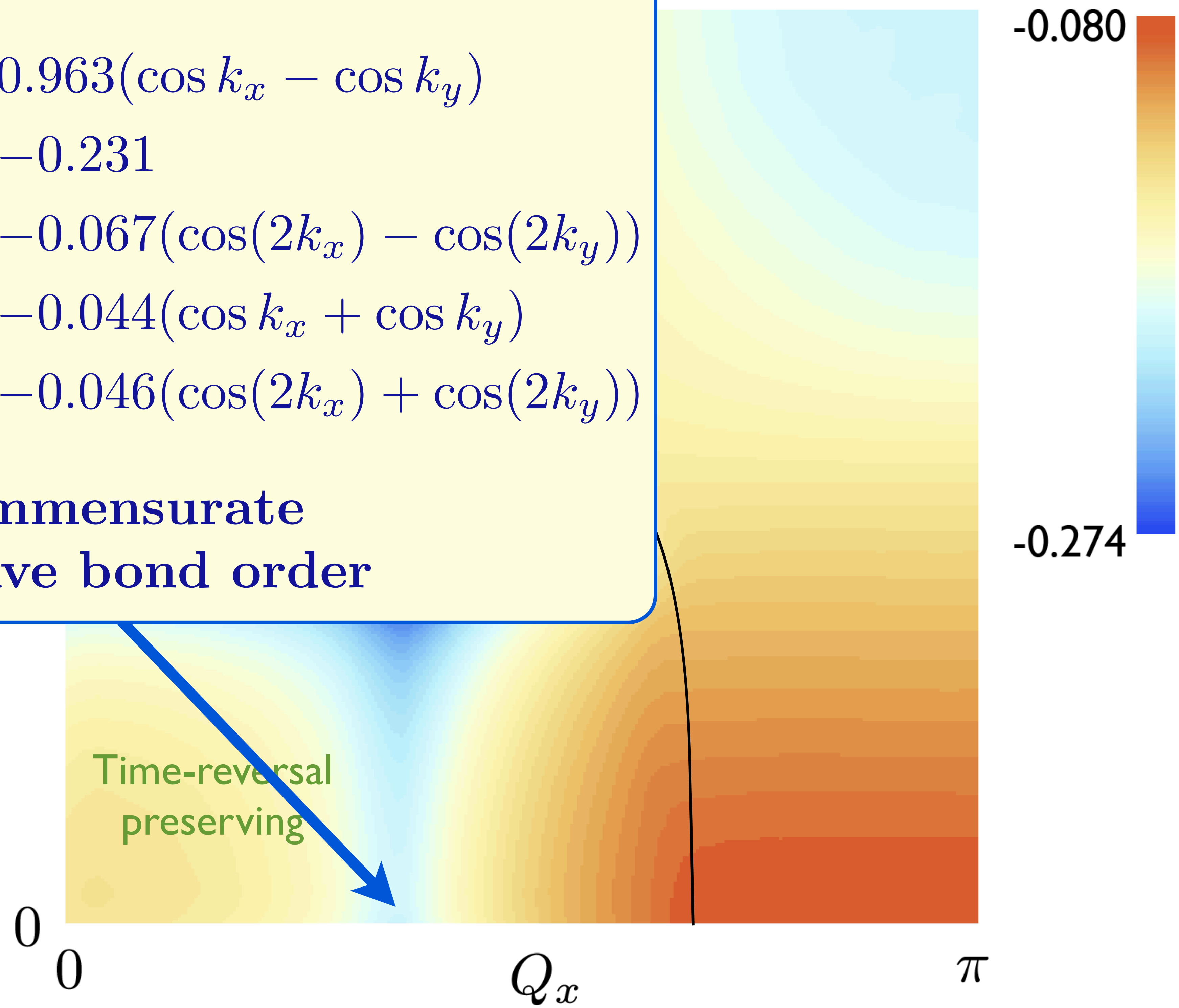
2013)

$\mathbf{Q} = (Q_m, 0)$ with

$$
\begin{aligned}
P_{\mathbf{Q}}(\mathbf{k}) \quad = \quad & 0.963(\cos k_x - \cos k_y) \\
& -0.231 \\
& -0.067(\cos(2k_x) - \cos(2k_y)) \\
& -0.044(\cos k_x + \cos k_y) \\
& -0.046(\cos(2k_x) + \cos(2k_y))
\end{aligned}
$$

