Ultra-quantum metals

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Simons Foundation, New York

PHYSICS
HARVARD

Ubiquitous

"Strange",

"Bad",

"Incoherent",

or "Ultra-quantum"

metal has a resistivity, ρ, which obeys

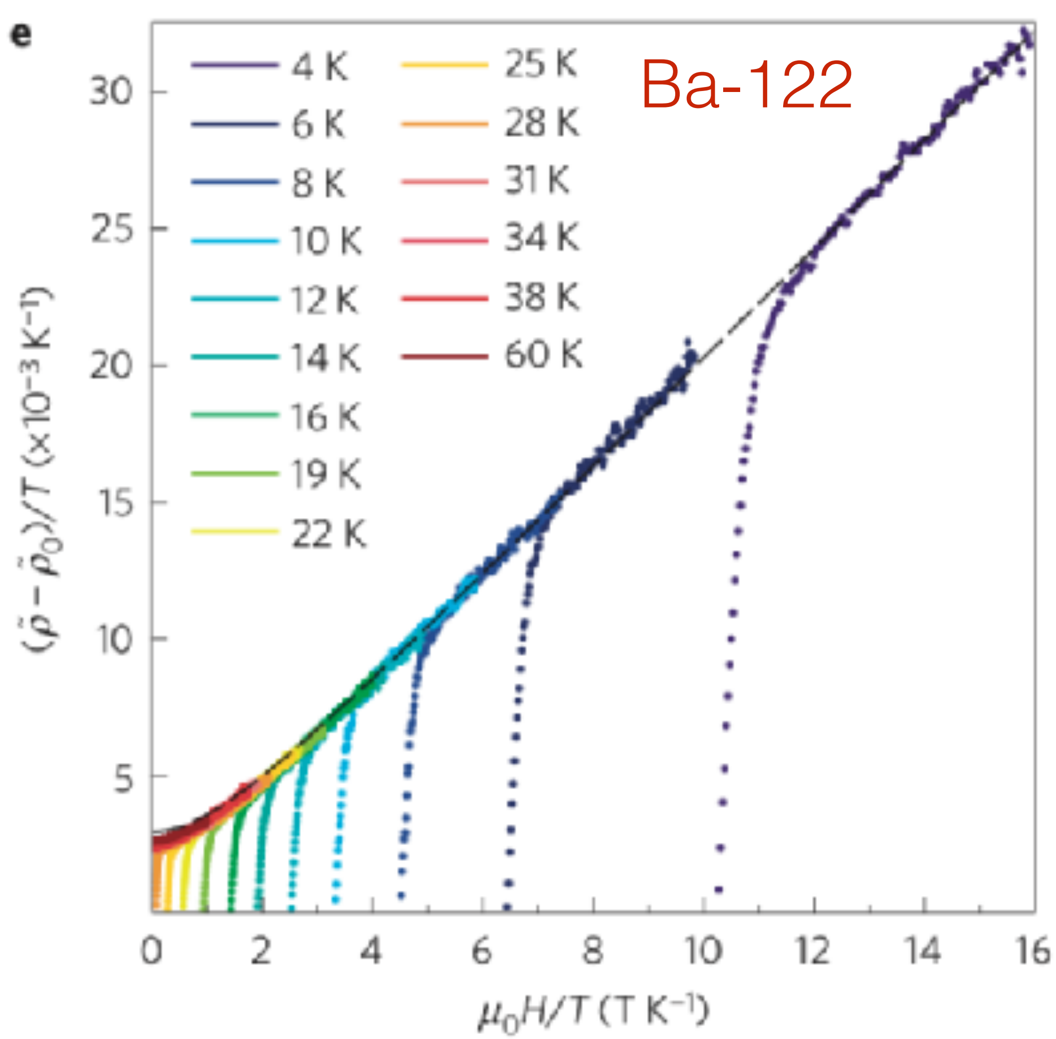
$$\rho \sim T,$$

and

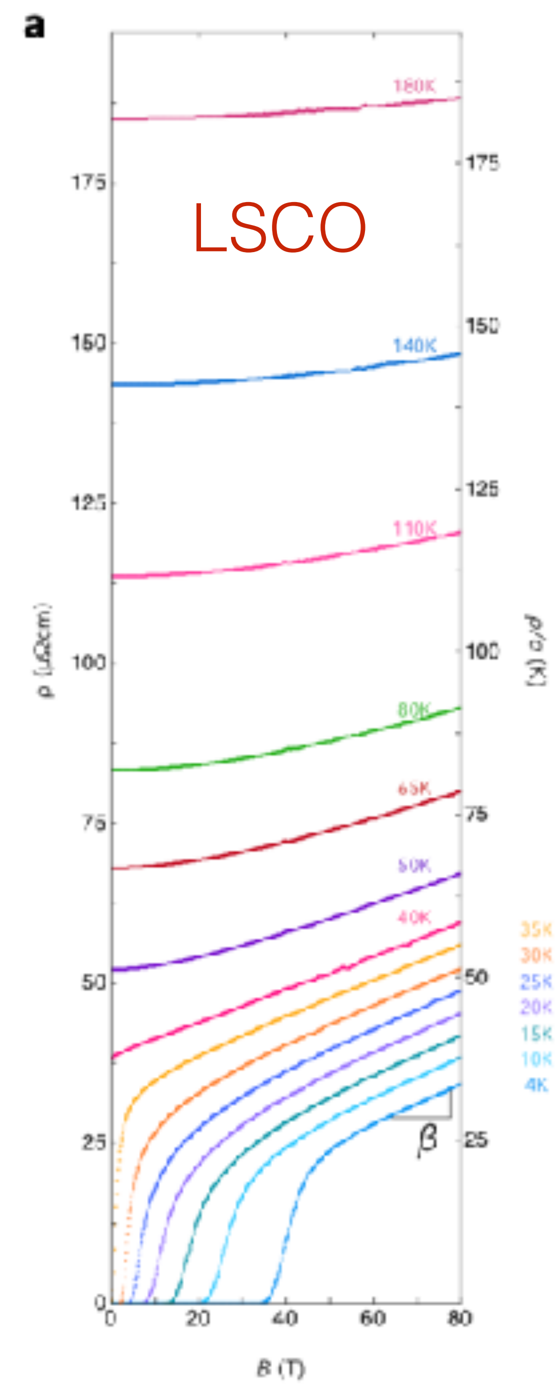
in some cases $\rho \gg h/e^2$
(in two dimensions),
where h/e^2 is the quantum unit of resistance.

Ultra-quantum metals just got stranger…

B-linear magnetoresistance!?



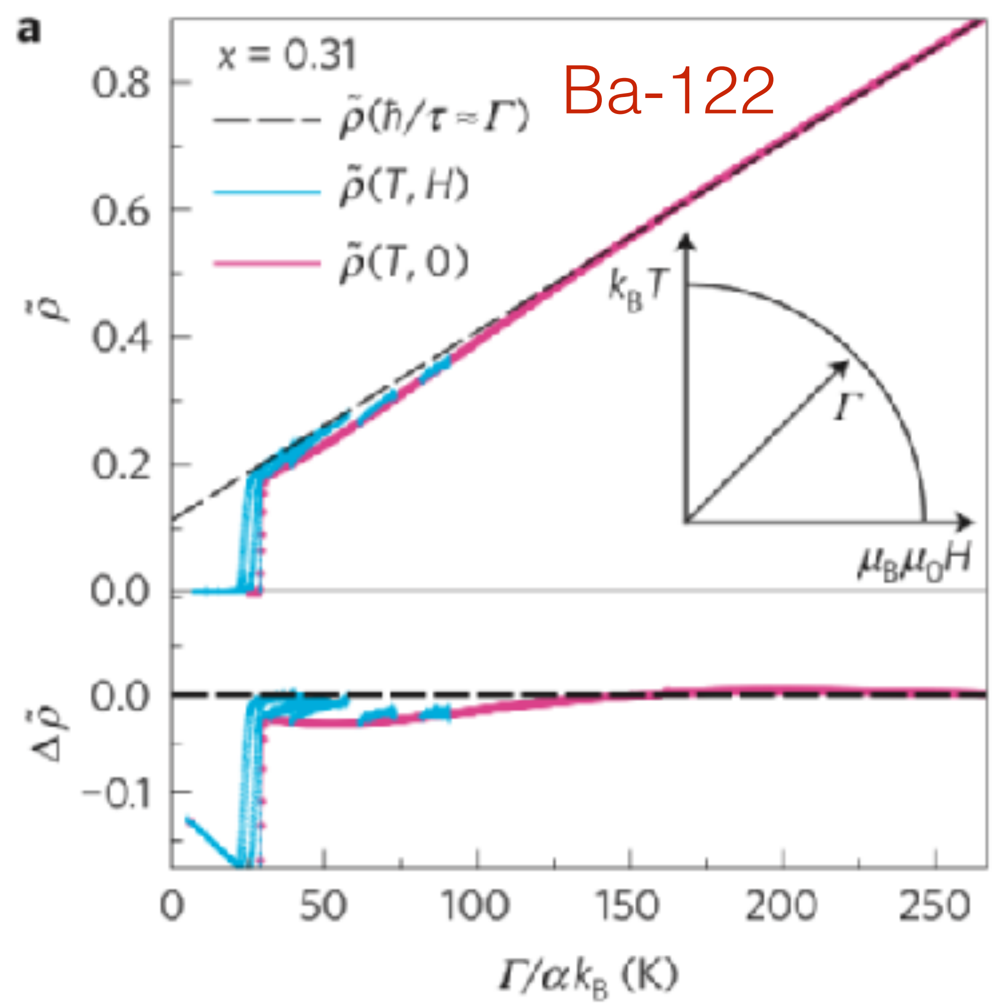
I. M. Hayes et. al., Nat. Phys. 2016



P. Giraldo-Gallo et. al., arXiv:1705.05806

Ultra-quantum metals just got stranger…

Scaling between B and T !?



$$\rho(H,T) - \rho(0,0) \propto \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2} \equiv \Gamma$$

I. M. Hayes et. al., Nat. Phys. 2016

Interactions between L. Balents, J. McGreevy, S. Sachdev, T. Senthil (partly nucleated by student, Aavishkar Patel)

- **Quantum spin liquids and the metal-insulator transition in doped semiconductors**, Andrew C. Potter, Maissam Barkeshli, John McGreevy, T. Senthil, Phys. Rev. Lett. **109**, 077205 (2012).

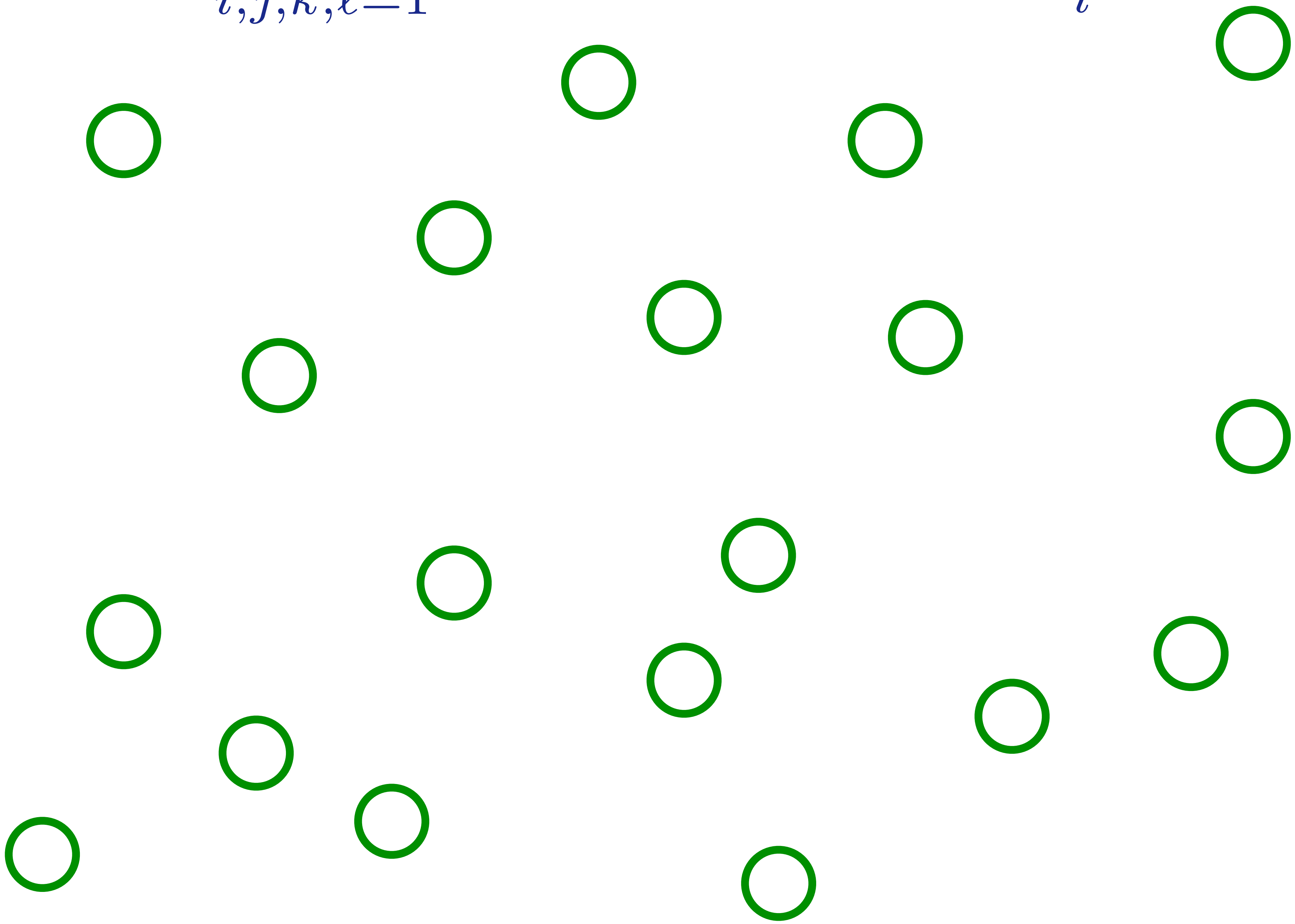
- **A strongly correlated metal built from Sachdev-Ye-Kitaev models**, Xue-Yang Song, Chao-Ming Jian, Leon Balents, Phys. Rev. Lett. **119**, 216601 (2017).

