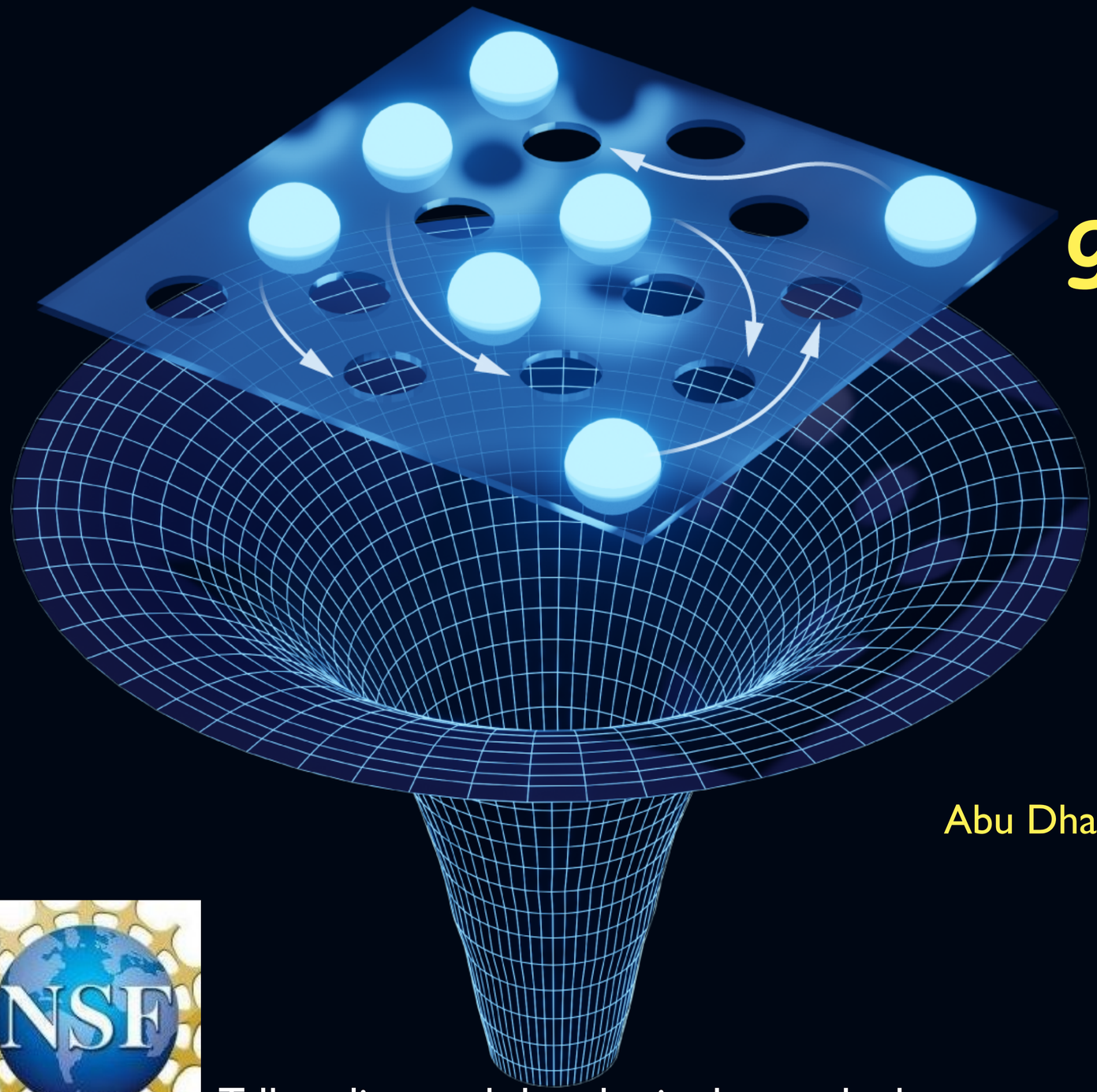

Experiments on graphene flakes and strange metals: insights from the SYK model

Abu Dhabi Meeting on Theoretical Physics
NYU Abu Dhabi
January 10, 2023

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS
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HARVARD

The SYK model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

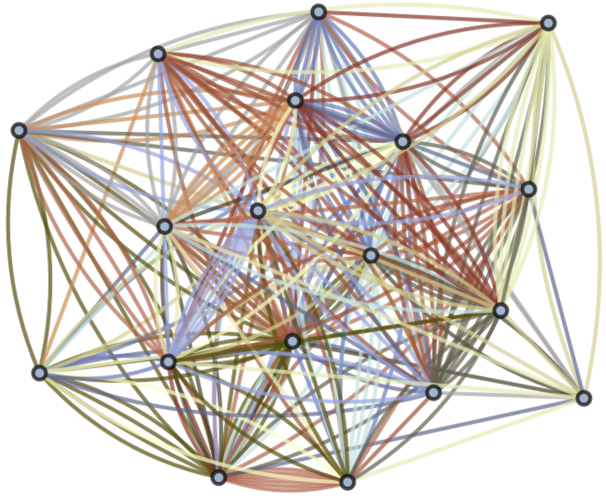
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \, ; \quad [\mathcal{H}, \mathcal{Q}] = 0 \, ; \quad 0 \leq \mathcal{Q} \leq 1$$

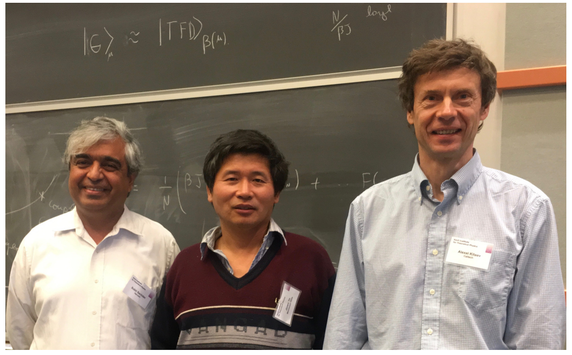
$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





Graphene flakes and the SYK model

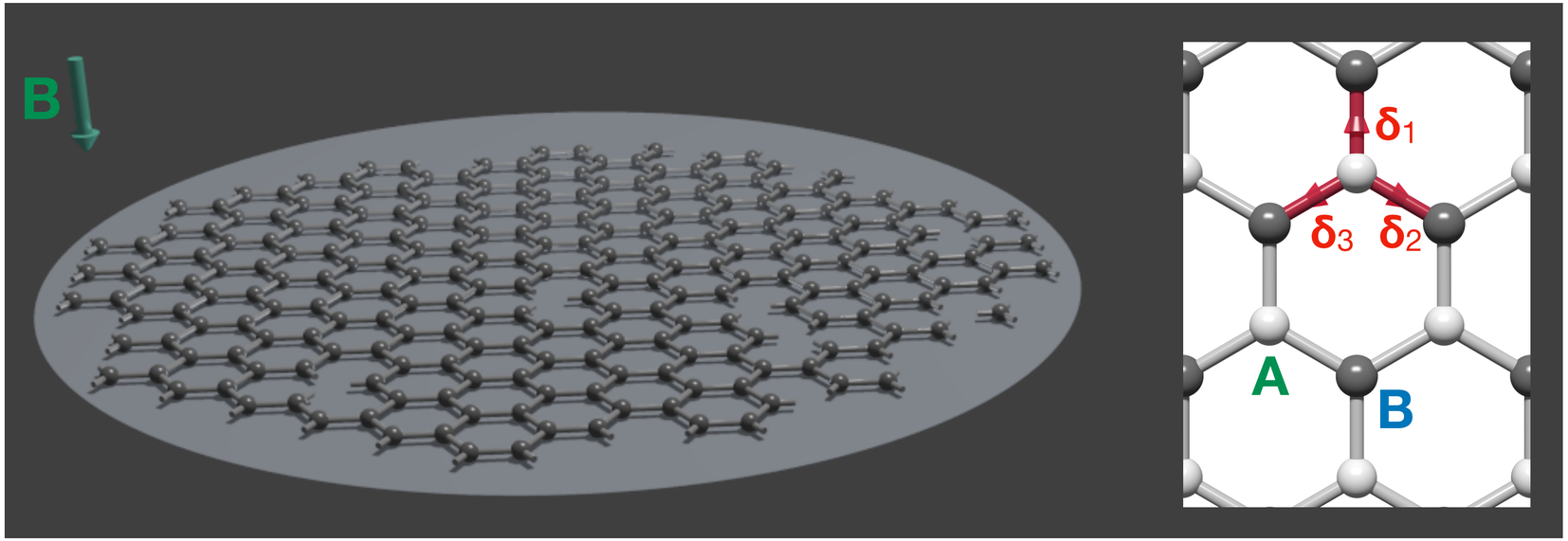
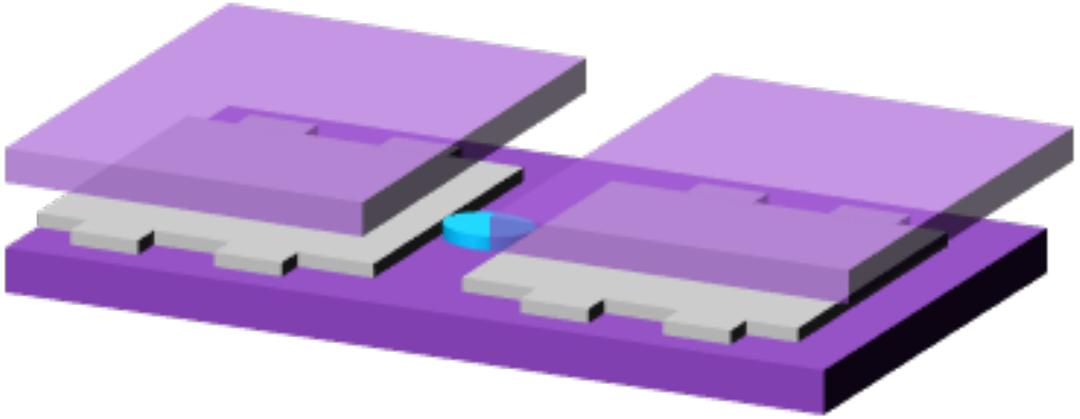