Incommensurate
d_{+s} **-wave bond order**



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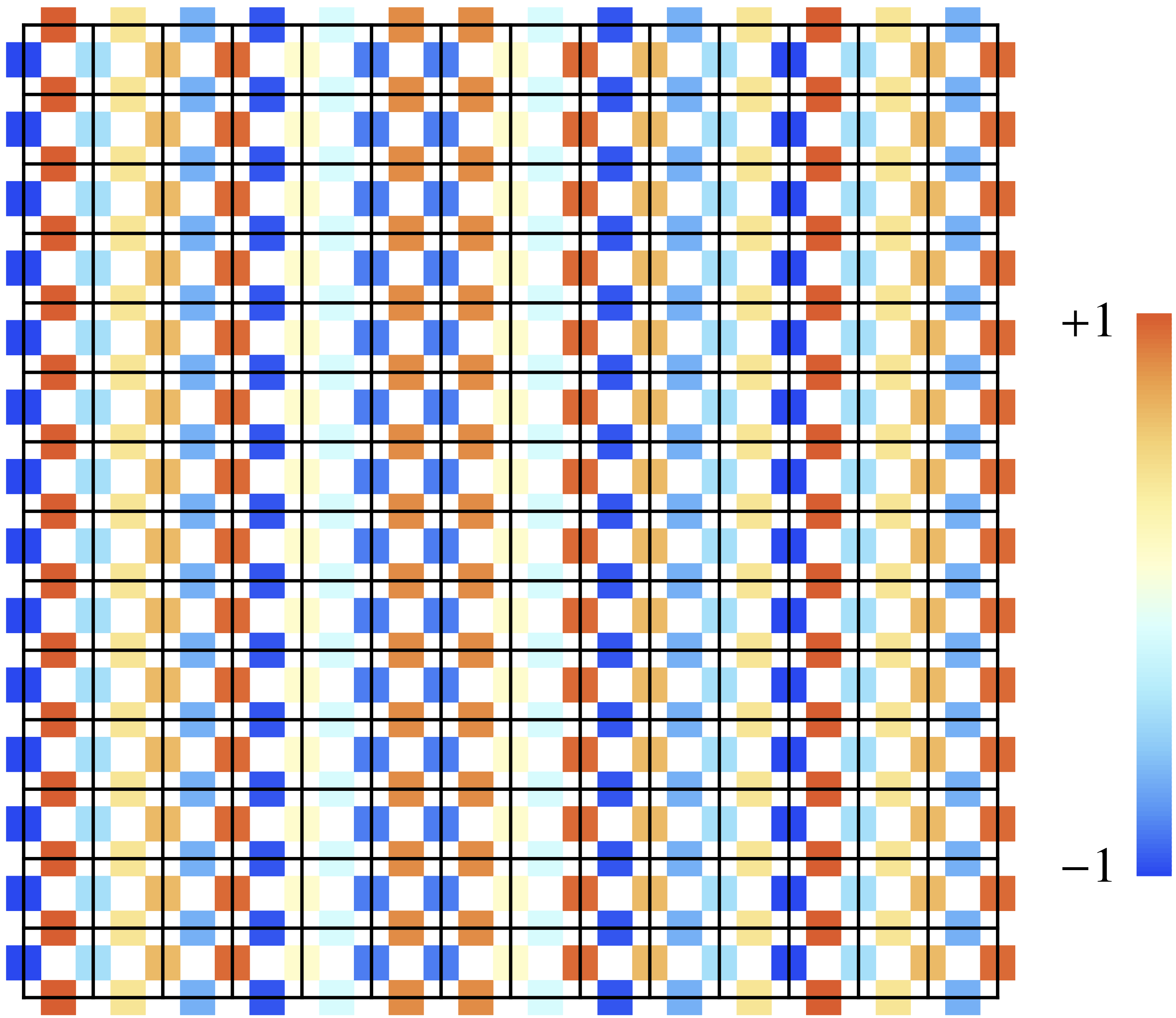
Incommensurate *d*-wave bond order