- **Magnetotransport in a model of a disordered strange metal**, Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, arXiv:1712.05026

- **Translationally invariant non-Fermi liquid metals with critical Fermi-surfaces: Solvable models**, Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, arXiv:1801.06178

The Sachdev-Ye-Kitaev (SYK) model

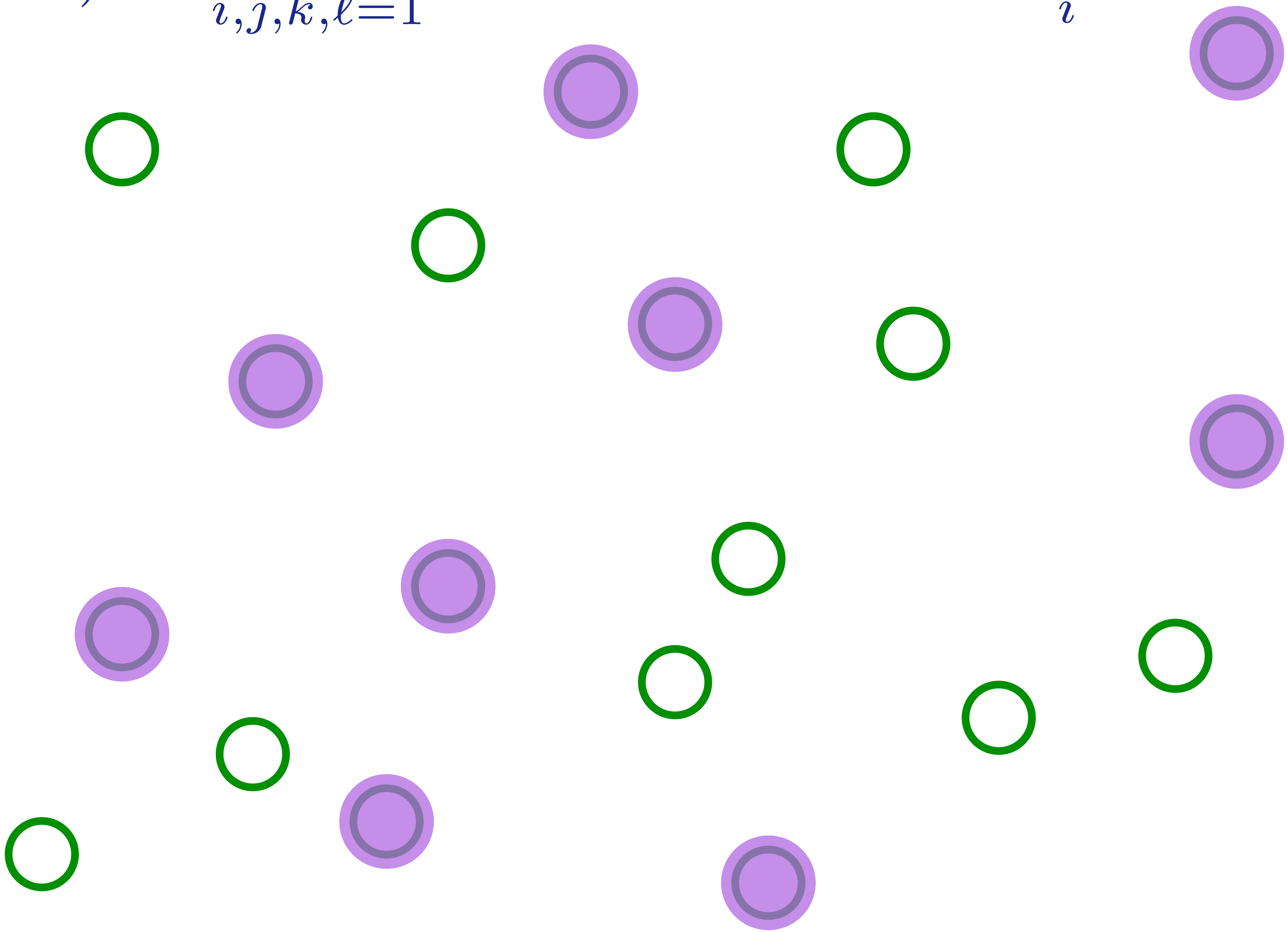
$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell} \, f_i^\dagger f_j^\dagger f_k f_\ell - \mu \sum_i f_i^\dagger f_i$$



Pick a set of random positions

The SYK model

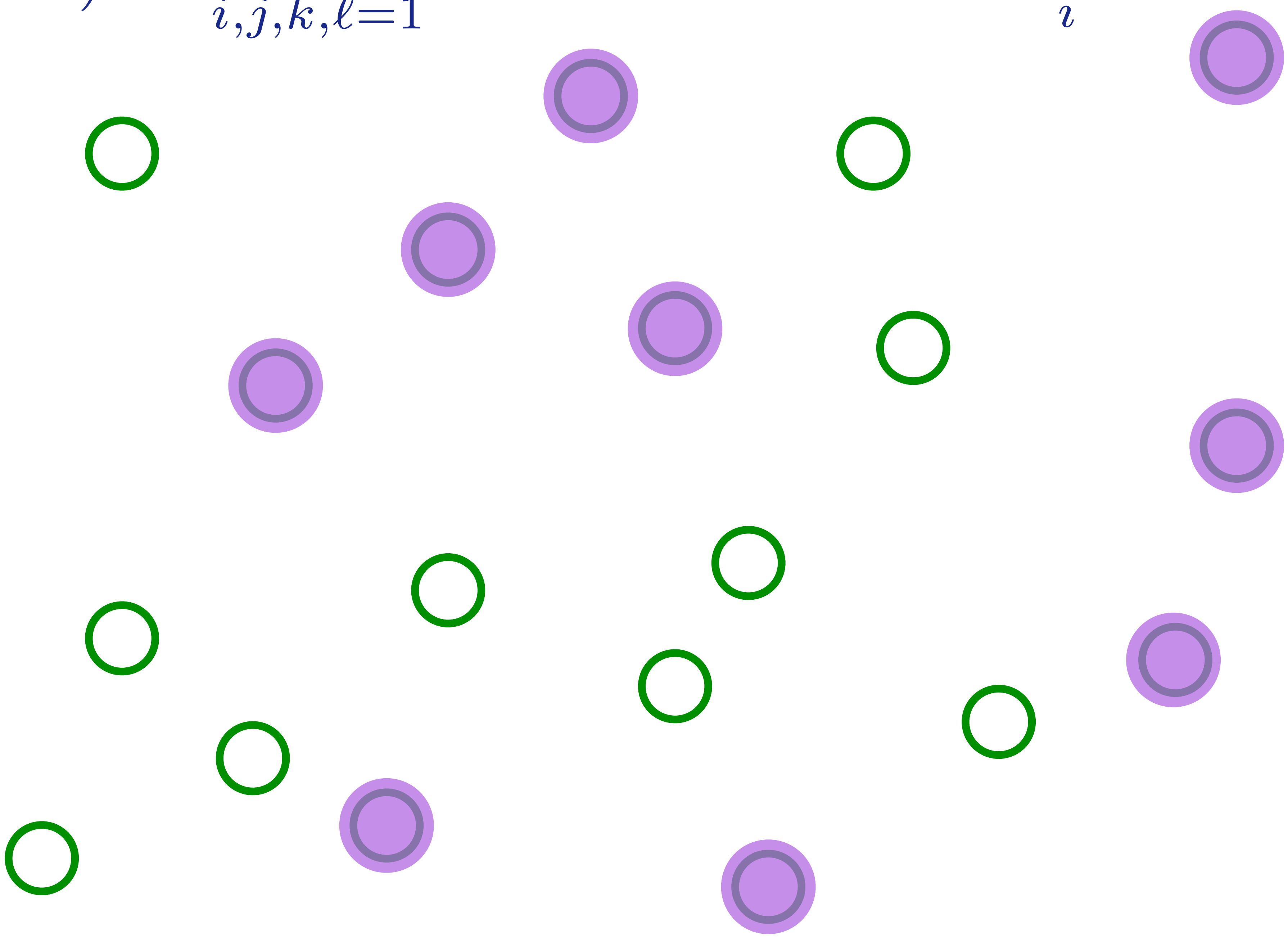
$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell} \, f_i^\dagger f_j^\dagger f_k f_\ell - \mu \sum_i f_i^\dagger f_i$$



Place electrons randomly on some sites

The SYK model

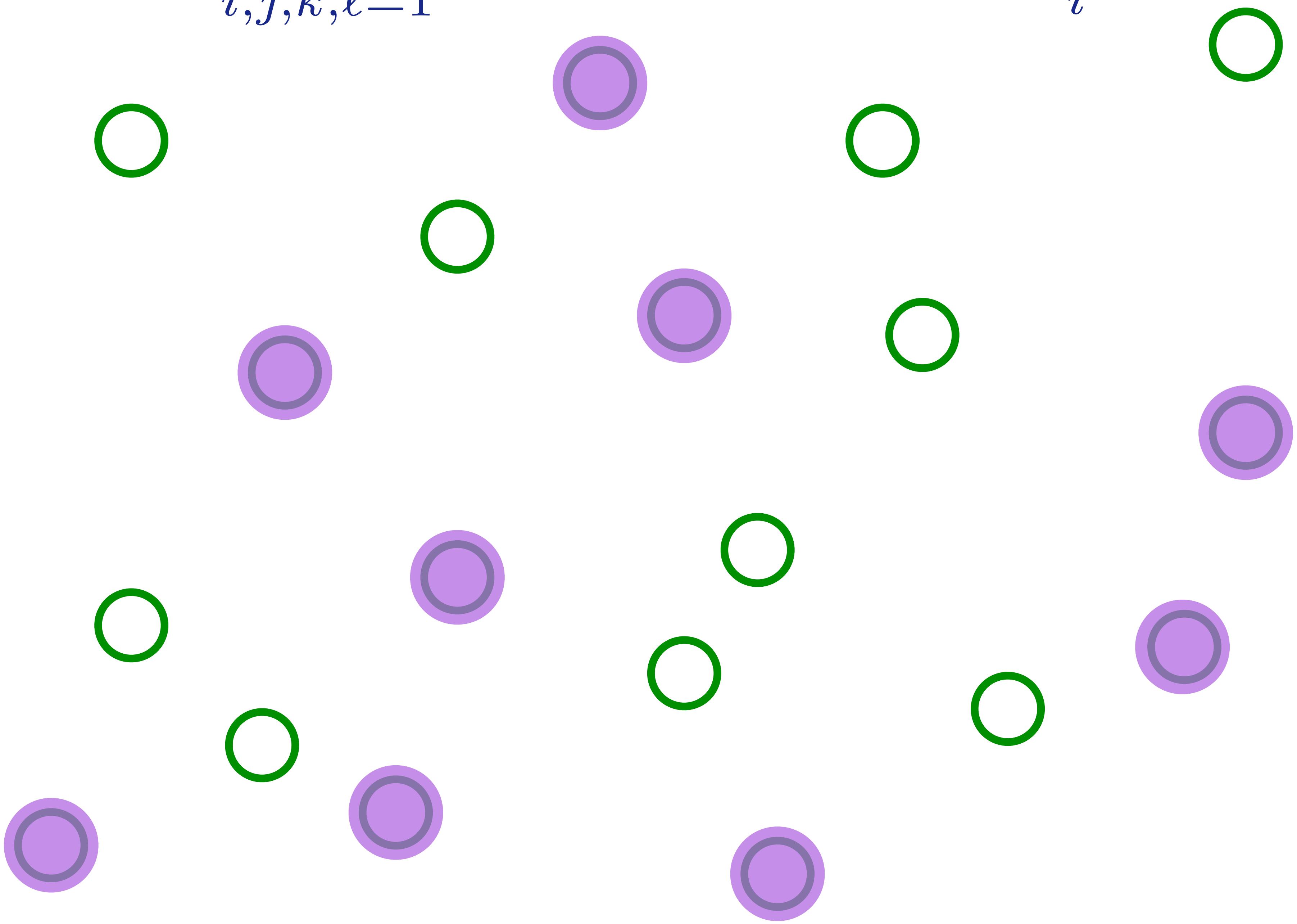
$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, f_i^\dagger f_j^\dagger f_k f_\ell - \mu \sum_i f_i^\dagger f_i$$



Entangle electrons pairwise randomly

The SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell} \, f_i^\dagger f_j^\dagger f_k f_\ell - \mu \sum_i f_i^\dagger f_i$$