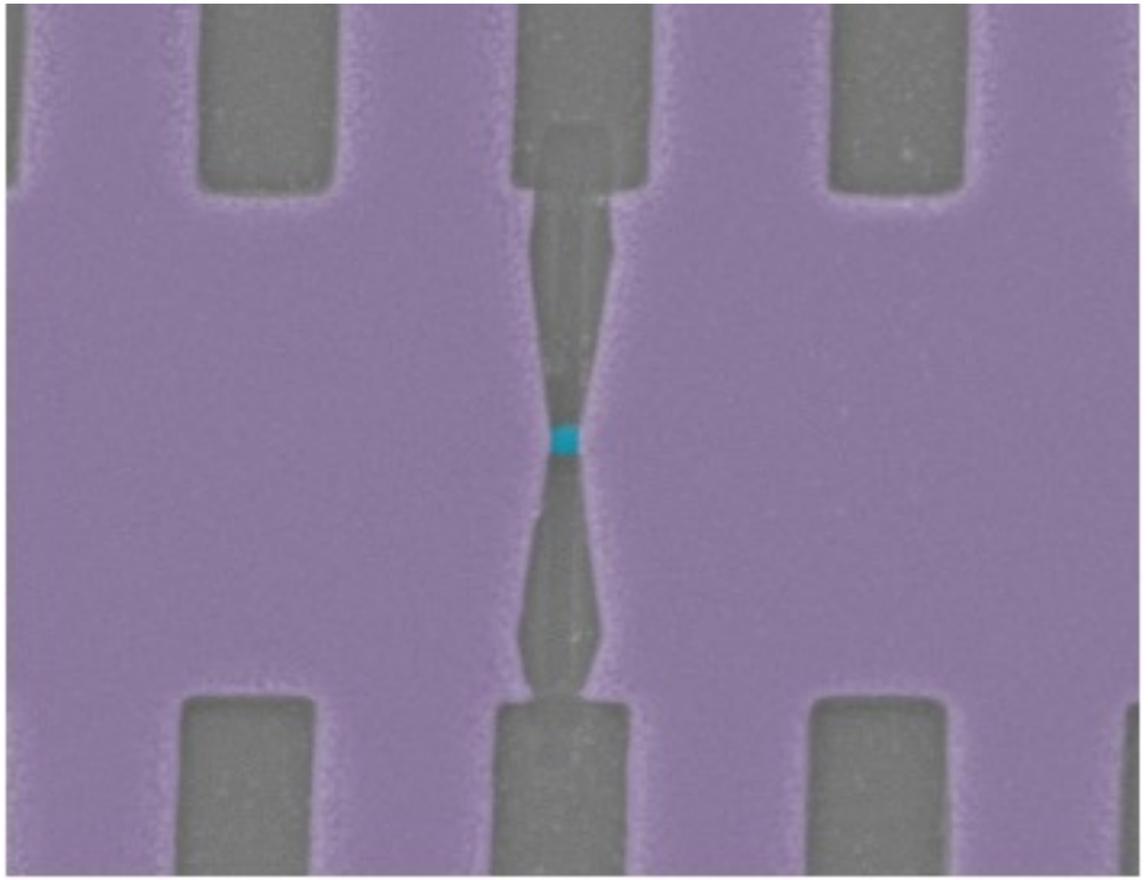
FIG. 1. Schematic depiction of the proposed device. Irregular shaped graphene flake in applied magnetic field B forms the (0+1) dimensional many-body system equivalent to a black hole in (1+1) anti-de Sitter space. Inset: lattice structure of graphene with A and B sublattices marked and nearest neighbor vectors denoted by $\boldsymbol{\delta}_j$.

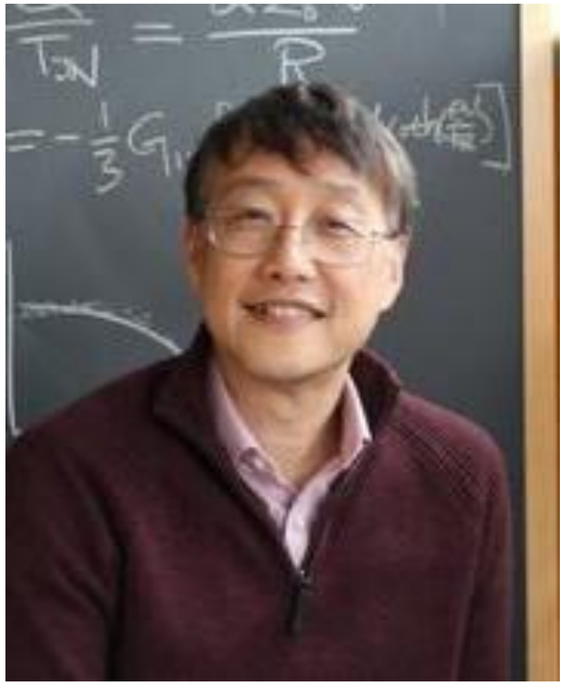
A. Chen, R. Ilan, F. de Juan, D. I. Pikulin, and M. Franz, *Phys. Rev. Lett.* **121**, 036403 (2018)



Laurel Anderson ……..
Philip Kim, to appear.

L. Anderson, Ph.D.
thesis, Harvard, 2022





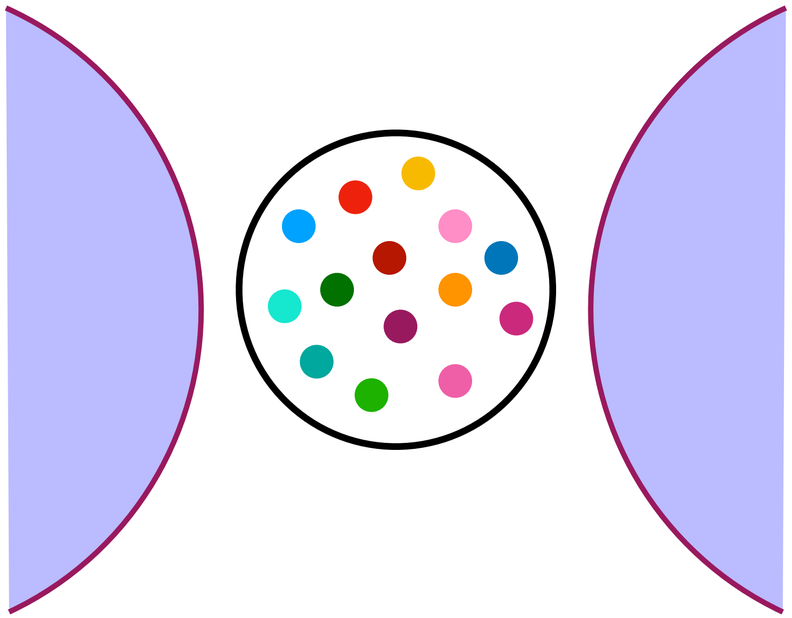
Thermopower

Apply a temperature difference ΔT, and measure the voltage difference ΔV, while no current is flowing. The thermopower is

$$\Theta = \frac{\Delta V}{\Delta T} = \frac{k_B}{e} \times (\text{a number})$$

Thermopower

A. Kruchkov, A. A. Patel, Philip Kim, and S. Sachdev, PRB **101**, 205148 (2020)



SYK metal island weakly coupled to normal metal leads

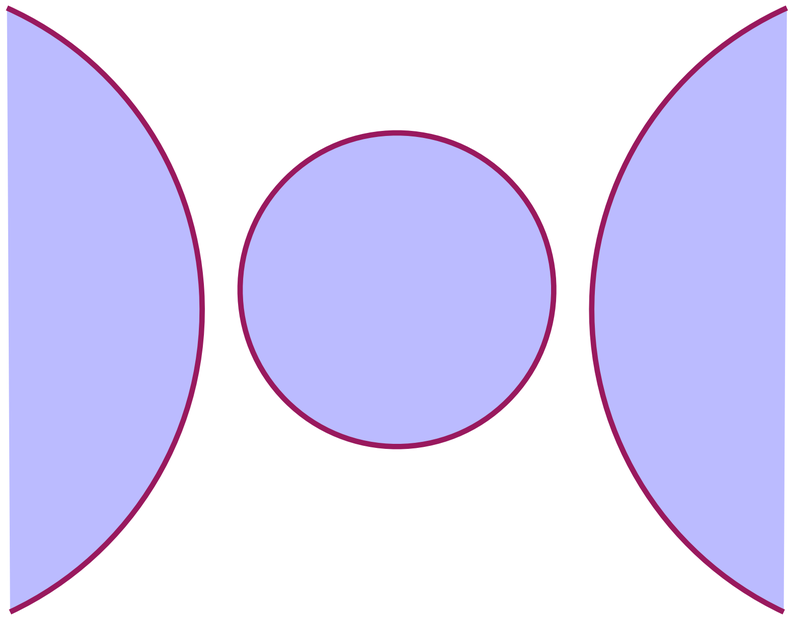
$\mathcal{E}$ is a dimensionless measure of the particle-hole asymmetry ($\mathcal{E} = 0$ for $\mathcal{Q} = 1/2$)

$$f(\omega) = \frac{1}{e^{\hbar\omega/k_B T} + 1}$$

$$\Theta = \frac{\hbar}{eT} \frac{\int d\omega\, \omega\, (-\partial f/\partial\omega) \left[\mathrm{Im}\overline{G(\omega)}\right]}{\int d\omega\, (-\partial f/\partial\omega) \left[\mathrm{Im}\overline{G(\omega)}\right]} = \frac{k_B}{e} \frac{4\pi\mathcal{E}}{3} = \frac{2}{3e} \frac{dS_0}{d\mathcal{Q}}$$