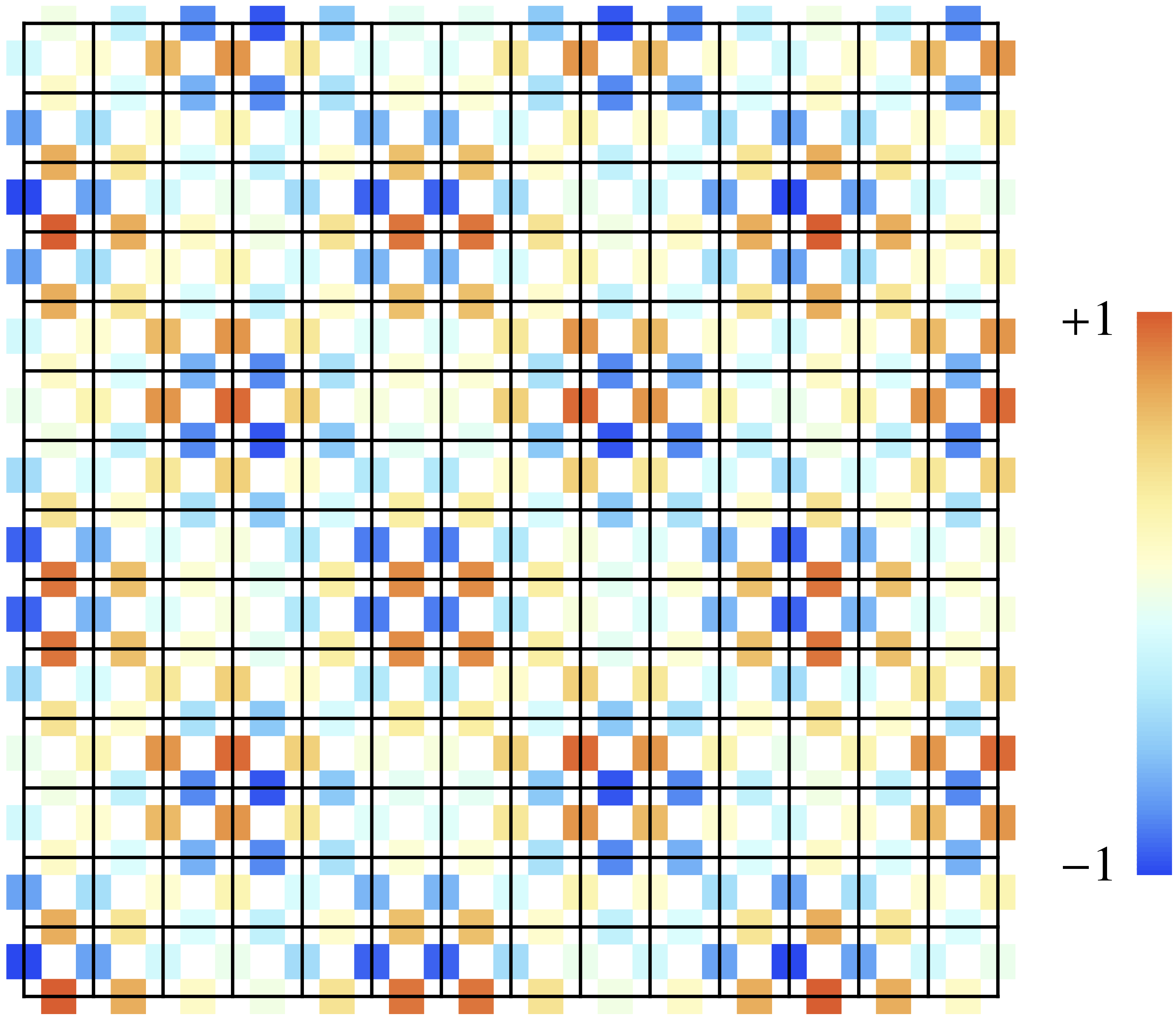
Plot of $P_{ij} = \left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle$ for i, j nearest neighbors.