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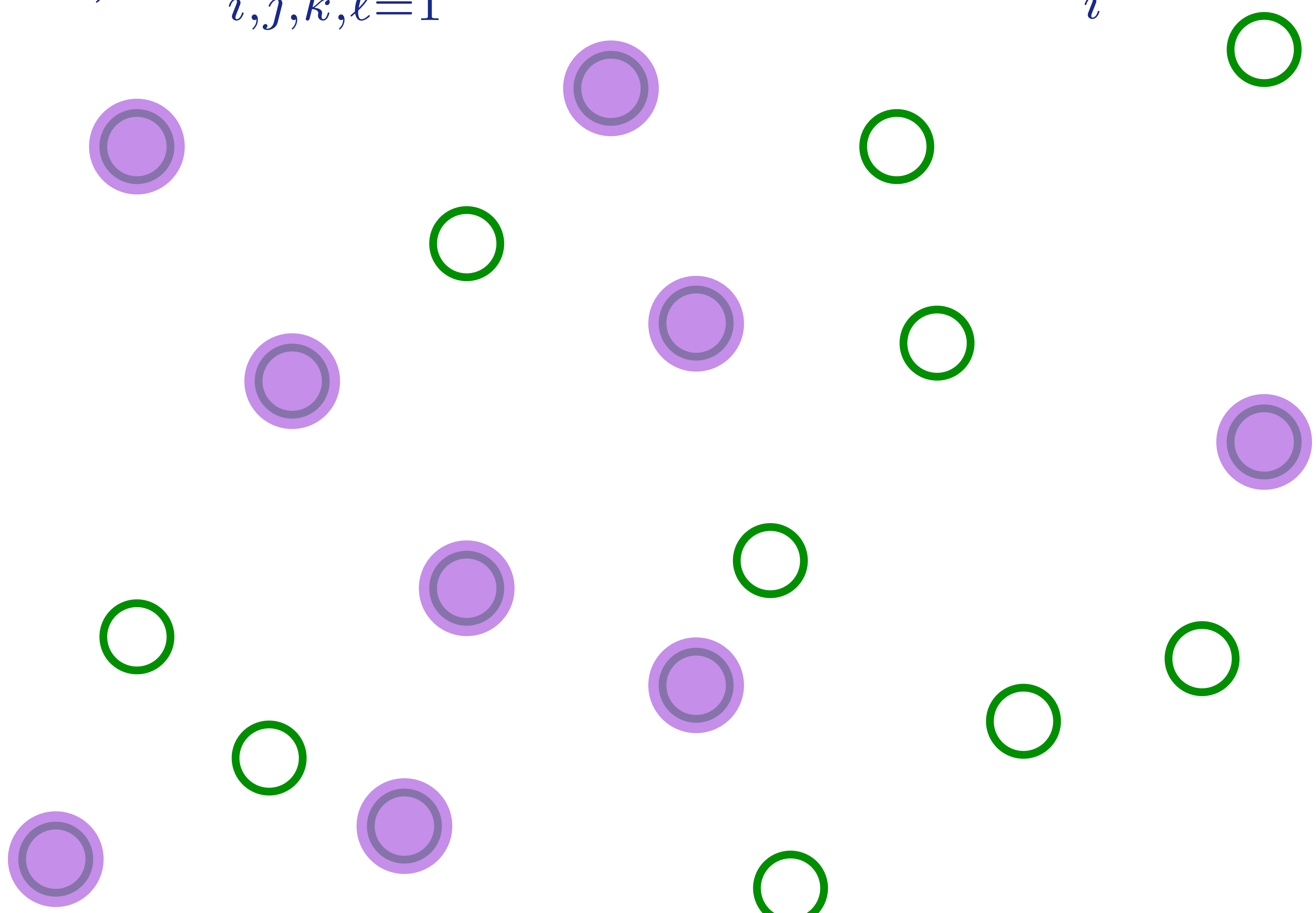

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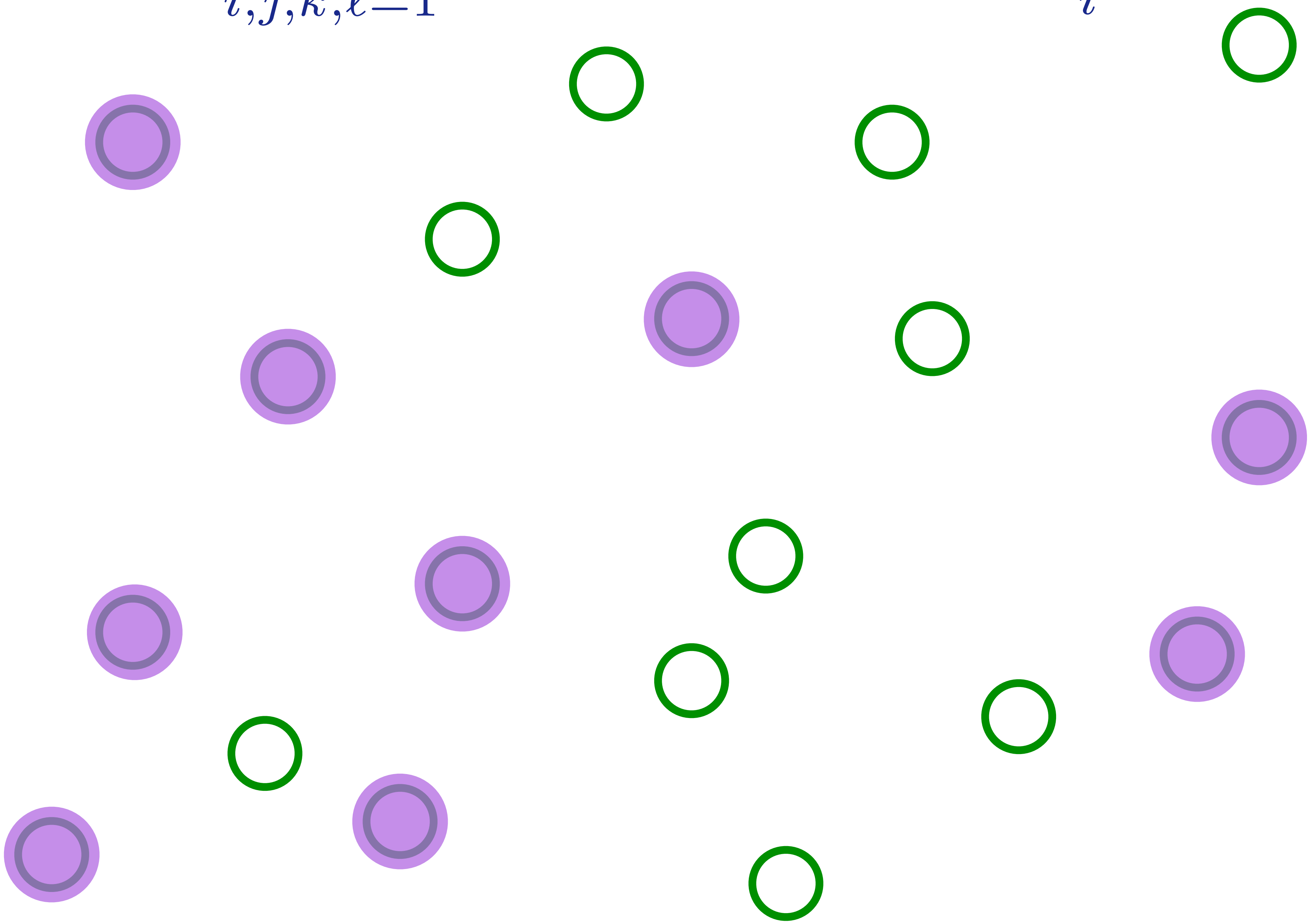

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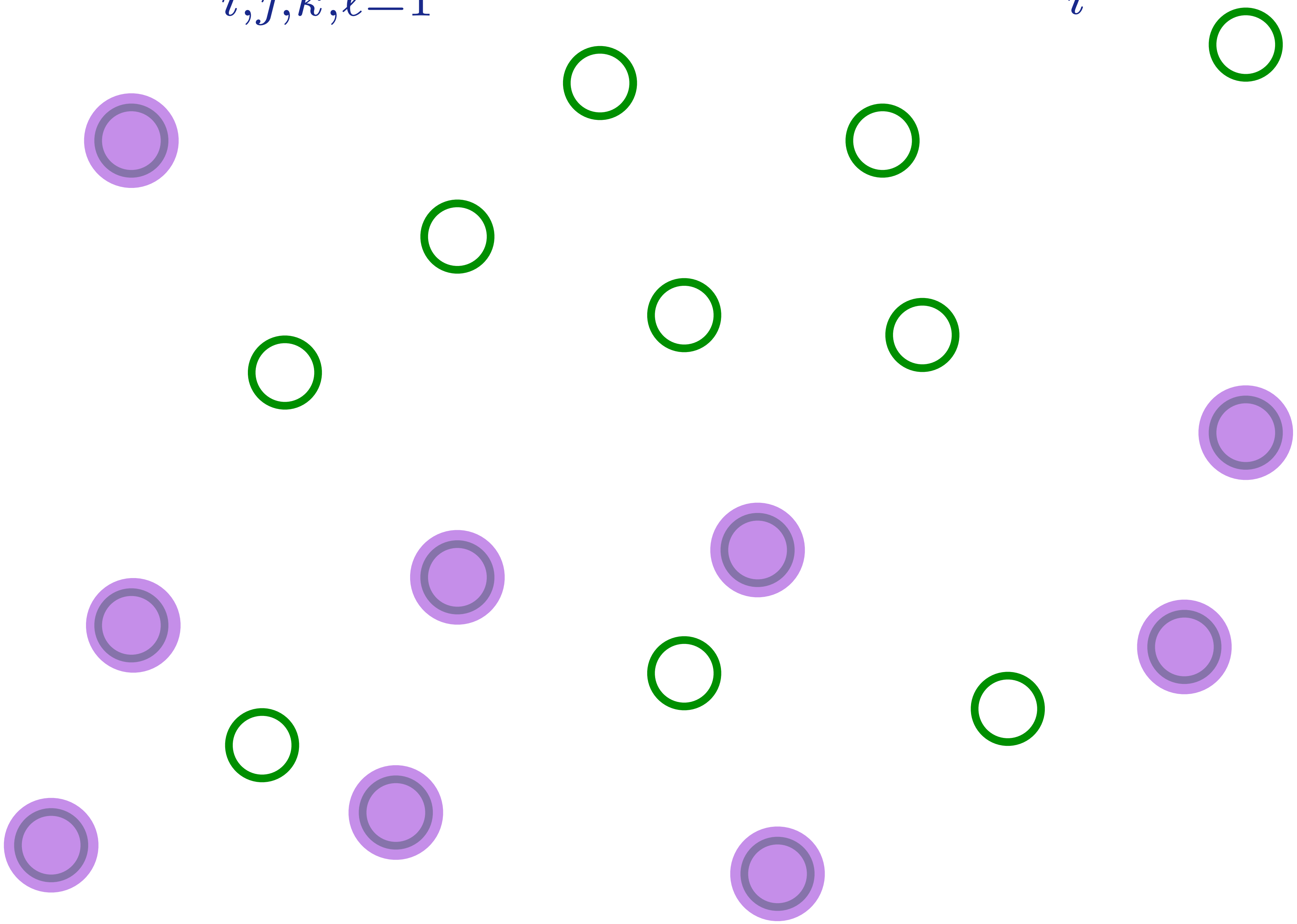

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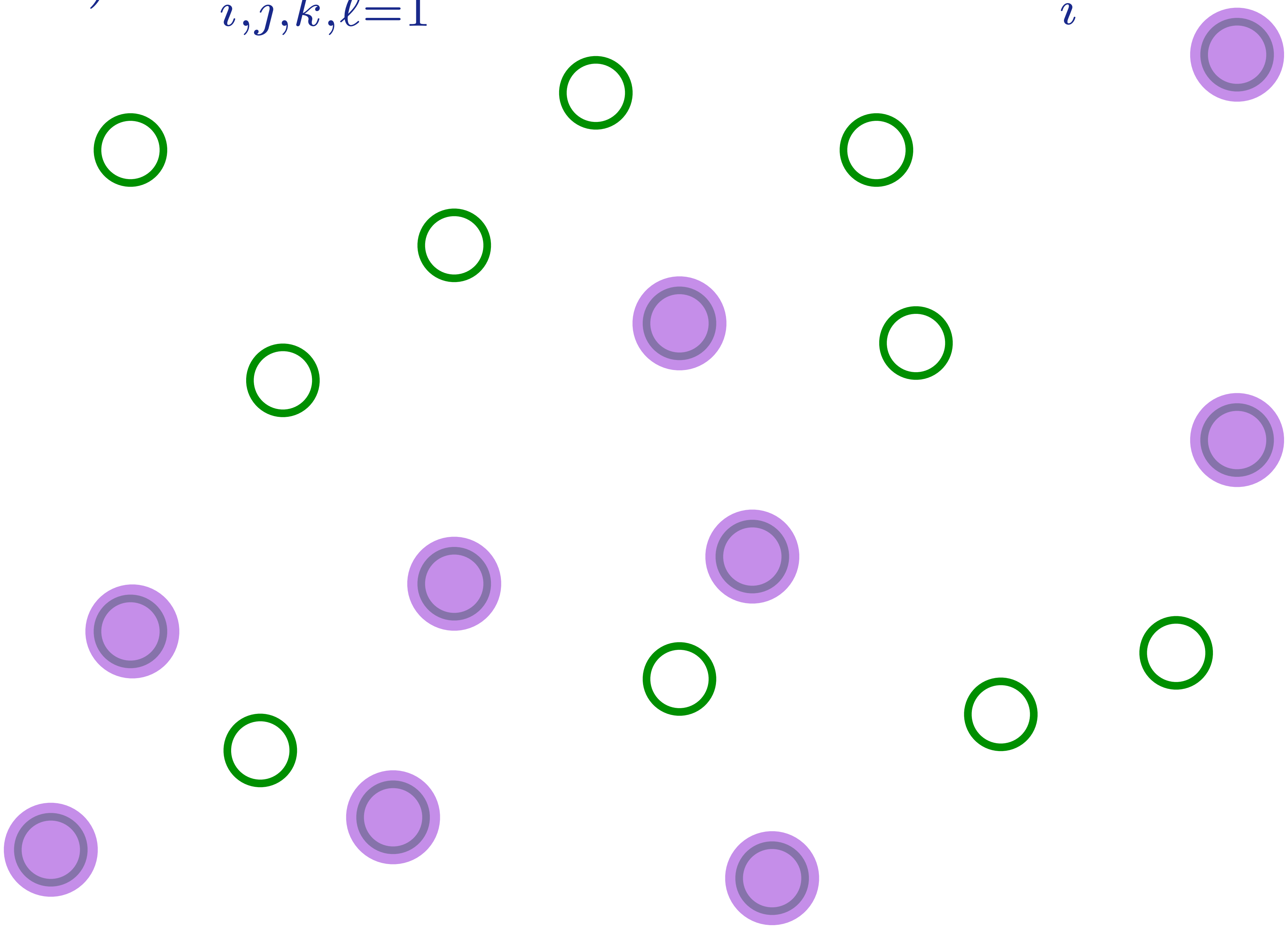