Thermopower is non-zero as $T \to 0$

Thermopower



Normal metal island
weakly coupled to
normal metal leads

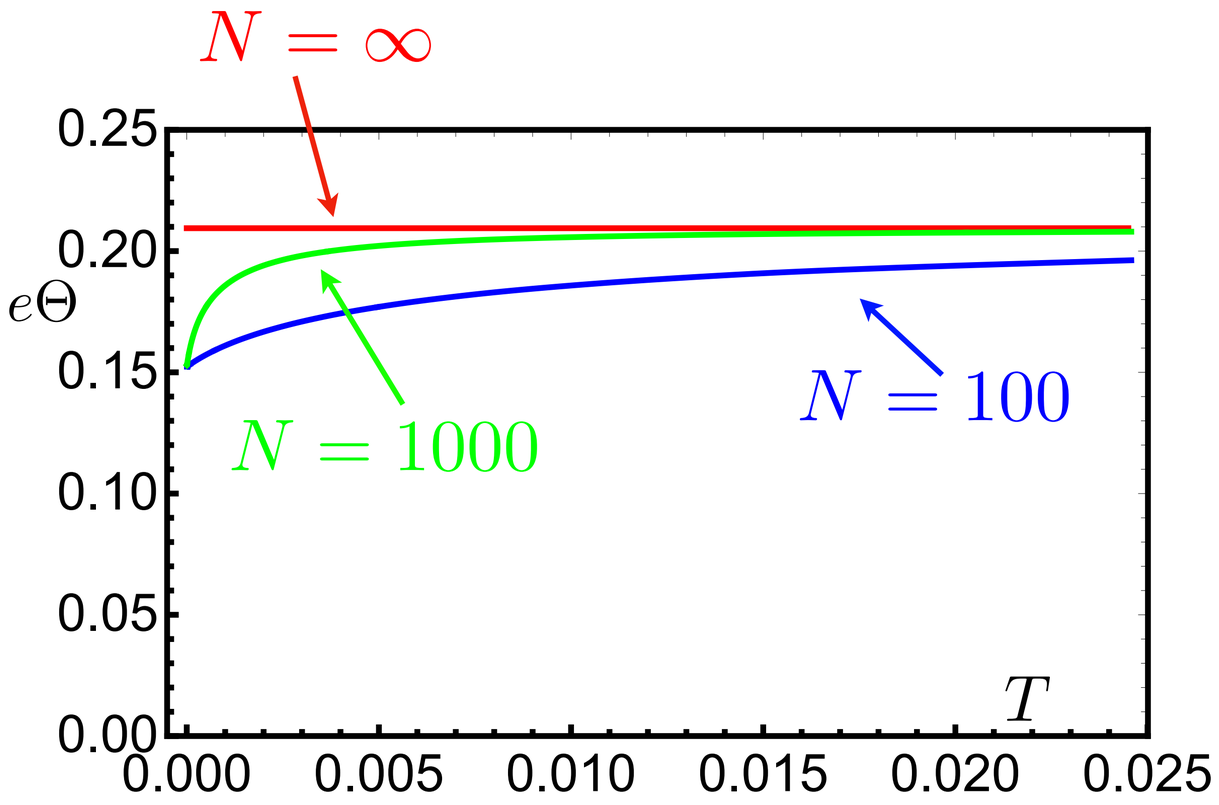
$$\Theta = \frac{\hbar}{eT}\frac{\int d\omega\, \omega\, (-\partial f/\partial\omega)\left[\mathrm{Im}\overline{G(\omega)}\right]}{\int d\omega\, (-\partial f/\partial\omega)\left[\mathrm{Im}\overline{G(\omega)}\right]} = \frac{k_B}{e}\frac{\pi^2\rho'(E_F)k_BT}{3\rho(E_F)} = \frac{1}{e}\frac{dS}{dQ}$$

$$f(\omega) = \frac{1}{e^{\hbar\omega/k_BT}+1}$$

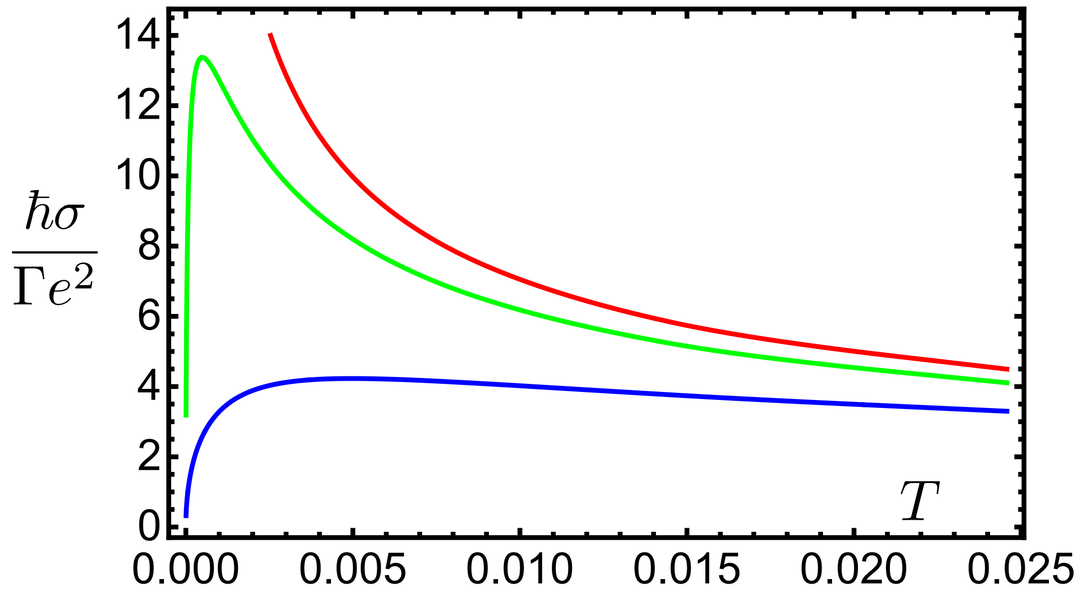
Thermopower vanishes linearly as $T \to 0$

Finite N corrections from Schwarzian theory of SYK island

Thermopower



Conductance



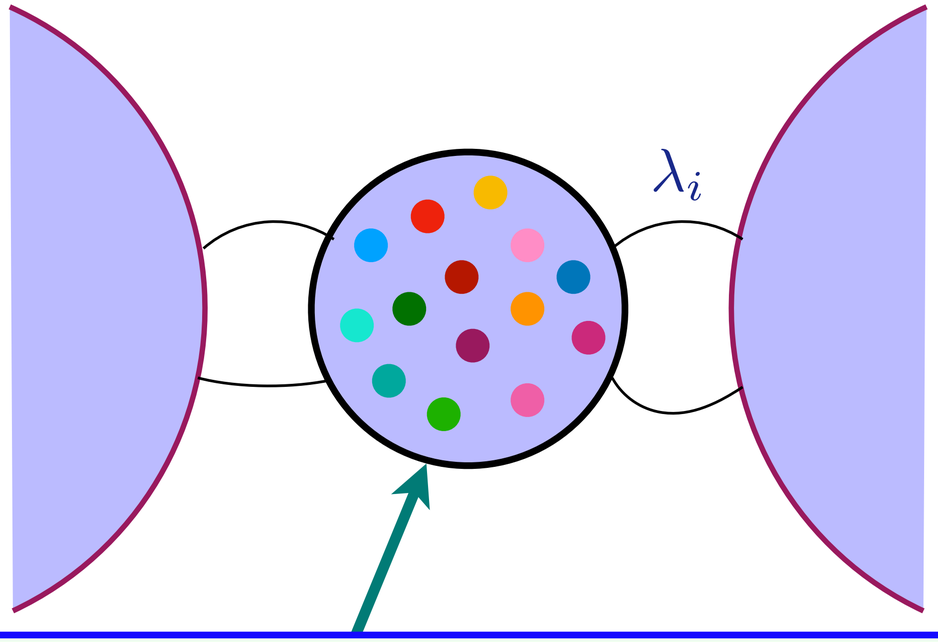
$$\sigma = \frac{4\Gamma e^2}{h}\int d\omega\,(-\partial f/\partial\omega)\left[\mathrm{Im}\overline{G(\omega)}\right]$$

$$\sim \frac{1}{\sqrt{T}}, \text{ for SYK island at } N=\infty$$

$$\sim \text{constant, for normal metal island at } N=\infty$$

A. Kruchkov, A.A. Patel, Philip Kim, and S. Sachdev, PRB **101**, 205148 (2020)

A more realistic model



$$\Gamma = \pi \rho_{\rm lead} \sum_i |\lambda_i|^2$$

$$\overline{|h_{\alpha\beta}|^2} = h^2$$

$$H = \frac{1}{\sqrt{N}} \sum_{\alpha\beta=1}^{N} h_{\alpha\beta} c_\alpha^\dagger c_\beta + \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

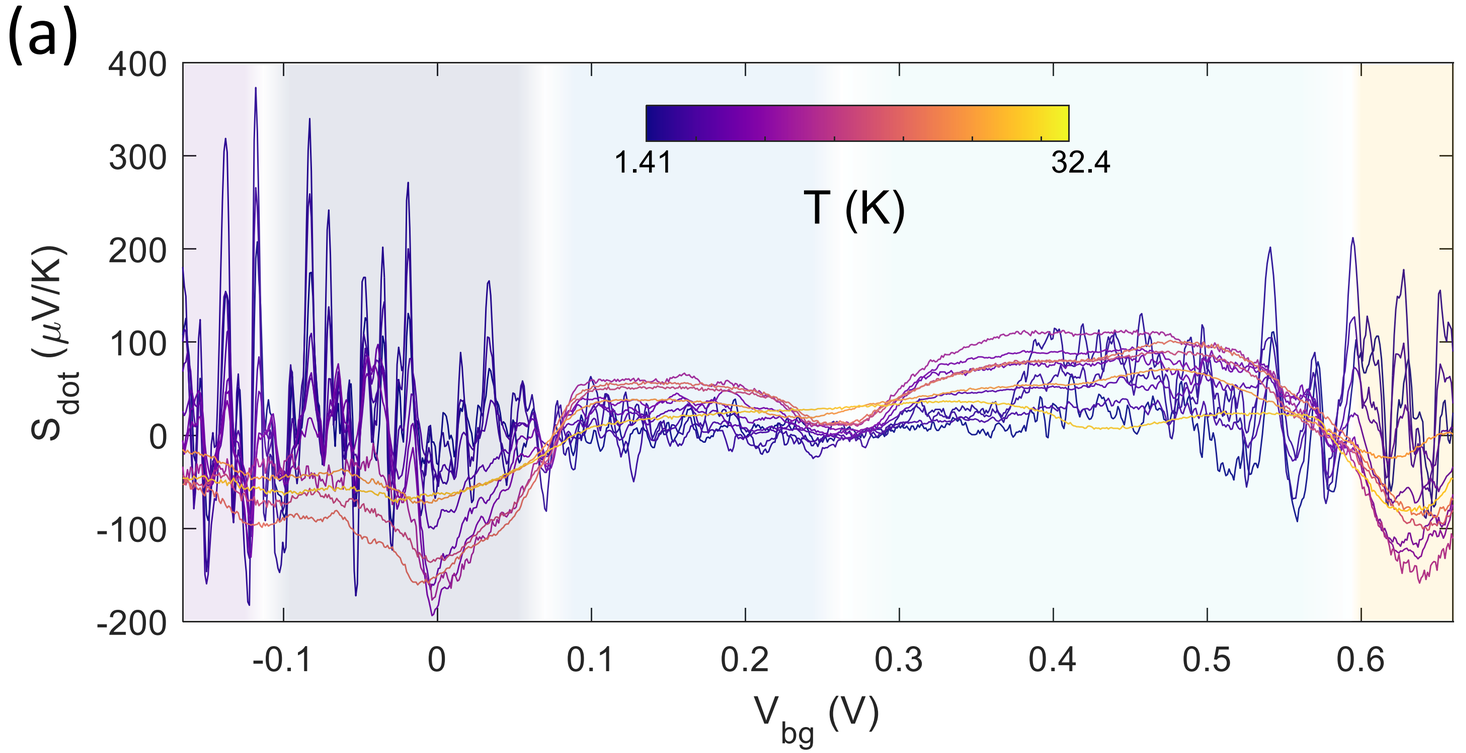
Normal metal behavior for $T < T_{\rm coh} \sim h^2/U$