Unidirectional order along $\mathbf{Q} = (4\pi/11, 0)$

Incommensurate *d*-wave bond order

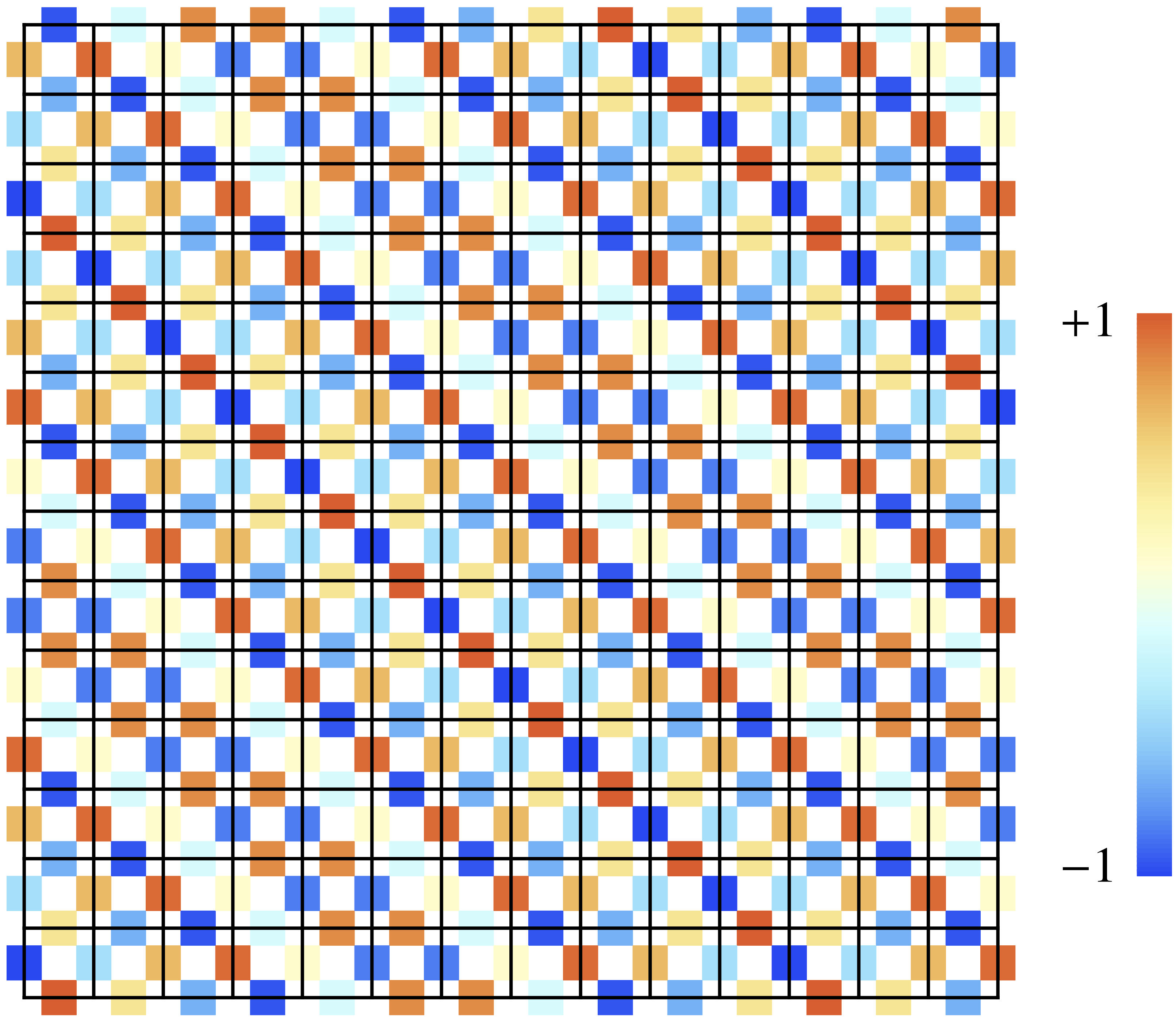
Plot of $P_{ij} = \left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle$ for i, j nearest neighbors.



Bi-directional order along $\mathbf{Q} = (4\pi/11, 0)$ and $(0, 4\pi/11)$.