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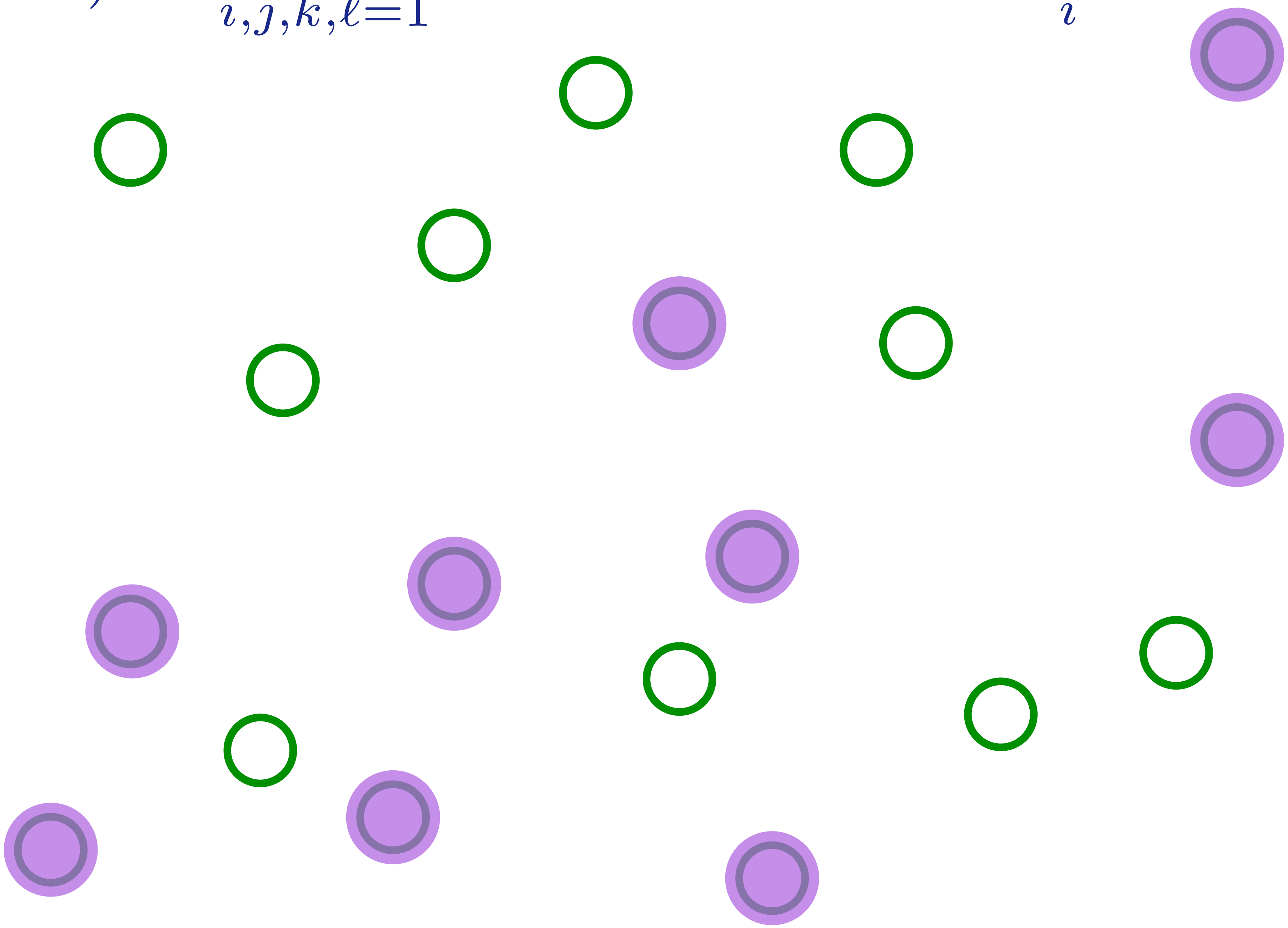

This describes both a strange metal and a black hole!

The SYK model

Feynman graph expansion in $U_{ijk\ell}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

The SYK model

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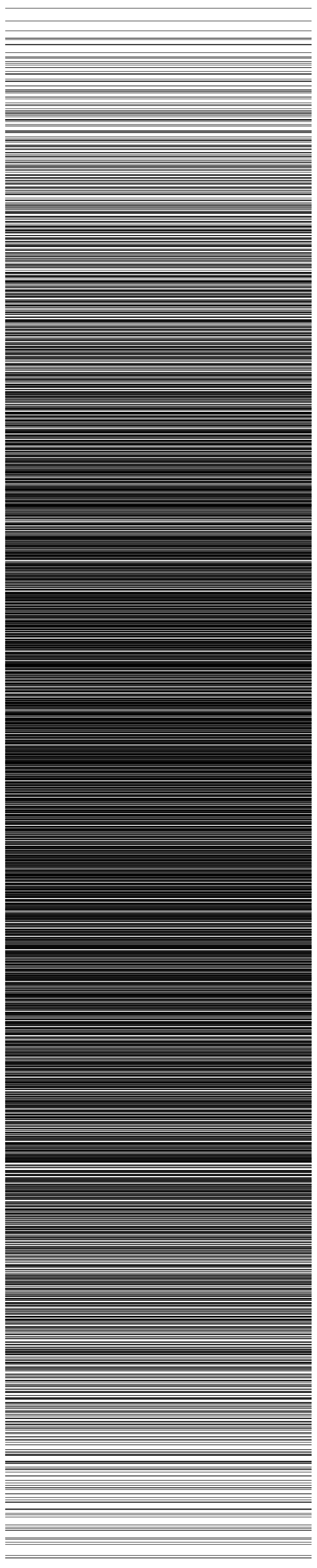
Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

where $A = e^{-i\pi/4}(\pi/U^2)^{1/4}$ at half-filling. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

The SYK model



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$ with

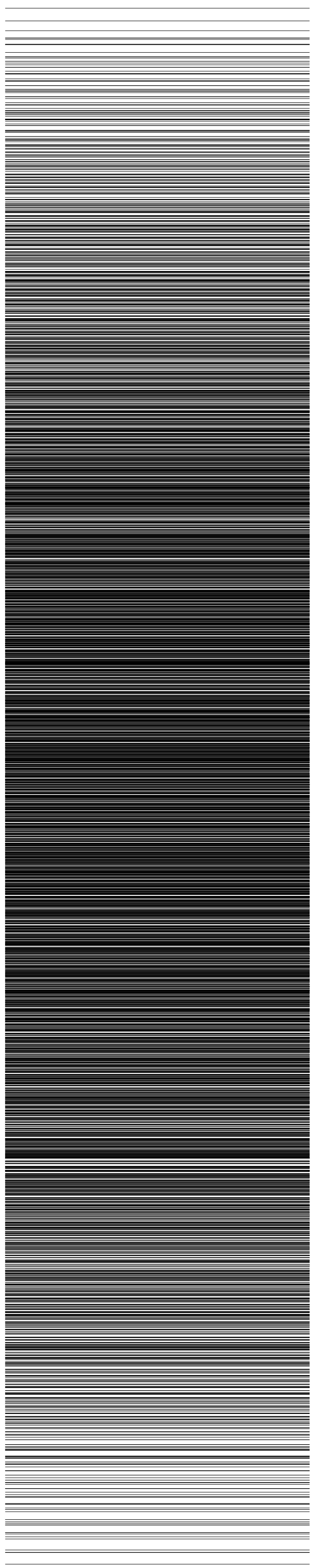
$$
\begin{aligned}
s_0 &= \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots \\
&< \ln 2
\end{aligned}
$$

where G is Catalan's constant, for the half-filled case $Q = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model



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For SYK models with $\mathcal{N} = 2$ supersymmetry, GPS=BPS

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model

- Low energy, many-body density of states $\rho(E) \sim e^{N s_0} \sinh(\sqrt{2(E-E_0)N\gamma})$

D. Stanford and E. Witten, 1703.04612
A. M. Garica-Garcia, J.J.M. Verbaarschot, 1701.06593
D. Bagrets, A. Altland, and A. Kamenev, 1607.00694

- Low temperature entropy $S = N s_0 + N\gamma T + \ldots$.

- $T = 0$ fermion Green's function $G(\tau) \sim \tau^{-1/2}$ at large τ. (Fermi liquids with quasiparticles have $G(\tau) \sim 1/\tau$)

- $T > 0$ Green's function has conformal invariance $G \sim (T/\sin(\pi k_B T\tau/\hbar))^{1/2}$

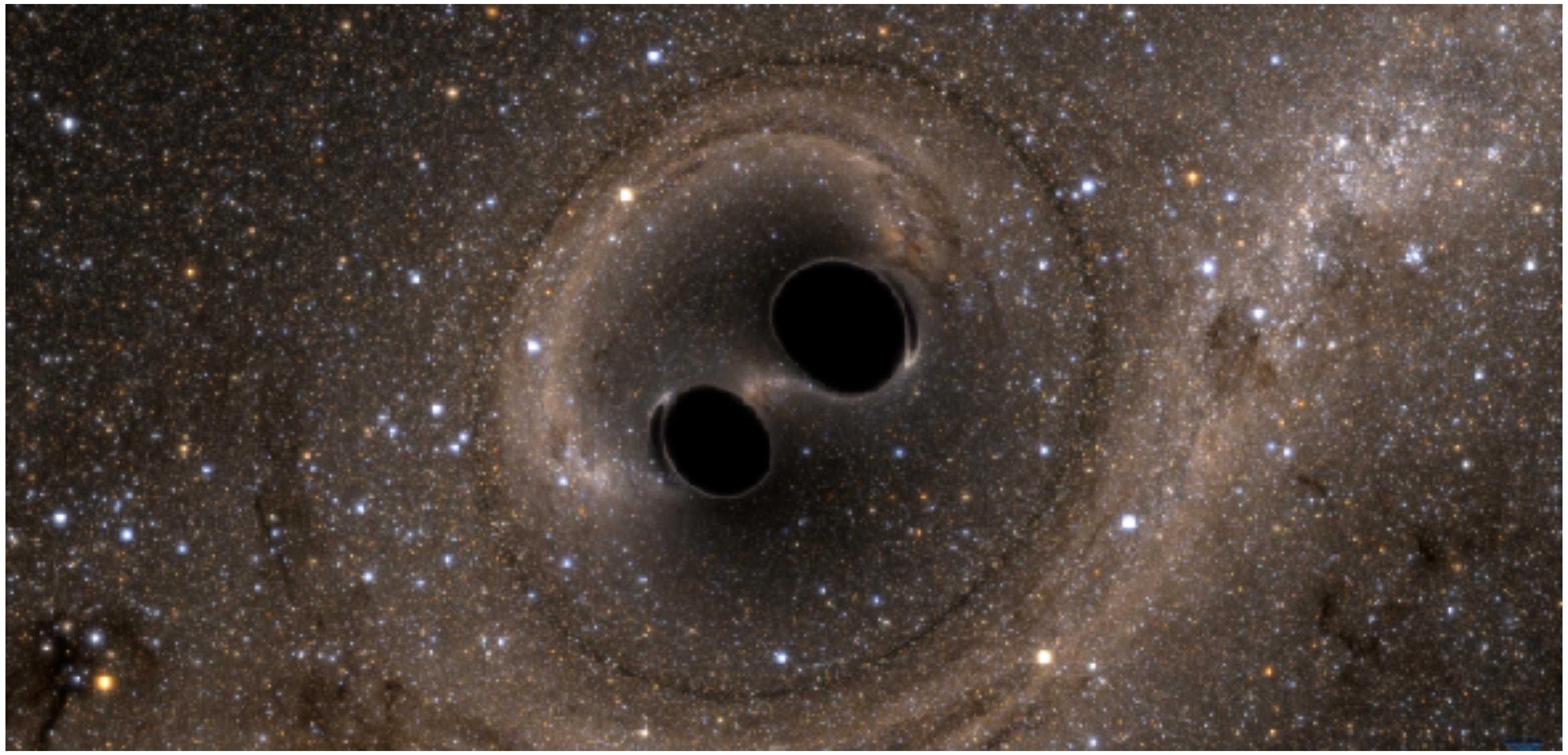
A. Georges and O. Parcollet PRB **59**, 5341 (1999)

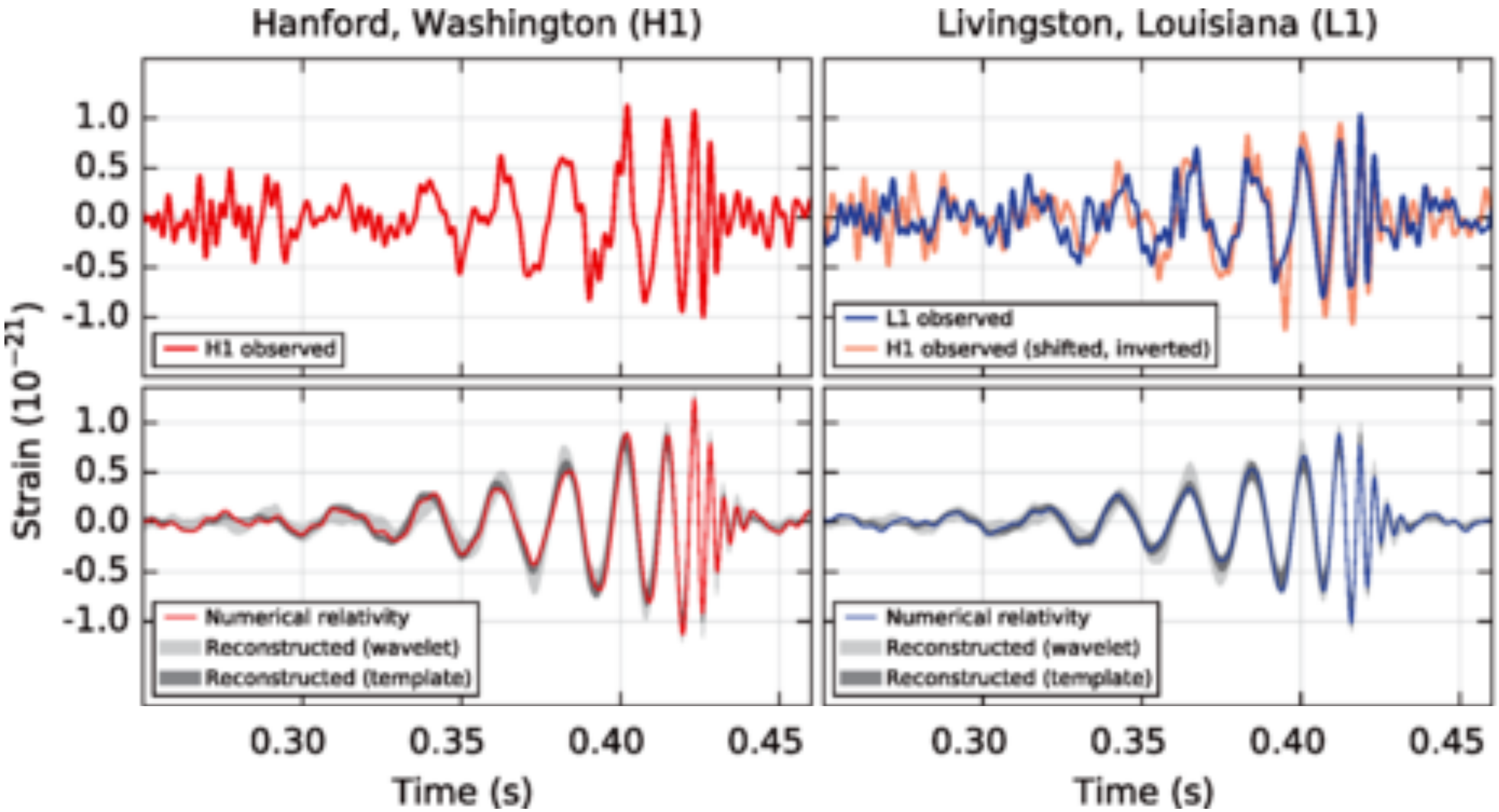
- The last property indicates $\tau_{\text{eq}} \sim \hbar/(k_B T)$, and this has been found in a recent numerical study.