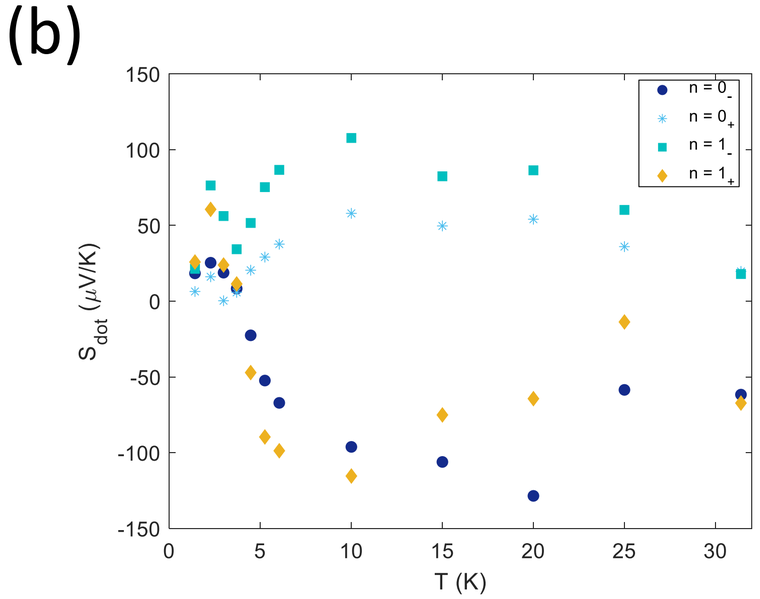
SYK behavior for $T > T_{\rm coh} \sim h^2/U$

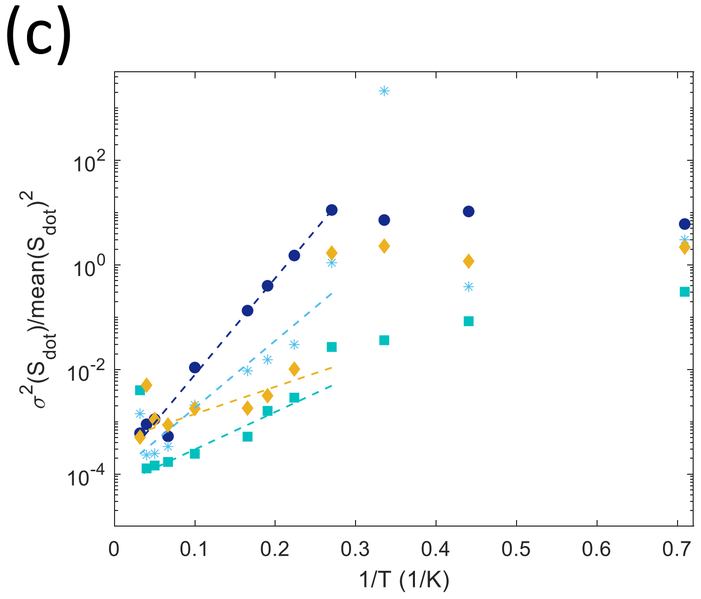
Notation: $S \to \Theta$

Laurel Anderson ……..
Philip Kim, to appear.

L. Anderson, Ph.D. thesis, Harvard, 2022



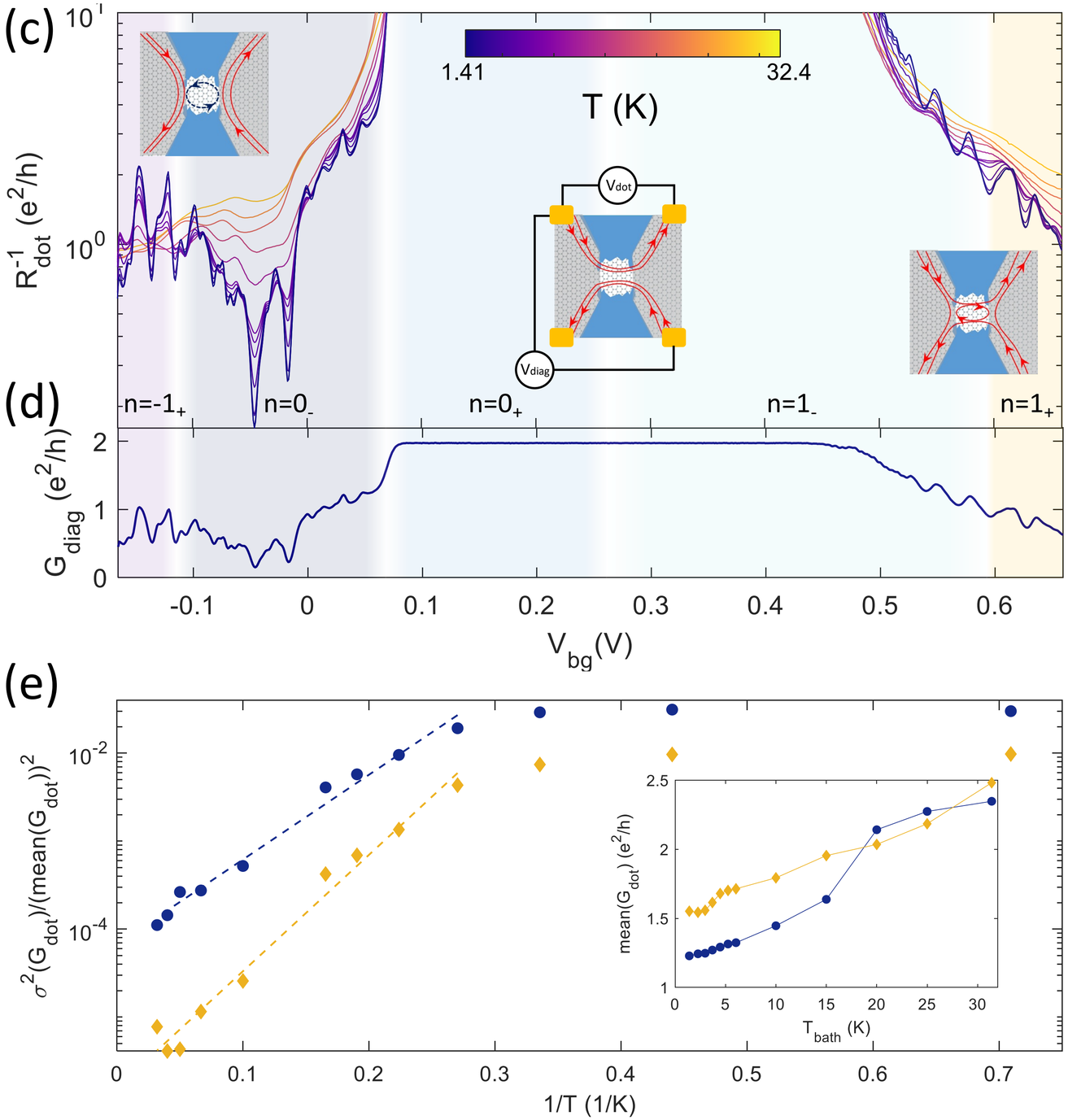




Notation: $G \to \sigma$

Laurel Anderson ……..
Philip Kim, to appear.

L. Anderson, Ph.D. thesis, Harvard, 2022



Striking feature of observations at finite N:
fluctuations from random couplings are much larger
for $T < T_{\rm coh}$ (normal metal regime)
than for $T > T_{\rm coh}$ (possible SYK regime)

Striking feature of observations at finite N:
fluctuations from random couplings are much larger
for $T < T_{\rm coh}$ (normal metal regime)
than for $T > T_{\rm coh}$ (possible SYK regime)

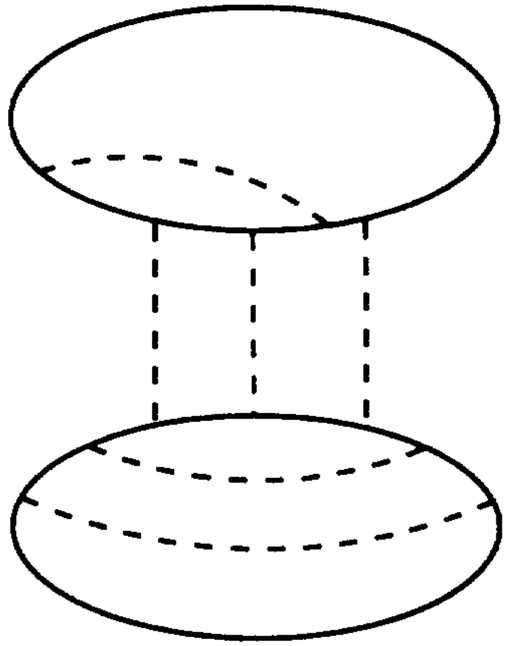
Henry Shackleton

Normal metal regime:

$$\frac{\mathrm{var}(\sigma) = \overline{(\sigma - \overline{\sigma})^2}}{\overline{\sigma}^2} = \frac{3\xi(3)}{16\pi^3}\left(\frac{h}{TN}\right)^2$$

Follows from Dyson-Mehta formula
for single-particle RMT eigenvalue repulsion

$$\overline{\mathrm{Im}G^r(\omega)\mathrm{Im}G^r(\epsilon)} - \overline{\mathrm{Im}G^r(\omega)}\,\overline{\mathrm{Im}G^r(\epsilon)} = -\frac{1}{2(\omega-\epsilon)^2}$$