Incommensurate *d*-wave bond order

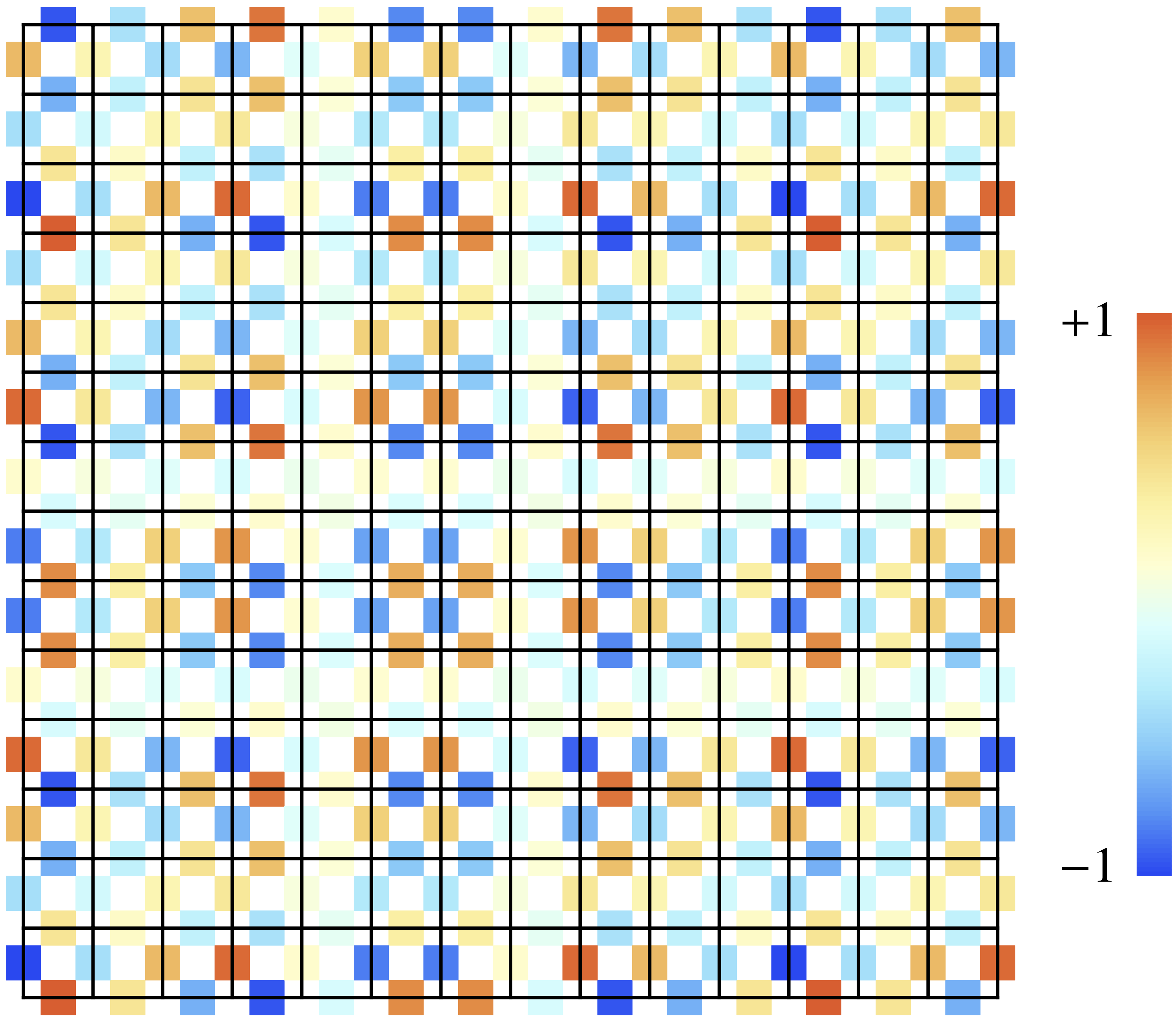
Plot of $P_{ij} = \left\langle c_{i\alpha}^\dagger c_{j\alpha} \right\rangle$ for i, j nearest neighbors.