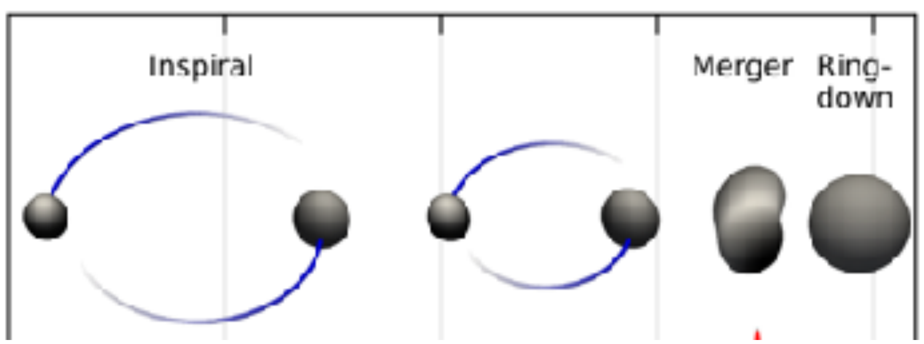
A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, arXiv:1706.07803

Black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a time $\sim \hbar/(k_B T_H)$.







LIGO
September 14, 2015

- The ring-down is predicted by General Relativity to happen in a time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously this happens to equal $\frac{\hbar}{k_B T_H}$: so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate!

The SYK model

- Low energy, many-body density of states $\rho(E) \sim e^{N s_0} \sinh(\sqrt{2(E - E_0)N\gamma})$

D. Stanford and E. Witten, 1703.04612
A. M. Garica-Garcia, J.J.M. Verbaarschot, 1701.06593
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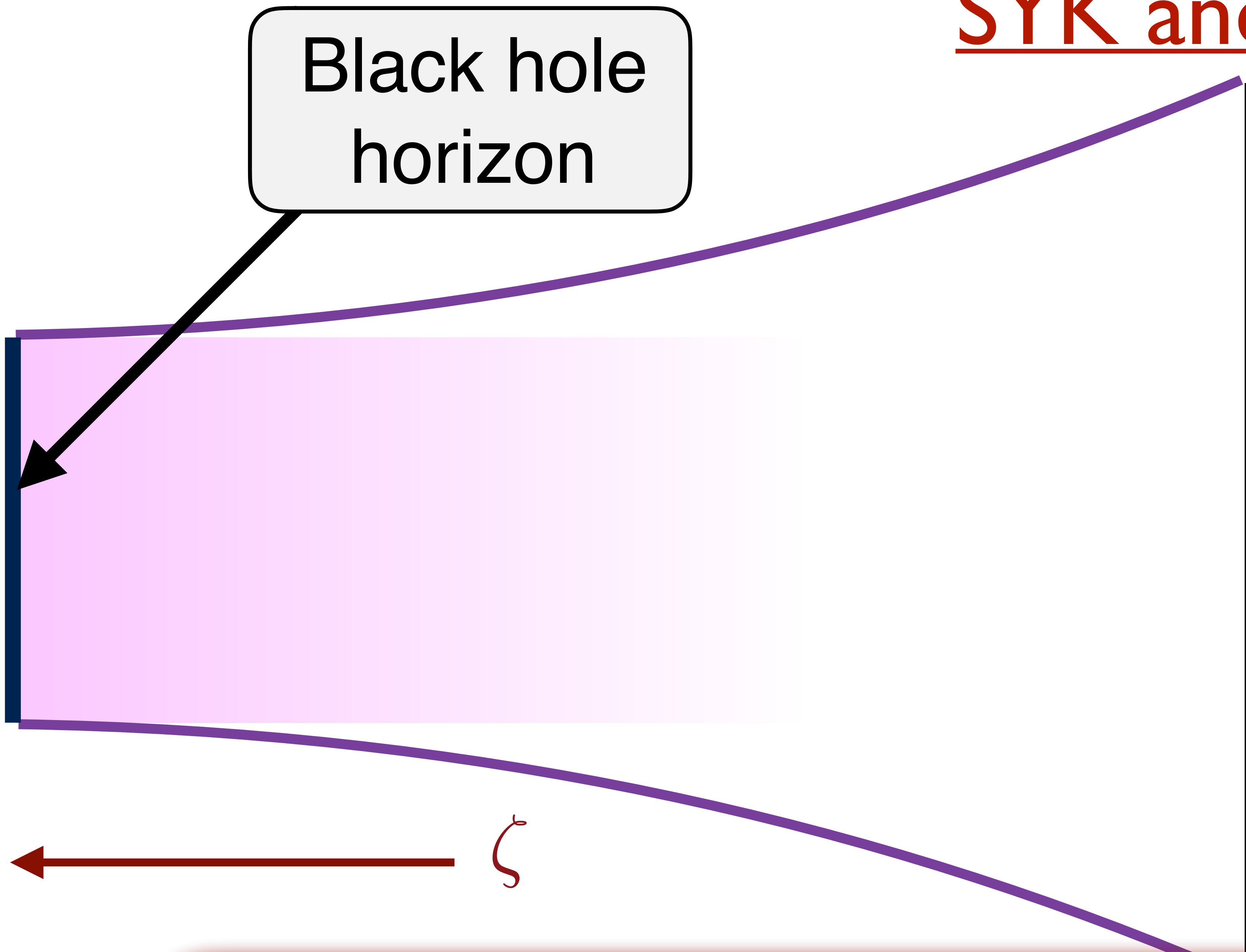
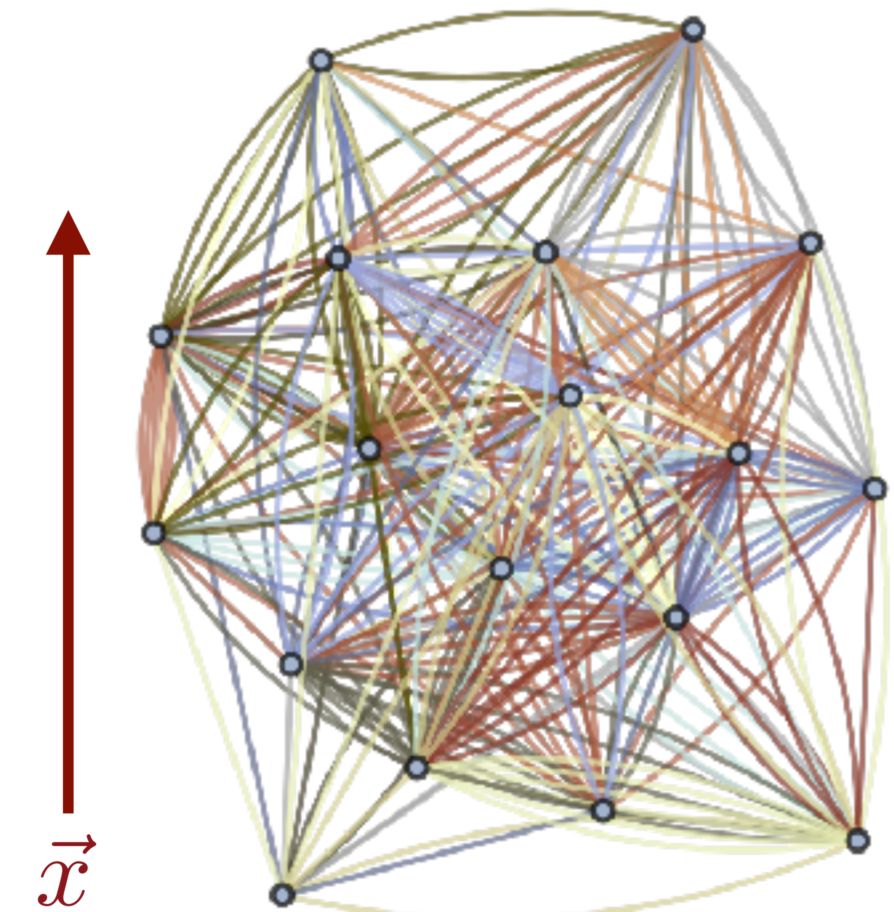
SYK and black holes





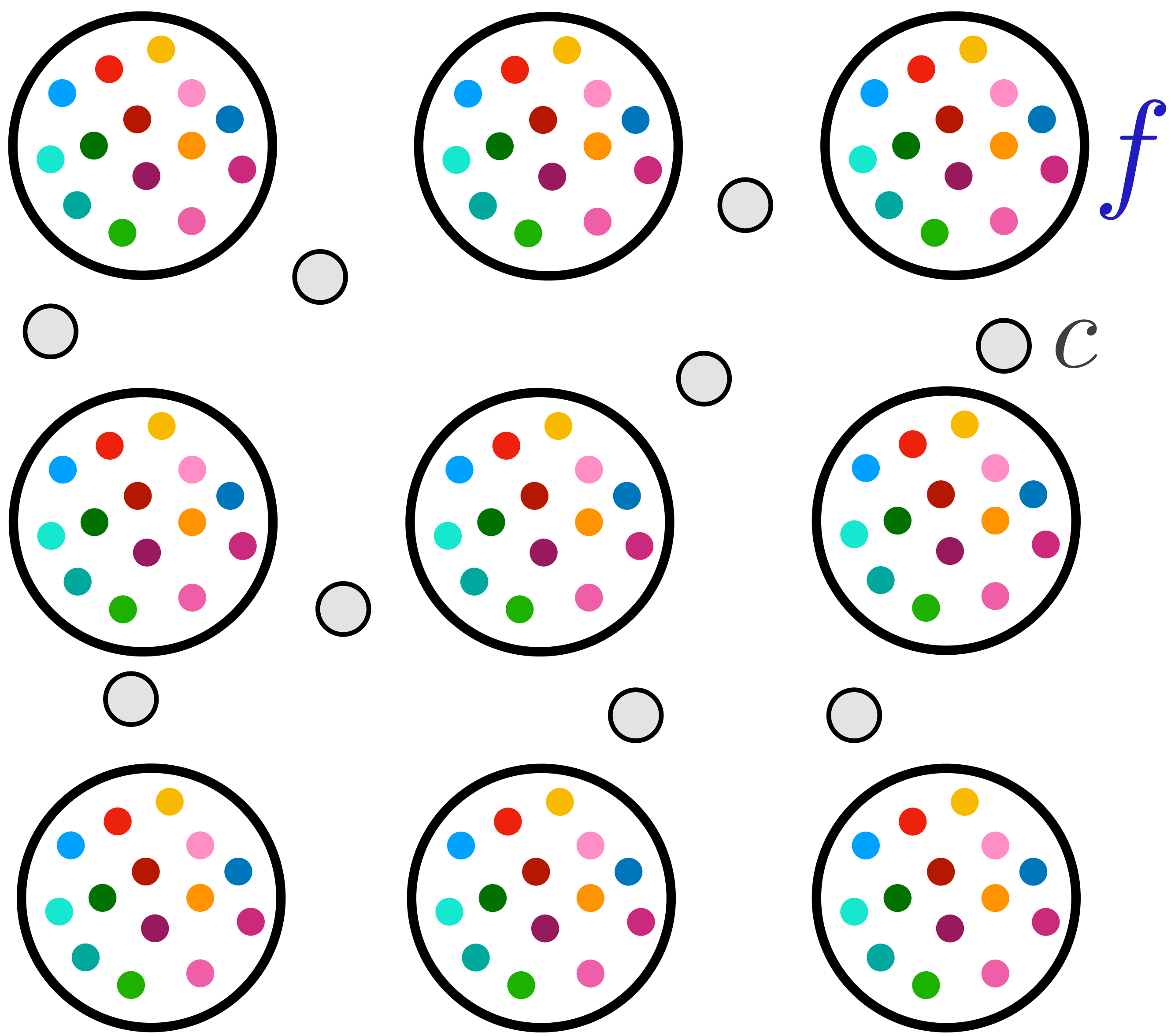
Black holes with a near-horizon AdS_2 geometry (described by quantum gravity in 1+1 spacetime dimensions) match the properties of the 0+1 dimensional SYK model in the previous slide: Ns_0 is the Bekenstein-Hawking entropy

S. Sachdev, PRL **105**, 151602 (2010); A. Kitaev (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857

Infecting a Fermi liquid and making it SYK

Mobile electrons (c) interacting with SYK quantum dots (f) with exchange interactions.