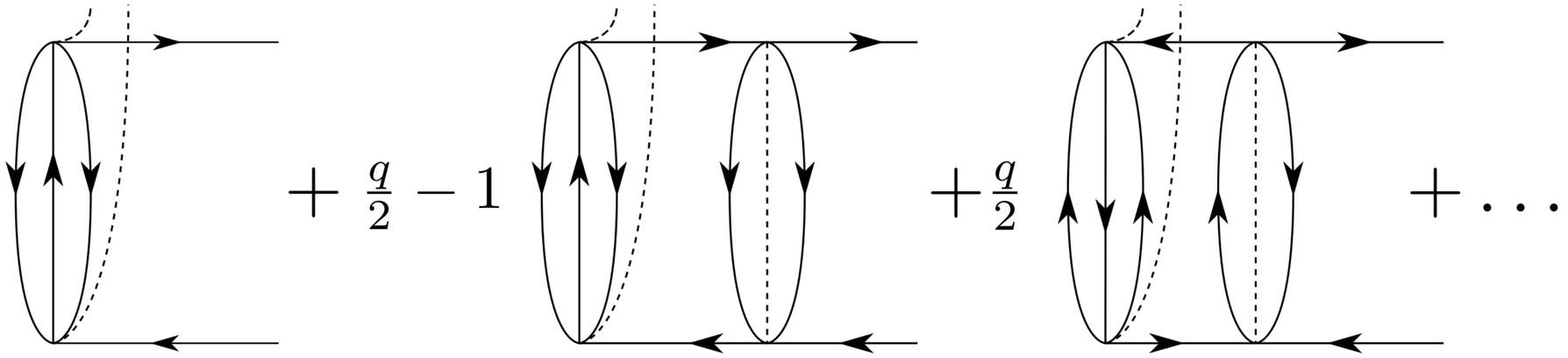


Striking feature of observations at finite N:
fluctuations from random couplings are much larger
for $T < T_{\rm coh}$ (normal metal regime)
than for $T > T_{\rm coh}$ (possible SYK regime)

Henry Shackleton

SYK regime:

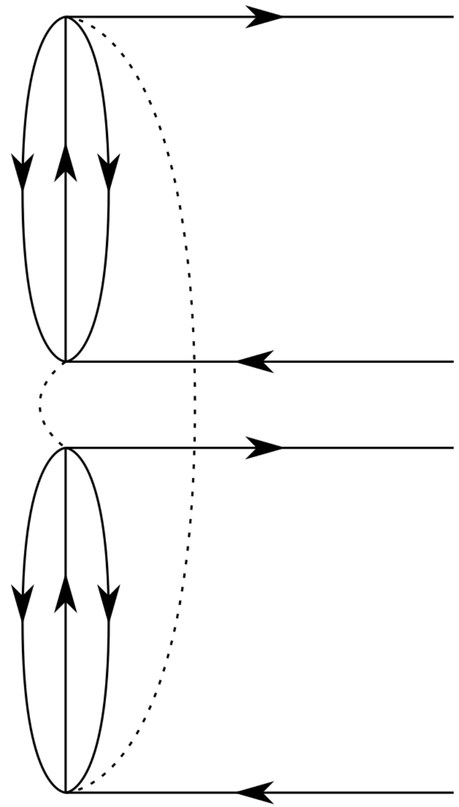
$$\frac{\mathrm{var}(\sigma) = \overline{(\sigma - \overline{\sigma})^2}}{\overline{\sigma}^2} = \frac{1}{8N^4}$$



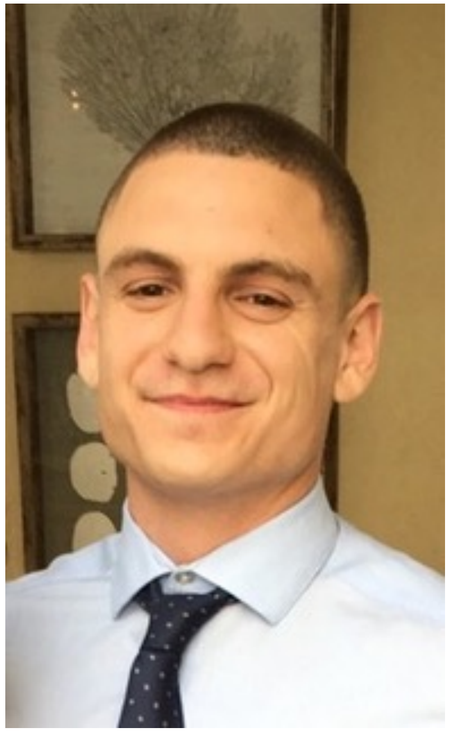
$+\ \frac{q}{2} - 1$

$+ \frac{q}{2}$

$+ \ldots$



From the Yukawa-SYK model to a universal theory of strange metals



Aavishkar Patel

Flatiron Institute, NYC

Haoyu Guo

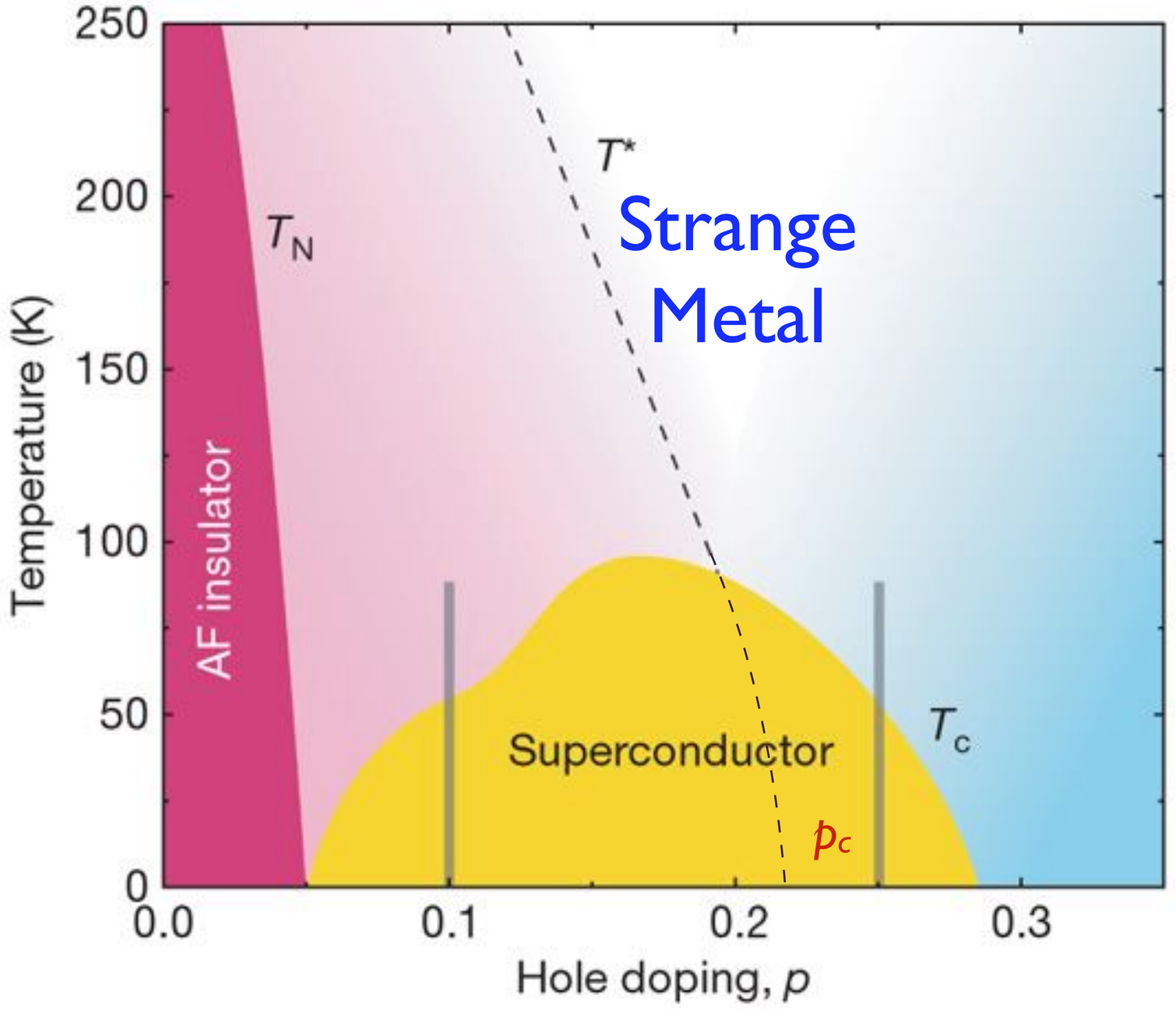
Harvard

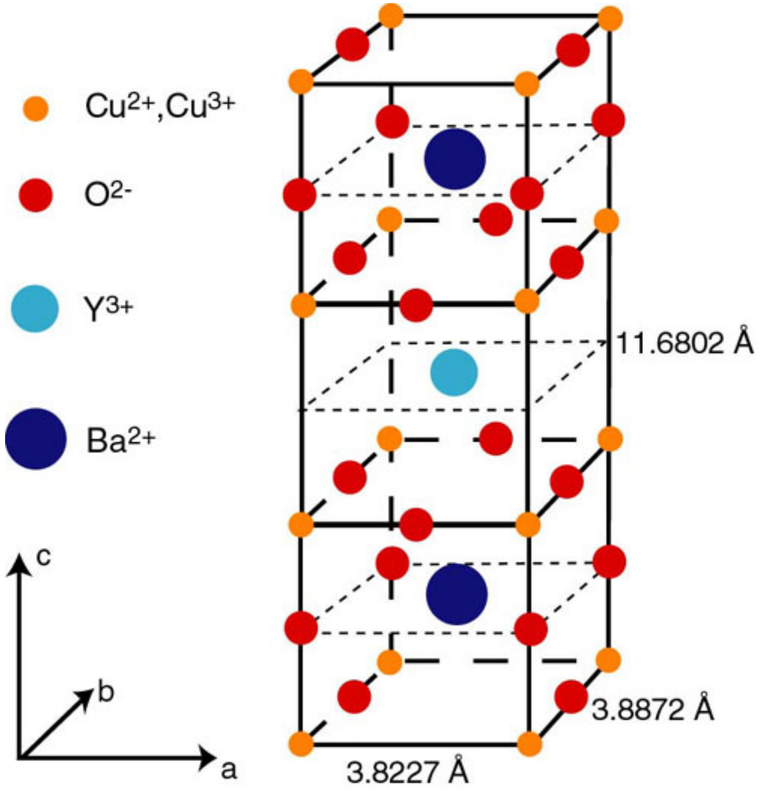
Ilya Esterlis

Harvard ➔ Wisconsin

arXiv: 2103.08615, 2203.04990, 2207.08841

E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, arXiv:2012.00763


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
T_N
T^*
T_c
p_c
AF insulator
Superconductor
Strange
Metal


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\frac{1}{\tau_{\rm trans}(\omega)} - i\omega \frac{m^*_{\rm trans}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon……A. Georges, arXiv:2205.04030

4. Photoemission: nearly "marginal Fermi liquid" electron spectral density:

$$\mathrm{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

T.J. Reber….D. Dessau, Nature Communications **10**, 5737 (2019)

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell \,,$$

$g_{ij\ell}$ independent random numbers with zero mean. Large N limit leads to Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell}\psi_i^\dagger \psi_j \phi_\ell \,,$$

$g_{ij\ell}$ independent random numbers with zero mean.