Unidirectional order along $\mathbf{Q} = (4\pi/11, 4\pi/11)$

Incommensurate *d*-wave bond order

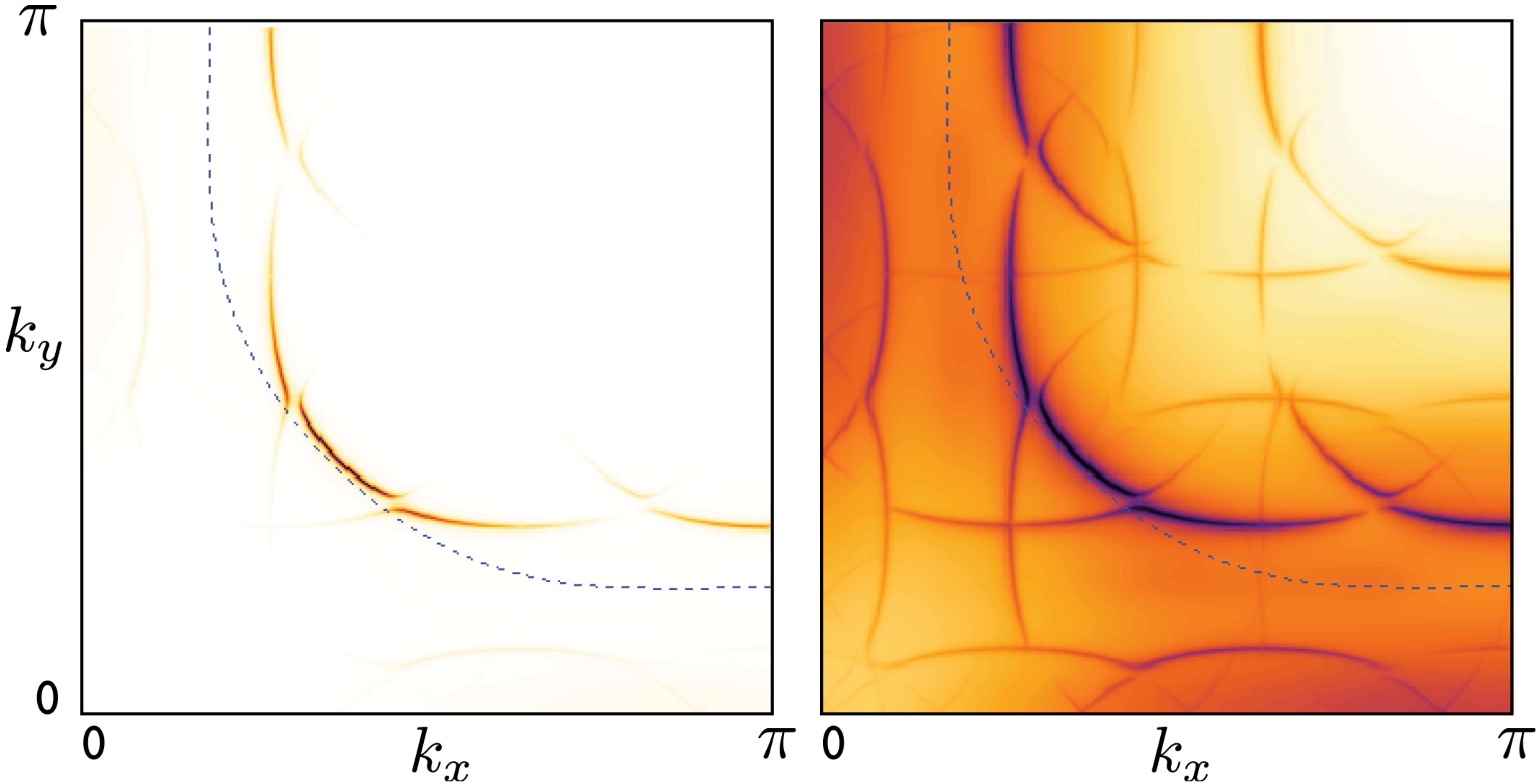
Plot of $P_{ij} = \left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle$ for i, j nearest neighbors.



Bi-directional order along $\mathbf{Q} = (4\pi/11, 4\pi/11)$ and $(4\pi/11, -4\pi/11)$