This yields the first model agreeing with magnetotransport in strange metals !



Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, arXiv:1801.06178

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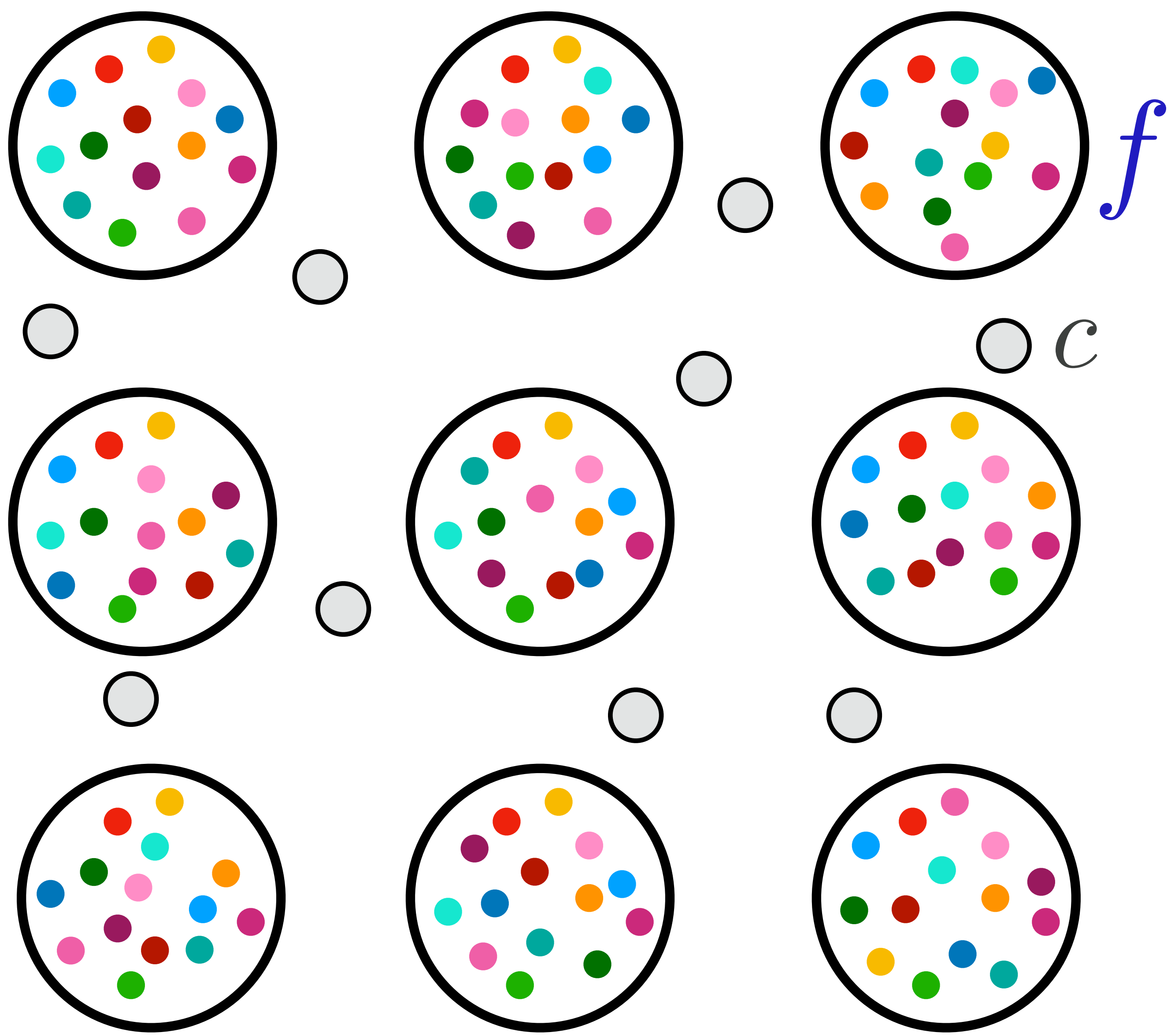


Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, arXiv:1712.05026

Infecting a Fermi liquid and making it SYK

Mobile electrons (*c*) interacting with SYK quantum dots (*f*) with exchange interactions.

Large N solution (with or without microscopic disorder) yields a 'marginal Fermi liquid' metal, with conductivities of the form:

$$
\begin{aligned}
\sigma_{xx}(B,T) &= \frac{1}{T}\Phi_L\left(\frac{B}{T}\right) \\
\sigma_{xy}(B,T) &= \frac{B}{T^2}\Phi_H\left(\frac{B}{T}\right)
\end{aligned}
$$

where the scaling functions interpolate as

$$
\Phi_{L,H}(b\to 0)\sim \text{constant} \quad ; \quad \Phi_{L,H}(b\to\infty)\sim 1/b^2
$$

This solution exhibits B/T scaling, but the magnetoresistance ρ_{xx} saturates for $B \gg T$.

Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, arXiv:1712.05026

Infecting a Fermi liquid and making it SYK

Need mesoscopic disorder to obtain linear-in-B magnetoresistance

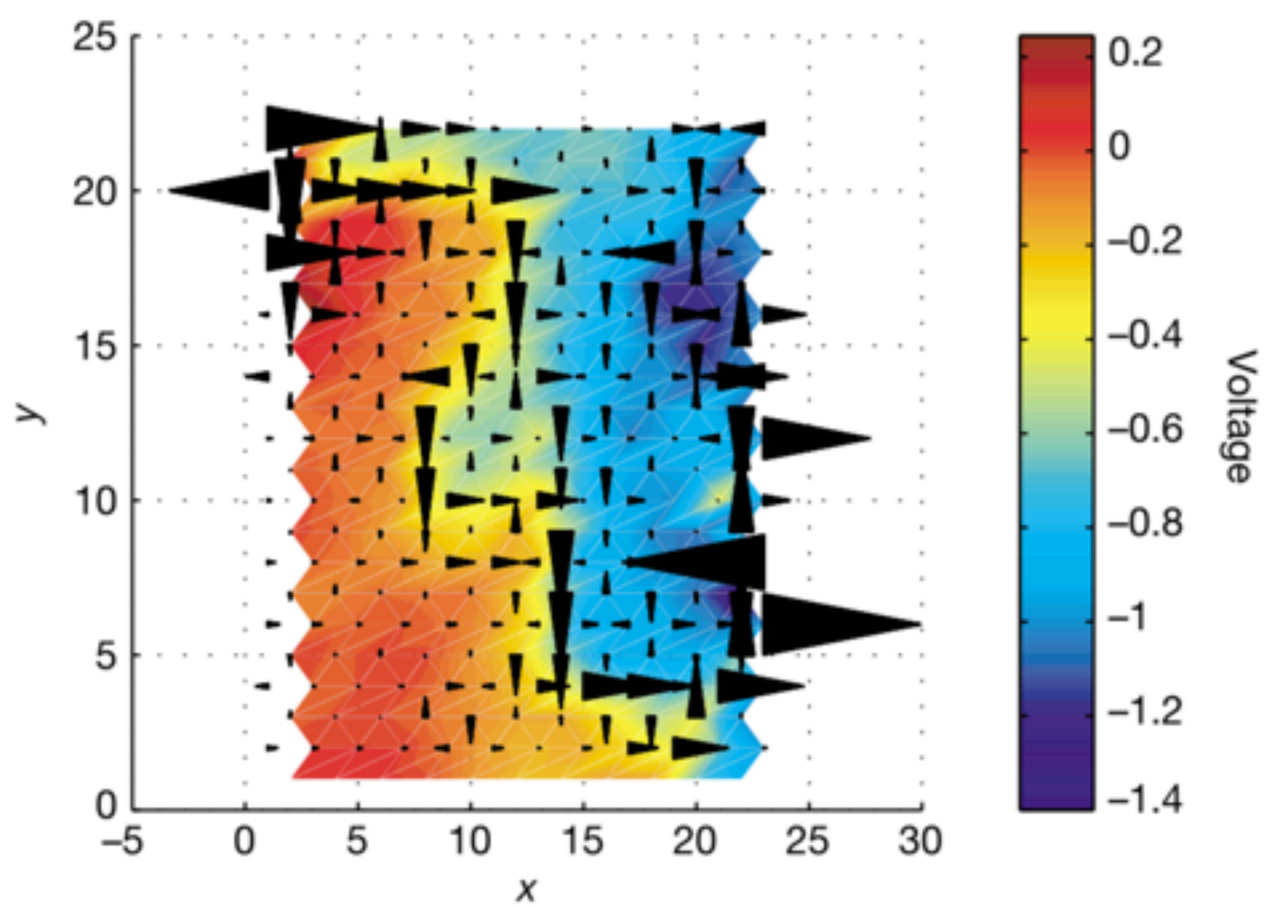
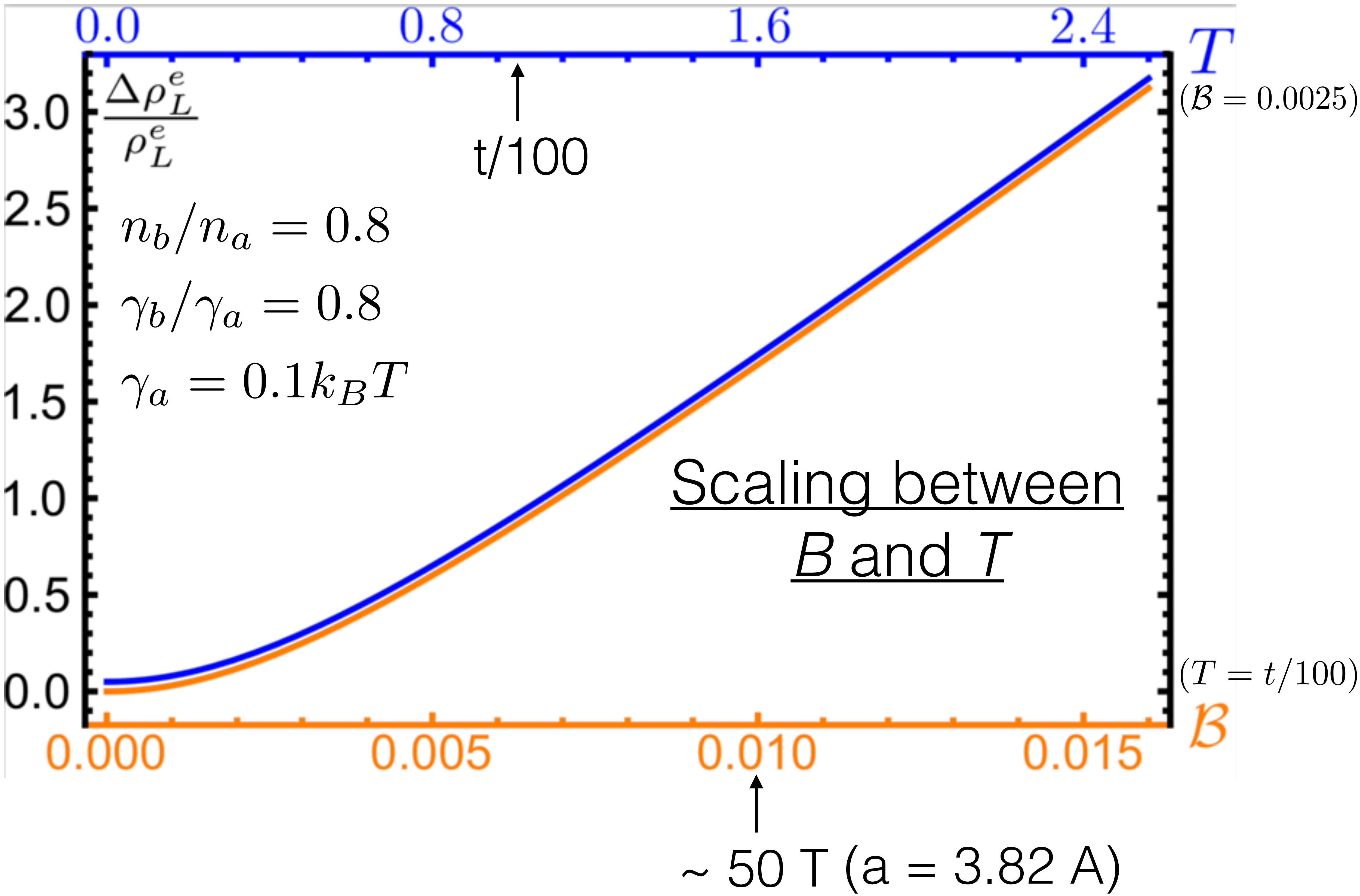


Figure 3 Visualization of currents and voltages at large magnetic field in a 10 × 10 random network of disks with radii 1 (arbitrary units), where the potential difference $U = -1$ V. The black arrows represent the currents, and arrow size depicts the magnitude of the current. The major current path is perpendicular to the applied voltage for a significant proportion of the time, which implies that the magnetoresistance is provided internally by the Hall effect, which is therefore linear in H.

- Current path length increases linearly with B at large B due to local Hall effect, which causes the global resistance to increase linearly with B at large B.

Exact numerical solution of charge-transport equations in a random-resistor network. (M. M. Parish and P. Littlewood, Nature 426, 162 (2003))

Infecting a Fermi liquid and making it SYK



Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, arXiv:1712.05026

- This simple two-component model describes a new state of matter which is realized by electrons in the presence of strong interactions and disorder.

- Can such a model be realized as a fixed-point of a generic theory of strongly-interacting electrons in the presence of disorder?

- Can we start from a single-band Hubbard model with (or without) disorder, and end up with such two-band fixed point, with emergent local conservation laws?