The disorder-averaged partition function is a G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\,\mathcal{D}\Sigma\,\mathcal{D}D\,\mathcal{D}\Pi \exp(-N S_{\rm all})$$

$$S_{\rm all} = -\ln\det(\partial_\tau - \mu + \Sigma) + \frac{1}{2}\ln\det(-\partial_\tau^2 + \omega_0^2 - \Pi)$$

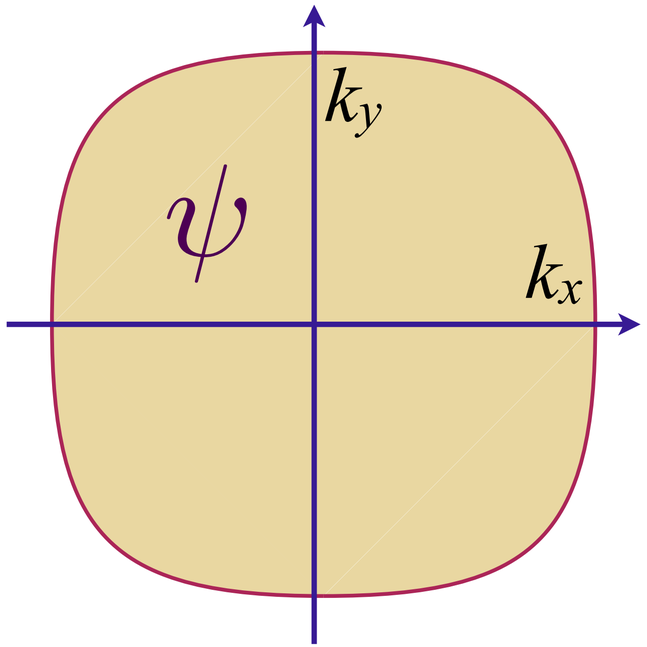
$$+ \int d\tau \int d\tau' \left[-\Sigma(\tau';\tau)G(\tau,\tau') + \frac{1}{2}\Pi(\tau'\tau)D(\tau,\tau')\right.$$

$$\left. + \frac{g^2}{2}G(\tau,\tau')G(\tau',\tau)D(\tau,\tau')\right] .$$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface coupled to a critical boson with disorder

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\,\mathcal{D}\Sigma\,\mathcal{D}D\,\mathcal{D}\Pi \exp(-N S_{\rm all})$$

$$\begin{aligned} S_{\rm all} = &- \ln\det(\partial_\tau + \varepsilon(\mathbf{k}) - \mu + \Sigma) + \frac{1}{2}\ln\det(-\partial_\tau^2 + \mathbf{q}^2 + m_b^2 - \Pi) \\ &+ \int d\tau d^2 r \int d\tau' d^2 r' \Big[-\Sigma(\tau', \mathbf{r}'; \tau, \mathbf{r}) G(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{1}{2}\Pi(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') \\ &+ \frac{g^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{v^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) \delta(\mathbf{r} - \mathbf{r}') \\ &+ \frac{g'^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') \delta(\mathbf{r} - \mathbf{r}') \Big] . \end{aligned}$$

Fermi surface coupled to a critical boson

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\,\mathcal{D}\Sigma\,\mathcal{D}D\,\mathcal{D}\Pi \exp(-N S_{\rm all})$$

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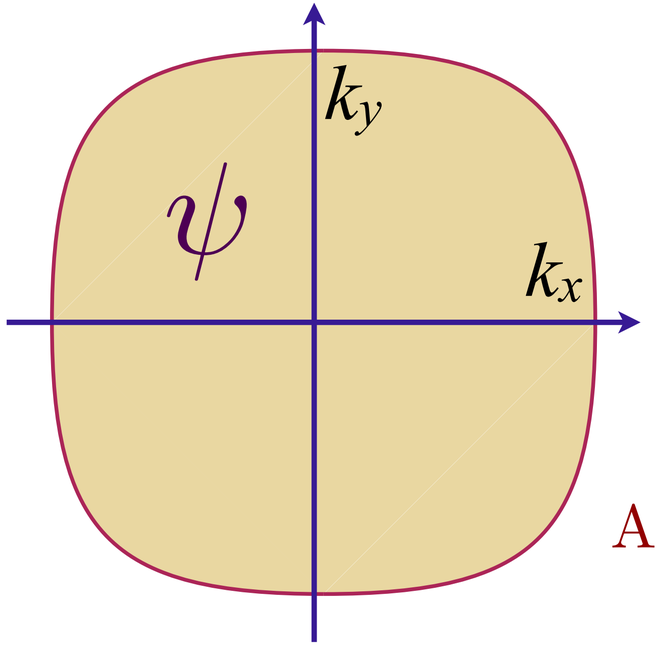
$$+ \int d\tau d^2 r \int d\tau' d^2 r' \left[-\Sigma(\tau', \mathbf{r}'; \tau, \mathbf{r}) G(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{1}{2}\Pi(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') \right.$$

$$\left. + \frac{g^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') \right.$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + [g \qquad\qquad]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ \qquad \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

A non-Fermi liquid but <u>not</u> a strange metal.
A 'perfect' metal without quasiparticles.

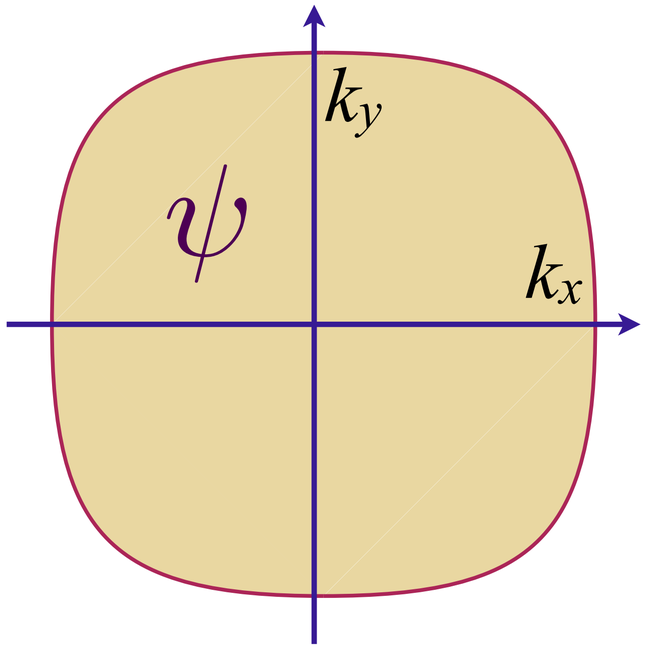
$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\mathrm{sgn}(\omega)|\omega|^{2/3}, \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

conductivity: $\sigma(\omega) \sim \dfrac{i}{\omega}$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface coupled to a critical boson with disorder

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\,\mathcal{D}\Sigma\,\mathcal{D}D\,\mathcal{D}\Pi \exp(-N S_{\rm all})$$

$$\begin{aligned} S_{\rm all} = &- \ln\det(\partial_\tau + \varepsilon(\mathbf{k}) - \mu + \Sigma) + \frac{1}{2}\ln\det(-\partial_\tau^2 + \mathbf{q}^2 + m_b^2 - \Pi) \\ &+ \int d\tau d^2 r \int d\tau' d^2 r' \Big[-\Sigma(\tau', \mathbf{r}'; \tau, \mathbf{r}) G(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{1}{2}\Pi(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') \\ &+ \frac{g^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{v^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) \delta(\mathbf{r} - \mathbf{r}') \\ &+ \frac{g'^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') \delta(\mathbf{r} - \mathbf{r}') \Big] . \end{aligned}$$

Fermi surface coupled to a critical boson with disorder

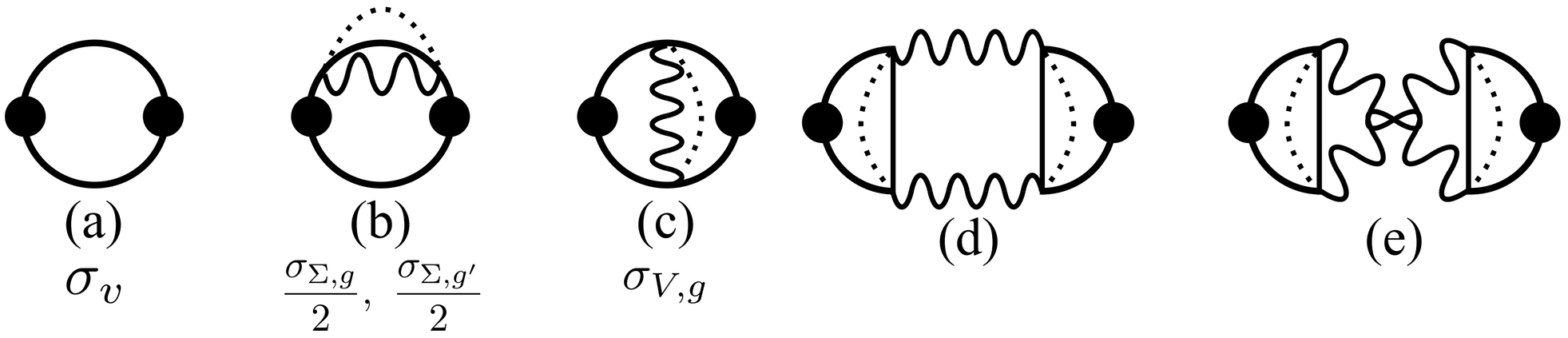
Electron Green's function: $G(\omega) \sim \dfrac{1}{\omega \dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i\left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\rm in}(\omega)}\right) \mathrm{sgn}(\omega)}$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2\right) |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi}\left(\frac{g^2}{v^2} + g'^2\right) \ln(\Lambda/\omega)$$

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

Fermi surface coupled to a critical boson with disorder

Conductivity:



+ all ladders and bubbles…..