Incommensurate *d*-wave bond order

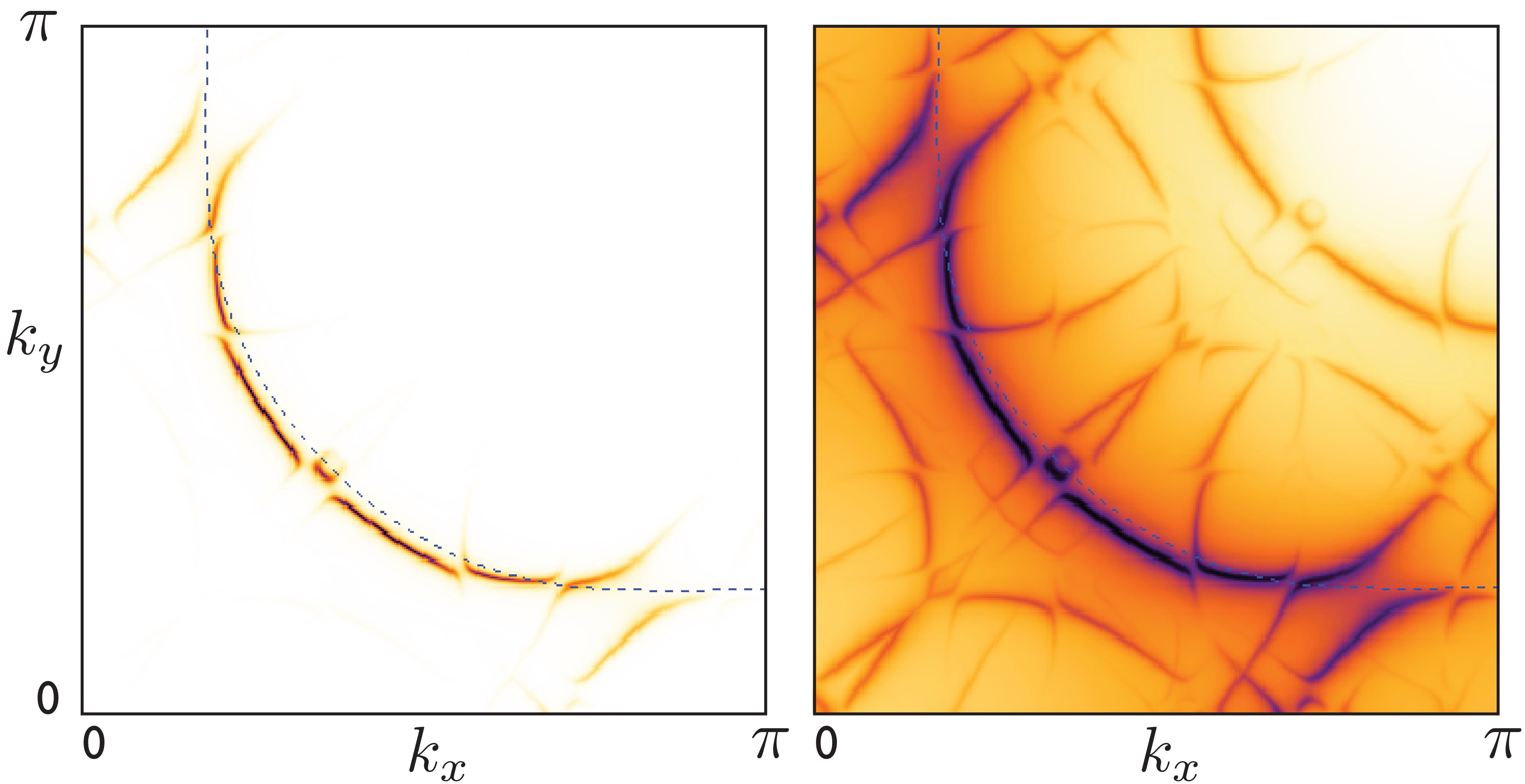
Photoemission spectrum $\mathrm{Im} G(\mathbf{k}, \omega = i0^+)$;
right panel shows $\log \mathrm{Im} G(\mathbf{k}, \omega = i0^+)$



Bi-directional order along $\mathbf{Q} = (4\pi/11, 0)$ and $(0, 4\pi/11)$.

Incommensurate *d*-wave bond order

Photoemission spectrum $\mathrm{Im}G(\mathbf{k}, \omega = i0^+)$;
right panel shows $\log \mathrm{Im}G(\mathbf{k}, \omega = i0^+)$



Bi-directional order along $\mathbf{Q} = (4\pi/11, 4\pi/11)$ and $(4\pi/11, -4\pi/11)$

- There is an approximate pseudospin symmetry in metals with antiferromagnetic spin correlations.

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- The pseudospin partner of *d*-wave superconductivity is an incommensurate *d*-wave bond order

- These orders form a pseudospin doublet, whose fluctuations lead to the "pseudogap" phase, described by the angular fluctuations of an order parameter with 6 real components.