- Electrons in doped silicon appear to separate into two components: localized spin moments and itinerant electrons

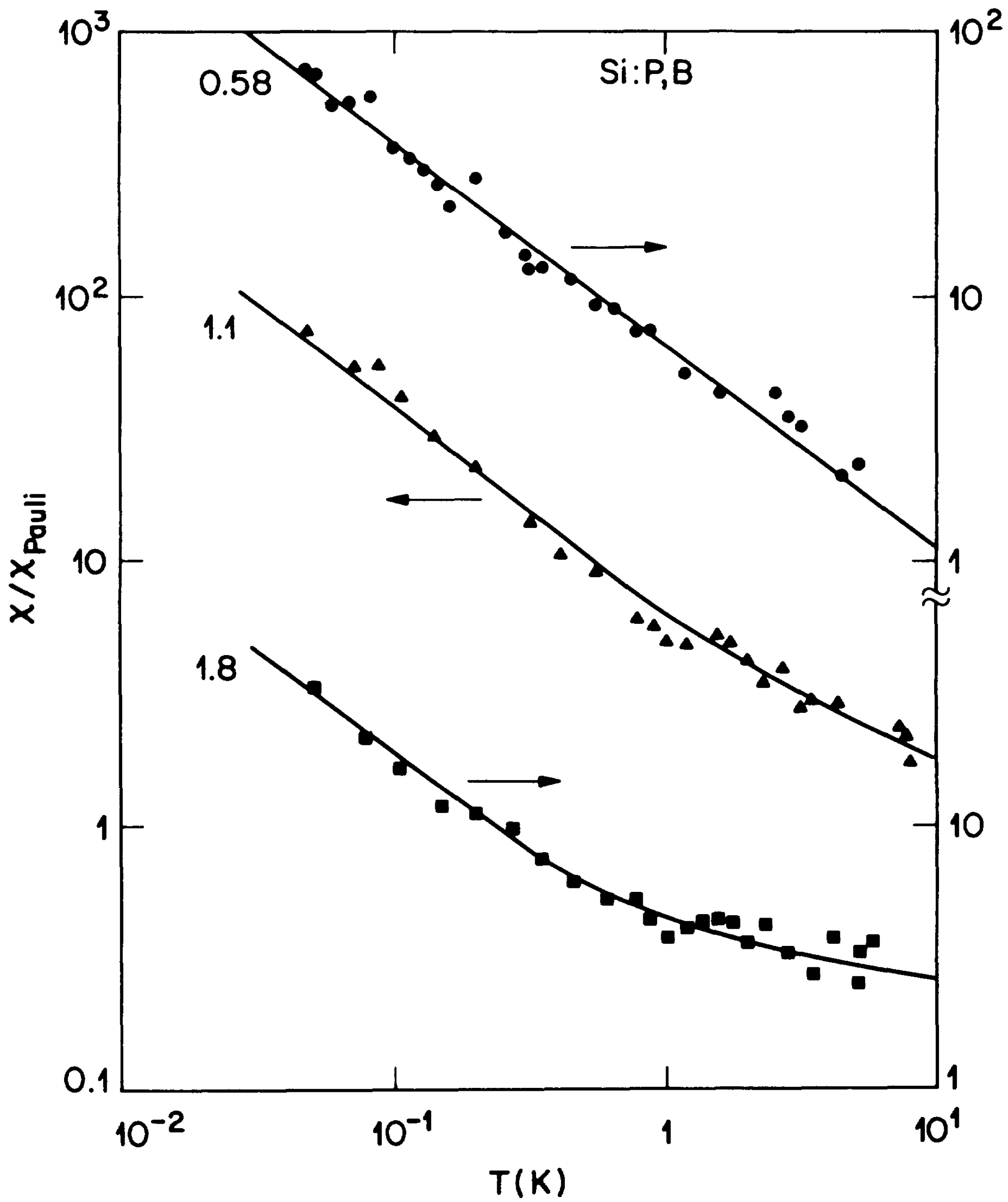
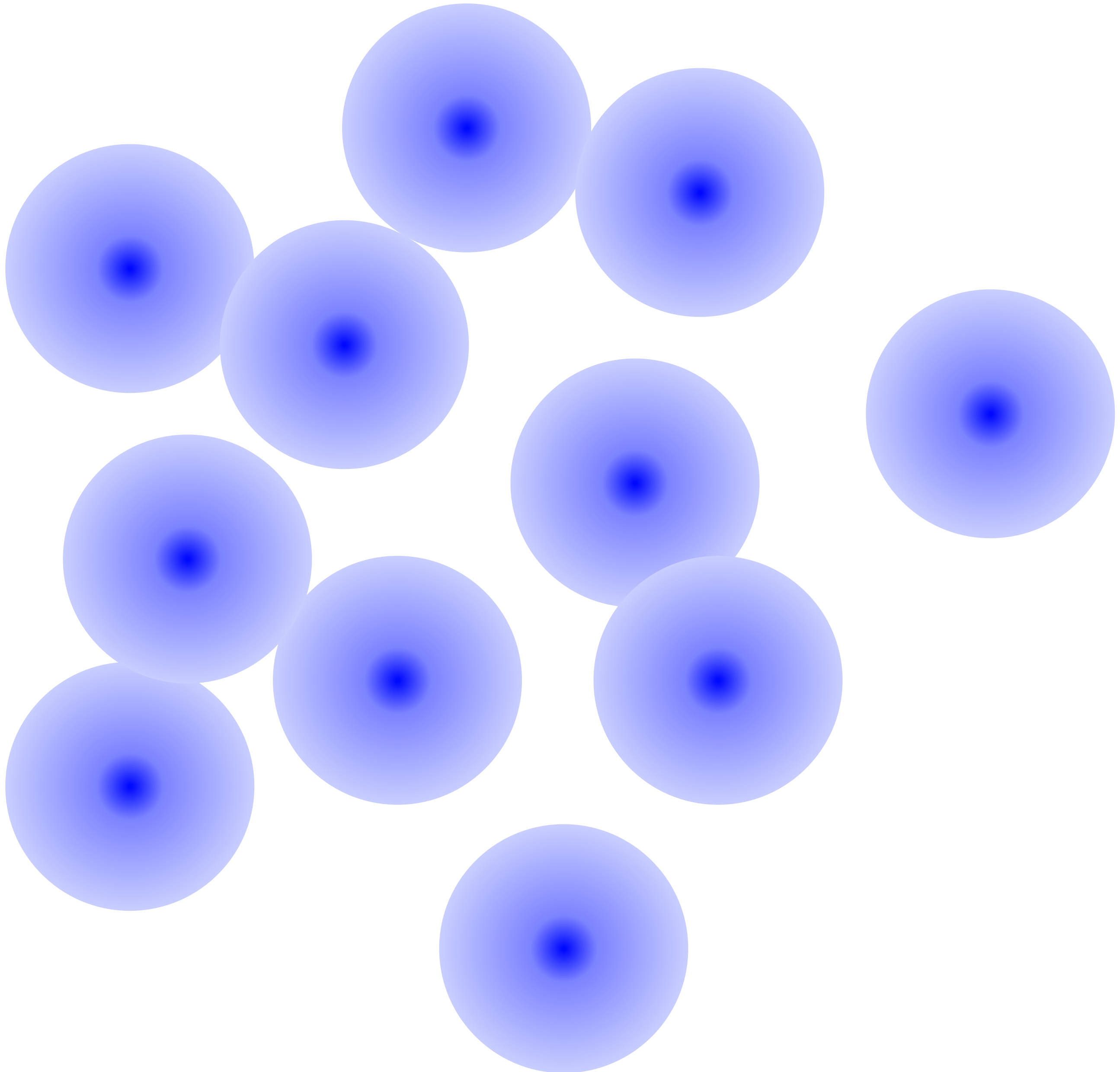
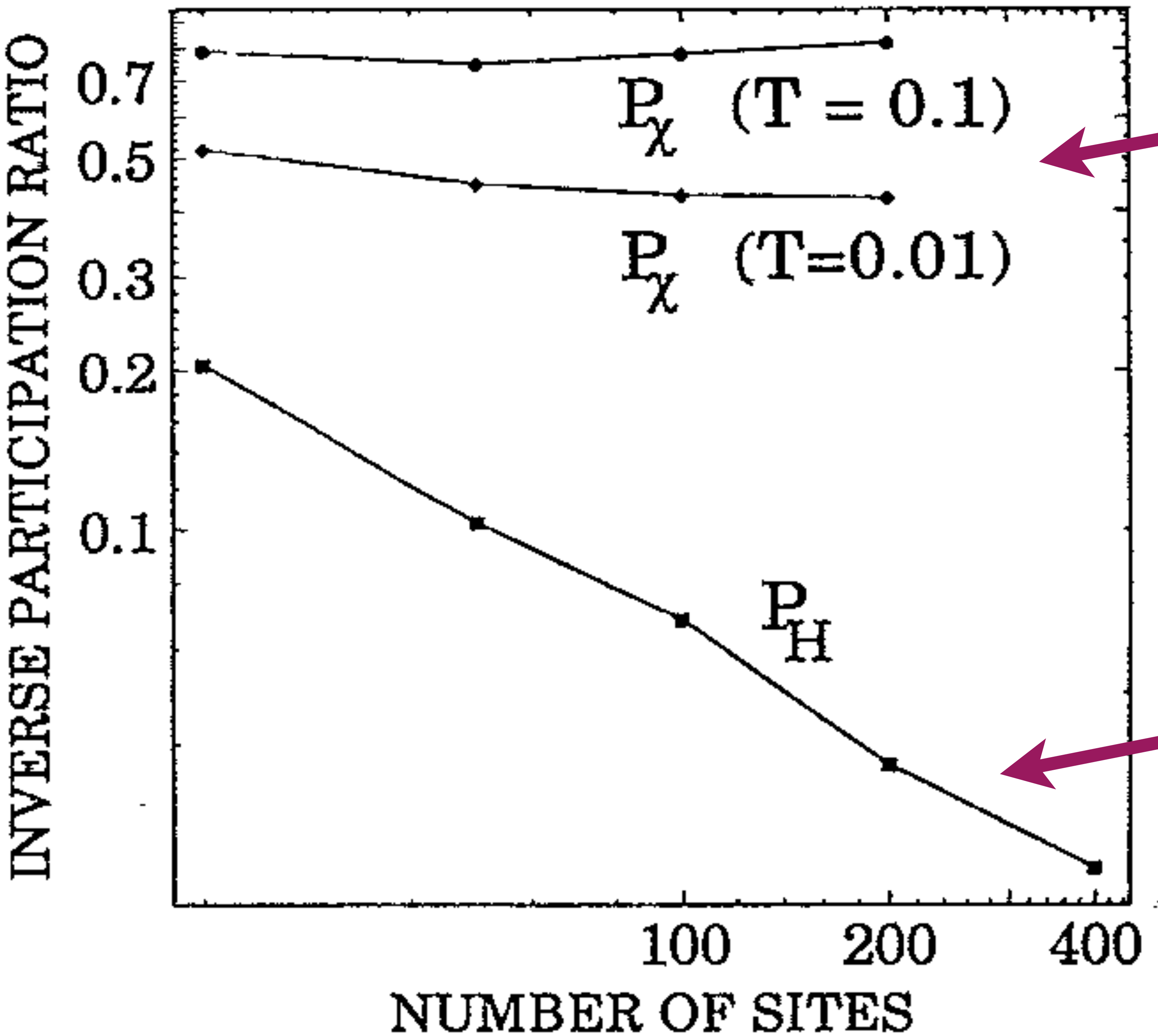


FIG. 1. Temperature dependence of normalized susceptibility χ/χ_{Pauli} of three Si:P,B samples with different normalized electron densities, n/n_c =0.58, 1.1, and 1.8. Solid lines through data are a guide to the eye.

M. J. Hirsch, D.F. Holcomb, R.N. Bhatt, and M.A. Paalanen
PRL **68**, 1418 (1992)



M. Milovanovic, S. Sachdev and R.N. Bhatt, PRL **63**, 82 (1989)
A.C. Potter, M. Barkeshli, J. McGreevy, T. Senthil, PRL **109**, 077205 (2012)



Magnetic excitations are localized (eigenmodes of the spin susceptibility)

Charged fermionic excitations are extended (eigenmodes of the electron Hamiltonian)

M. Milovanovic, S. Sachdev and R.N. Bhatt, PRL **63**, 82 (1989)

Many-body quantum chaos and transport

- Using holographic analogies, Shenker, Stanford, Maldacena introduced the "Lyapunov time", τ_L, the time over which a generic many-body quantum system loses memory of its initial state (defined via an "out-of-time-order" (OTOC) correlator), and established a shortest-possible time to reach quantum chaos

$$\tau_L \geq \frac{\hbar}{2\pi k_B T}$$

A related bound was proposed earlier (Sachdev, 1999). The SYK model, and black holes in Einstein gravity, saturate this bound.

Many-body quantum chaos and transport

- Much theoretical work has examined connections between this chaos bound, and measurable transport co-efficients. The most robust connection is to the *thermal diffusivity*, D_T

(Kapitulnik)

$$D_T \sim v_B^2 \tau_L$$

where v_B is the "butterfly velocity" at which chaos propagates.

A. A. Patel and S. Sachdev, PNAS **114**, 1844 (2017)
M. Blake, R. A. Davison, and S. Sachdev, PRD **96**, 106008 (2017)

Many-body quantum chaos and transport

A quantum hydrodynamical description for scrambling and many-body chaos

Mike Blake, Hyunseok Lee, and Hong Liu

arXiv:1801.00010

- A direct connection was established between the energy diffusion mode and the OTOCs measuring chaos. This assumes 'dominance' of the energy diffusion mode up to energies of order $k_B T$, which is beyond the hydrodynamic regime. Such a dominance holds in SYK models and holographic theories, and can be expected more generally in ultra-quantum metals without quasiparticle excitations.