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Fermi surface coupled to a critical boson with disorder

$$\text{Conductivity: } \sigma(\omega) \sim \frac{1}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega\,\dfrac{m^*_{\rm trans}(\omega)}{m}}$$

$$\frac{1}{\tau_{\rm trans}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*_{\rm trans}(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

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Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2;
Transport insensitive to g; Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat.

From the SYK model to a quantum theory of charged black holes

The SYK model

$$\mathcal{Z}(\mathcal{Q},T) = \mathrm{Tr}_{\mathcal{Q}} \exp\left(-\frac{\mathcal{H}}{T}\right) = \exp(-F/T)\,;\ \text{Entropy}\ S = -\frac{\partial F}{\partial T}.$$

$$\equiv \int_{E_0^-}^{\infty} D(E) e^{-E/T}\,; \quad D(E) = \sum_i \delta(E - E_i)\,; \quad \mathcal{H}\,|\Psi_i\rangle = E_i\,|\Psi_i\rangle$$

A CFT in d spatial dimensions has an entropy density $S \sim T^d$. The SYK model is a 0+1 dimensional CFT, and we obtain a T-independent entropy:

$$\lim_{T\to 0} \lim_{N\to\infty} \frac{S}{N} = s_0$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

The constant s_0 is a known universal number. At $\mathcal{Q} = 1/2$, we have

$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4} = 0.464847699170805107492692486833\ldots\,.$$

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$$\lim_{T\to 0}\lim_{N\to\infty} \frac{S}{N} = s_0 \quad \boxed{\Rightarrow \quad D(E) \stackrel{?}{=} e^{N s_0}\delta(E - E_0)}$$

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Conformal perturbations of saddle-point theory:

$$S = N(s_0 + \gamma T) \quad \Rightarrow \quad D(E) \sim \exp\left(N s_0 + \sqrt{2N\gamma E}\right) \ \text{for}\ E \sim NU$$

$\gamma = \#/U$ is non-universal.

Exact path integral over time reparameterizations:

$$S = N(s_0+\gamma T) - \frac{3}{2}\ln\left(\frac{U}{T}\right) - \frac{\ln N}{2} \quad \Rightarrow \quad D(E) \sim N^{-1} \exp\left(N s_0\right) \sinh(\sqrt{2N\gamma E})$$

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J. S. Cotler et al., JHEP 05 (2017) 118

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Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, JHEP 02 (2020) 157

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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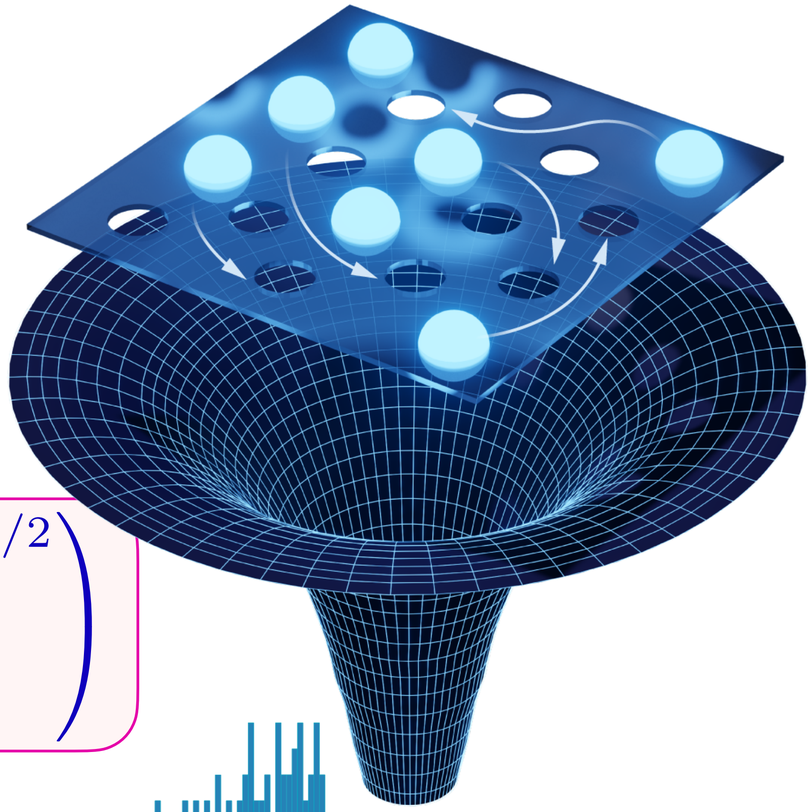
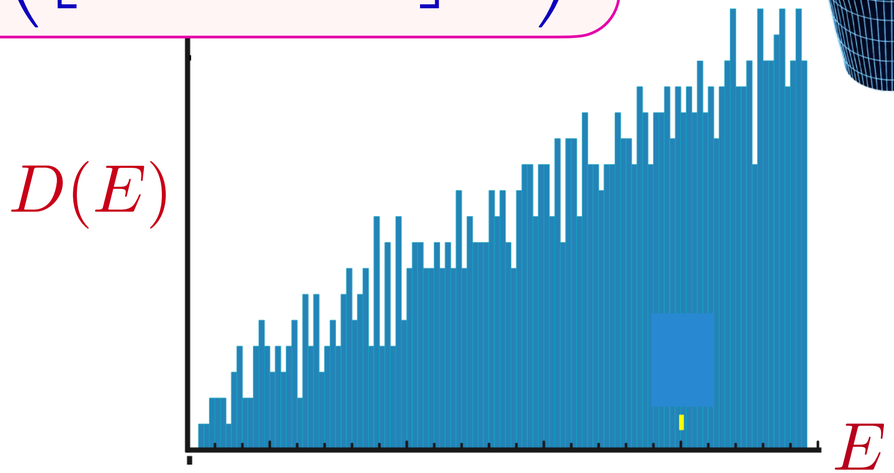
Quantum simulation of charged black holes by the SYK model



- For generic black holes with charge $\mathcal{Q}$ in asymptotically Minkowski 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2G\mathcal{Q}^2/c^4$, the horizon area at $T = 0$:

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

There is no degeneracy, but an exponentially small level spacing down to the ground state.



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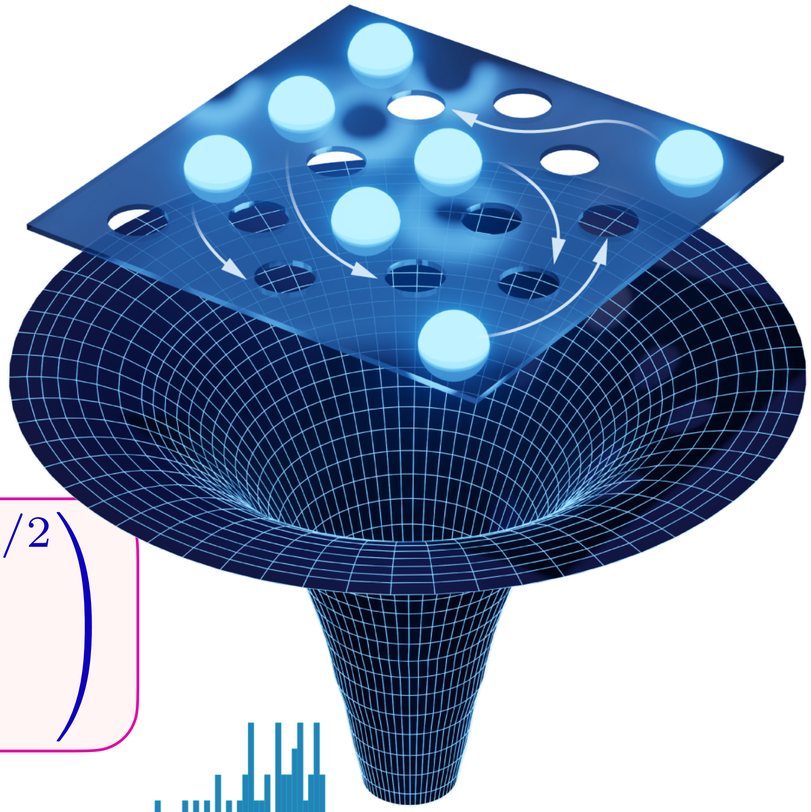

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Bekenstein-Hawking

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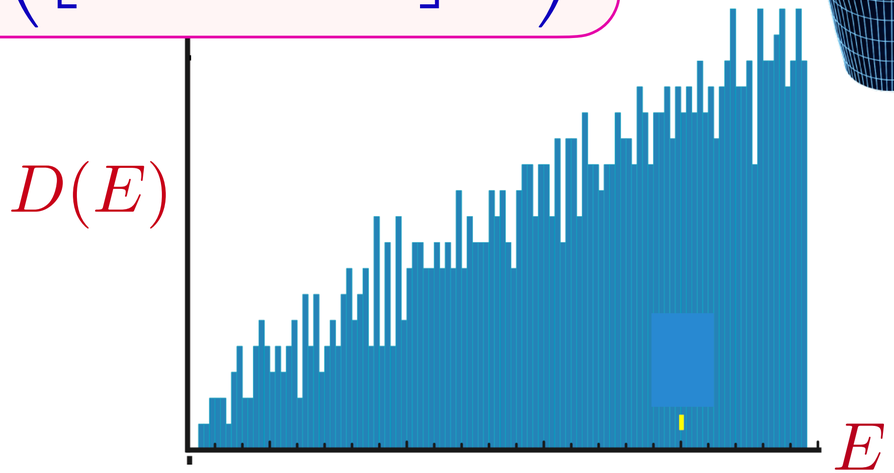

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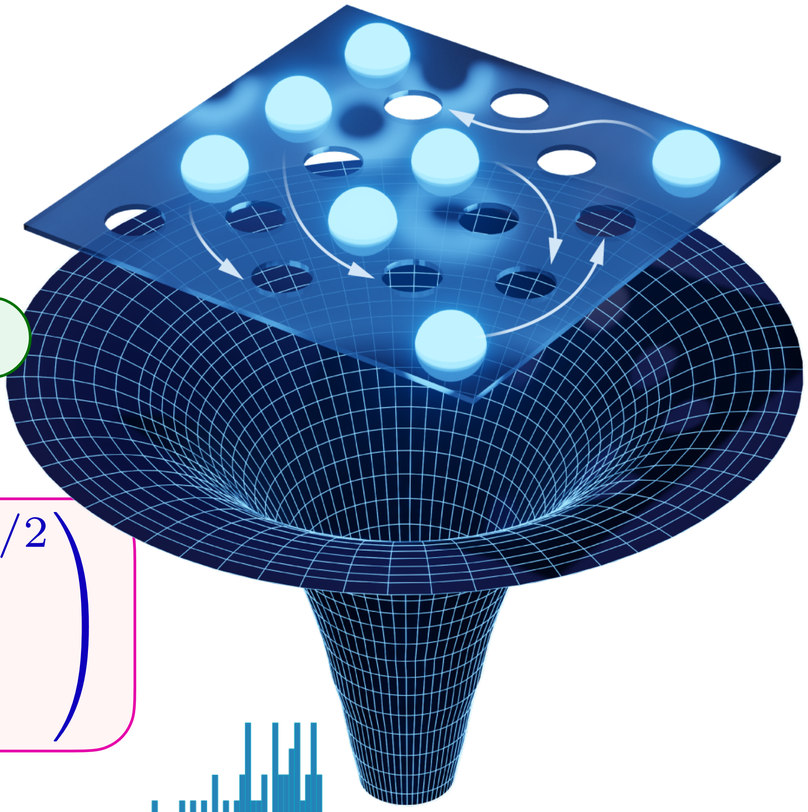

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Developments from the SYK model

Bekenstein-Hawking

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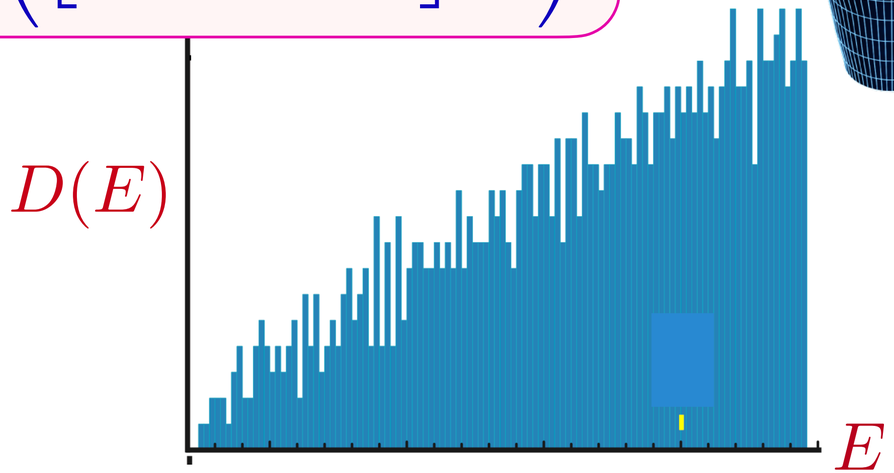

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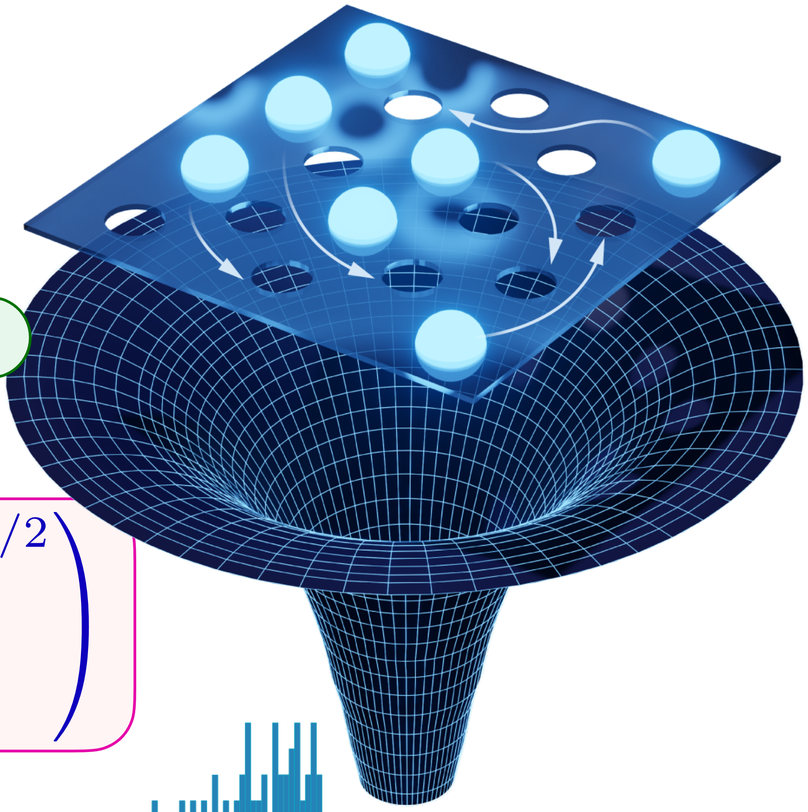

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Iliesiu, Murthy, Turiaci (2022)

Bekenstein-Hawking

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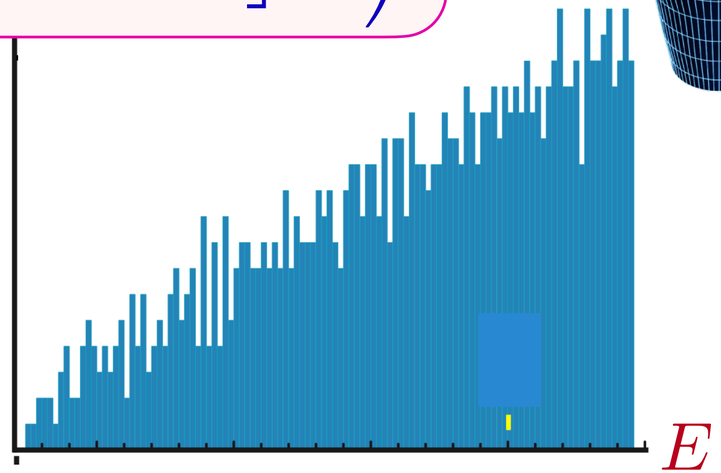
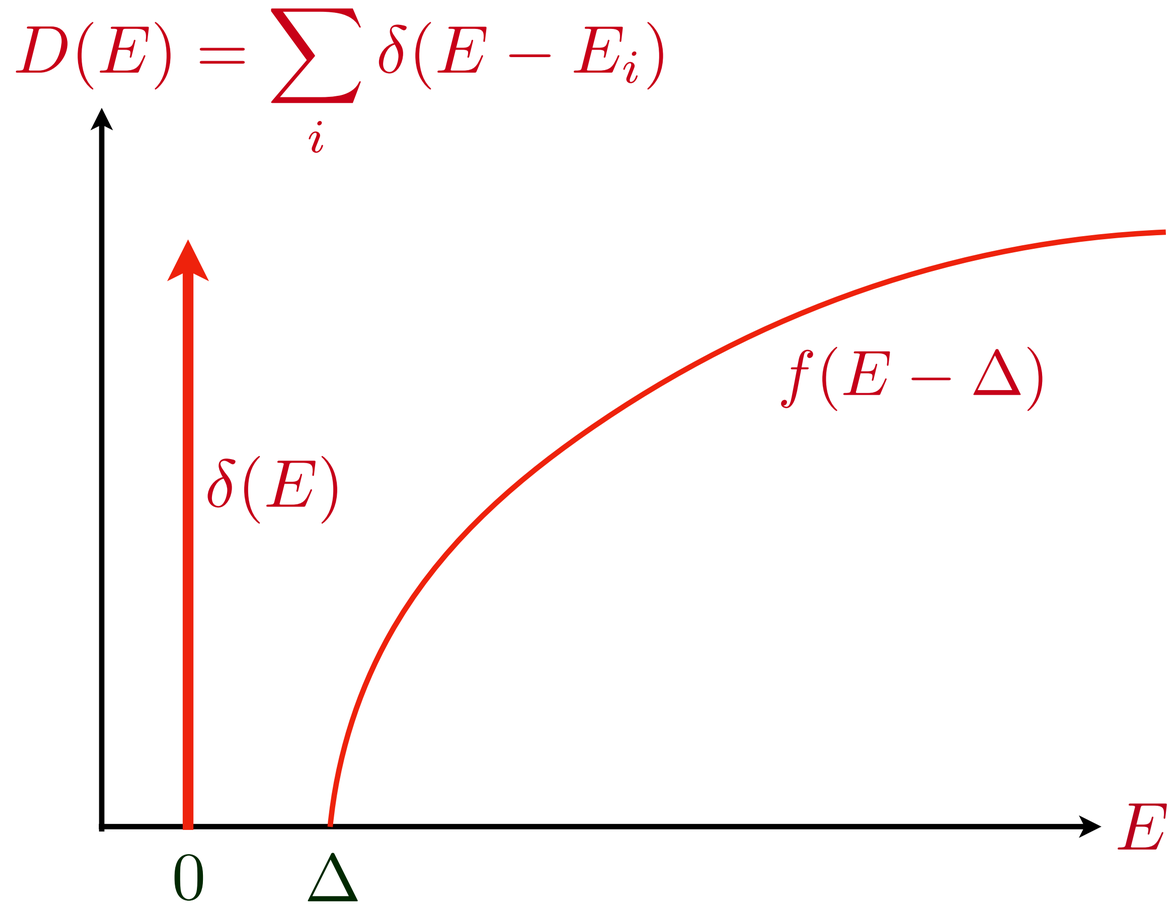


Strong theory of charged black holes

- With sufficient low energy supersymmetry:

$$D(E) = \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\Big[\delta(E) + \theta(E-\Delta) f(E-\Delta) + \ldots\Big]$$

There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state. Similar results for supersymmetric SYK models



M. Heydeman, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2020
L.V. Iliesiu, S. Murthy, G. J. Turiaci, 2022

Summary

- Graphene flakes display a crossover from quasiparticle behavior with large disorder-induced fluctuations at low T, to a higher T regime with weak fluctuations and a T-independent thermopower. The latter behavior matches that of the SYK model

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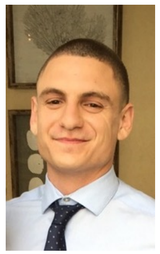

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990

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- Toy SYK model captures the correct universal low energy quantum theory of charged black holes, and provides a quantum simulation of black hole microstates.