Non-zero density of fermions strongly coupled to gauge fields,

(in the presence of disorder)

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(in the presence of disorder)

- Half-filled Landau level (Son)
- Surfaces of (correlated) topological insulators

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- Half-filled Landau level (Son)
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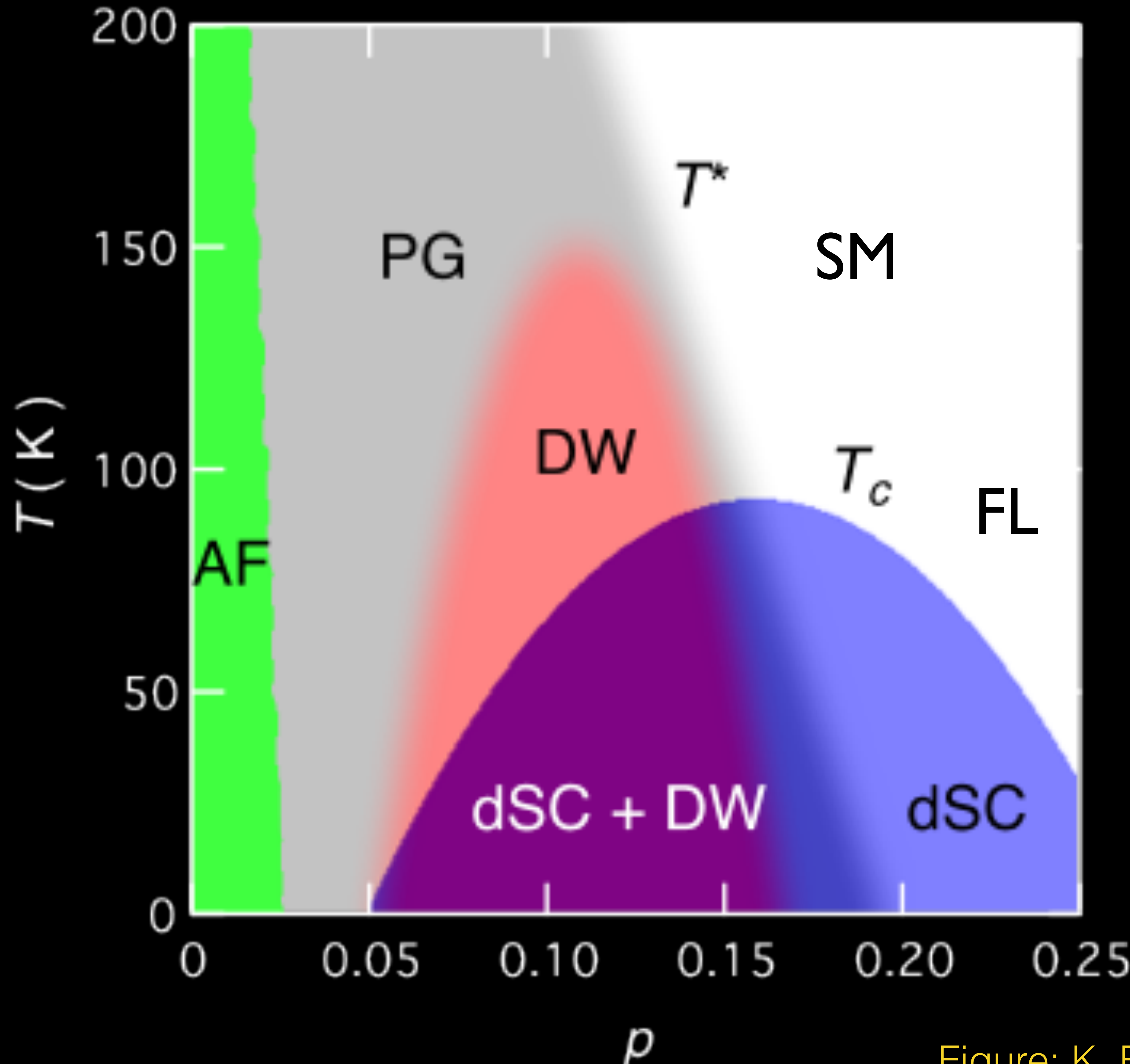
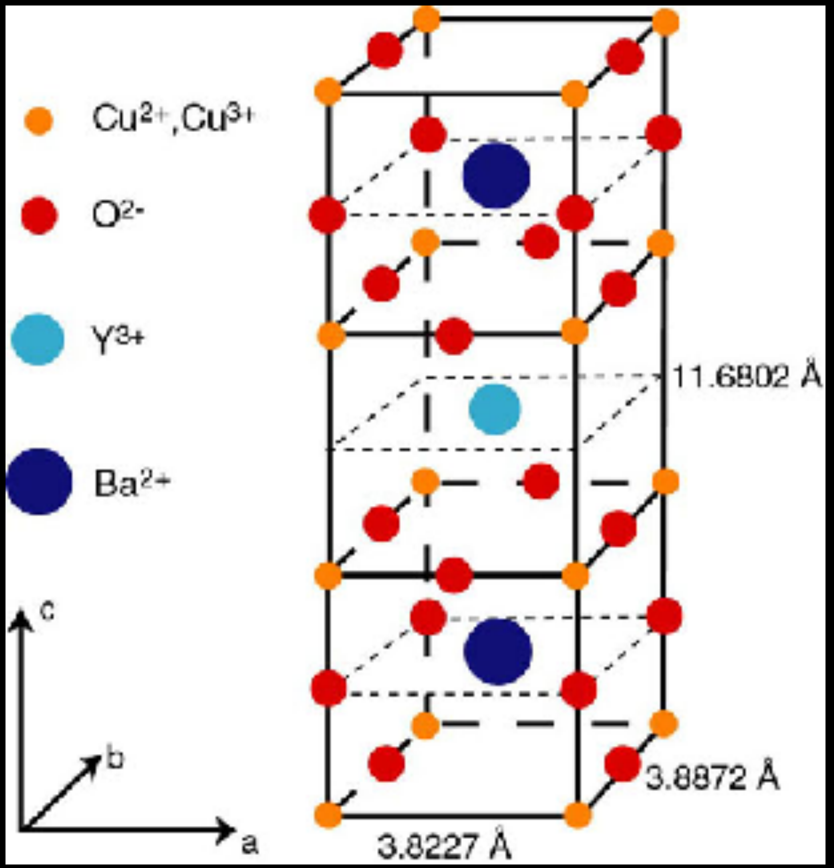
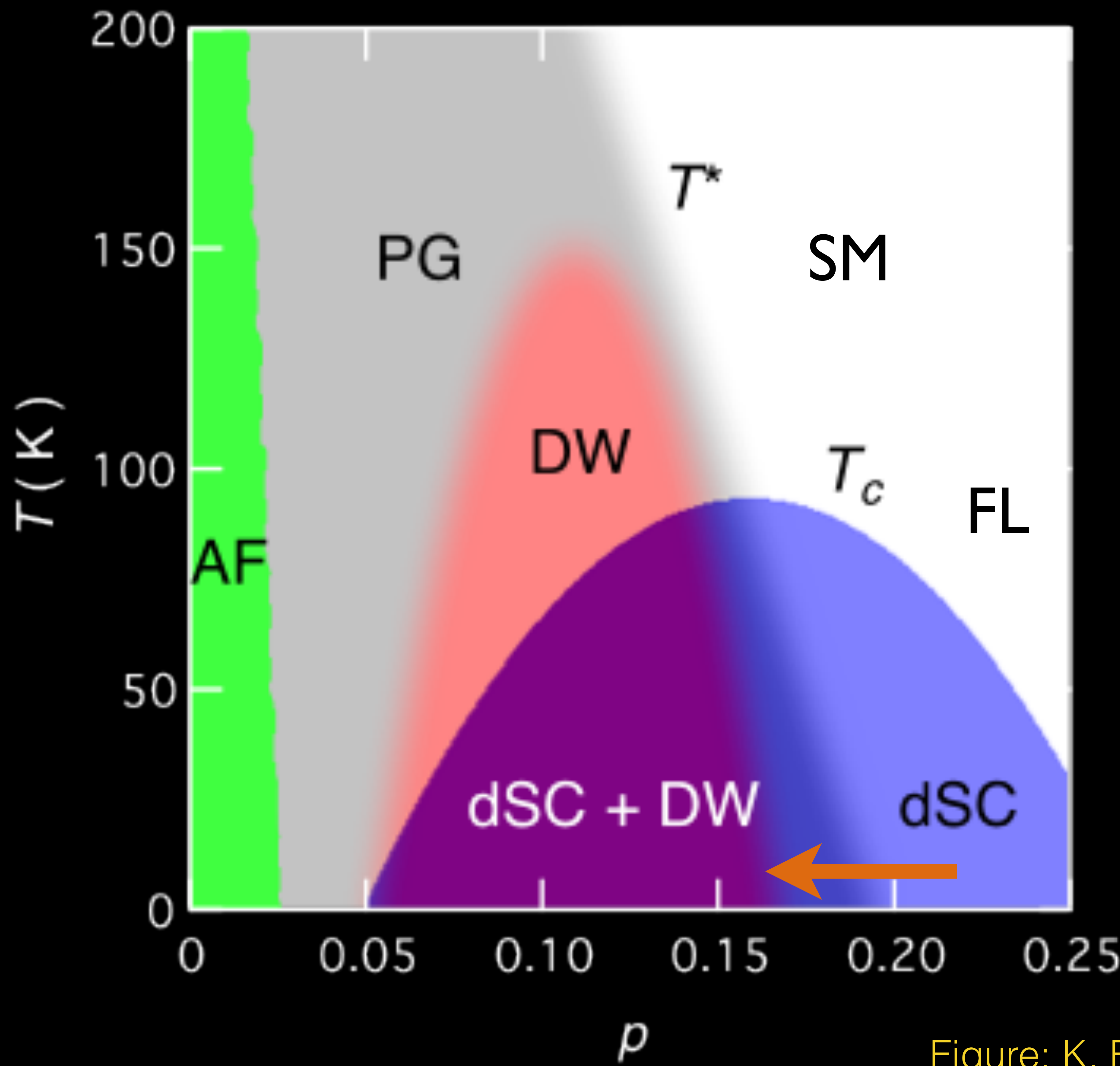
- Strange metal state of high temperature superconductors
- Pseudogap state of hole-doped cuprates

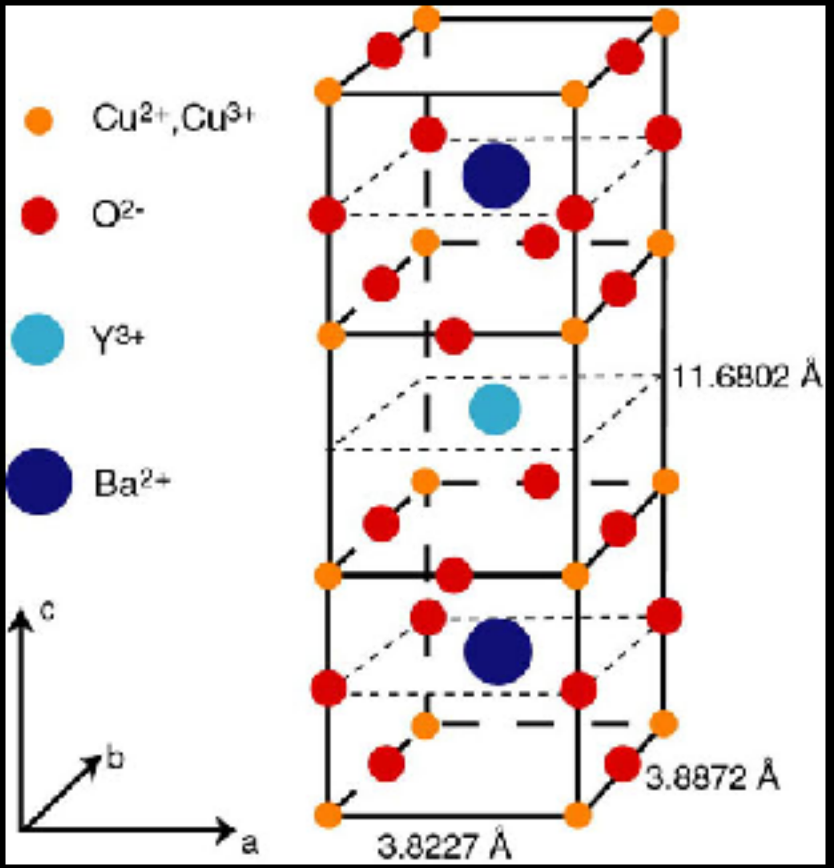




$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis





$YBa_2Cu_3O_{6+x}$

Deconfinement transition of a gauge theory ?

Figure: K. Fujita and J. C. Seamus Davis

Non-zero density of fermions strongly coupled to gauge fields,
(in the presence of disorder)

- Half-filled Landau level (Son)
- Surfaces of (correlated) topological insulators
- Strange metal state of high temperature superconductors
- Pseudogap state of hole-doped cuprates

Non-zero density of fermions strongly coupled to gauge fields,

(in the presence of disorder)

- Distinct phases of the gauge theory can be distinguished even in the presence of disorder

- Is the usual confinement/Higgs/deconfinement criterion general enough in the presence of disorder?

- Novel critical points or phases?

A solvable model

Fractionalize the electron ($c_{i\alpha}$, $i = 1 \ldots N$, $\alpha = 1 \ldots M$) into an "orthogonal fermion" $f_{i\alpha}$ and an Ising spin $\sigma_i^z = \pm 1$:

(Senthil, Metlitski, Vishwanath, Sachdev…)

$$c_{i\alpha} = \sigma_i^z \, f_{i\alpha}$$

This introduces a $\mathbb{Z}_2$ gauge invariance

$$\sigma_i^z \to \eta_i \sigma_i^z \quad , \quad f_{i\alpha} \to \eta_i f_{i\alpha}$$

A solvable model

Fractionalize the electron ($c_{i\alpha}$, $i = 1 \ldots N$, $\alpha = 1 \ldots M$) into an "orthogonal fermion" $f_{i\alpha}$ and an Ising spin $\sigma_i^z = \pm 1$:

(Senthil, Metlitski, Vishwanath, Sachdev…)

$$c_{i\alpha} = \sigma_i^z \, f_{i\alpha}$$

This introduces a $\mathbb{Z}_2$ gauge invariance

$$\sigma_i^z \to \eta_i \sigma_i^z \quad , \quad f_{i\alpha} \to \eta_i f_{i\alpha}$$

The solvable model is (closely related to) the $N \to \infty$, $M \to \infty$ limit of

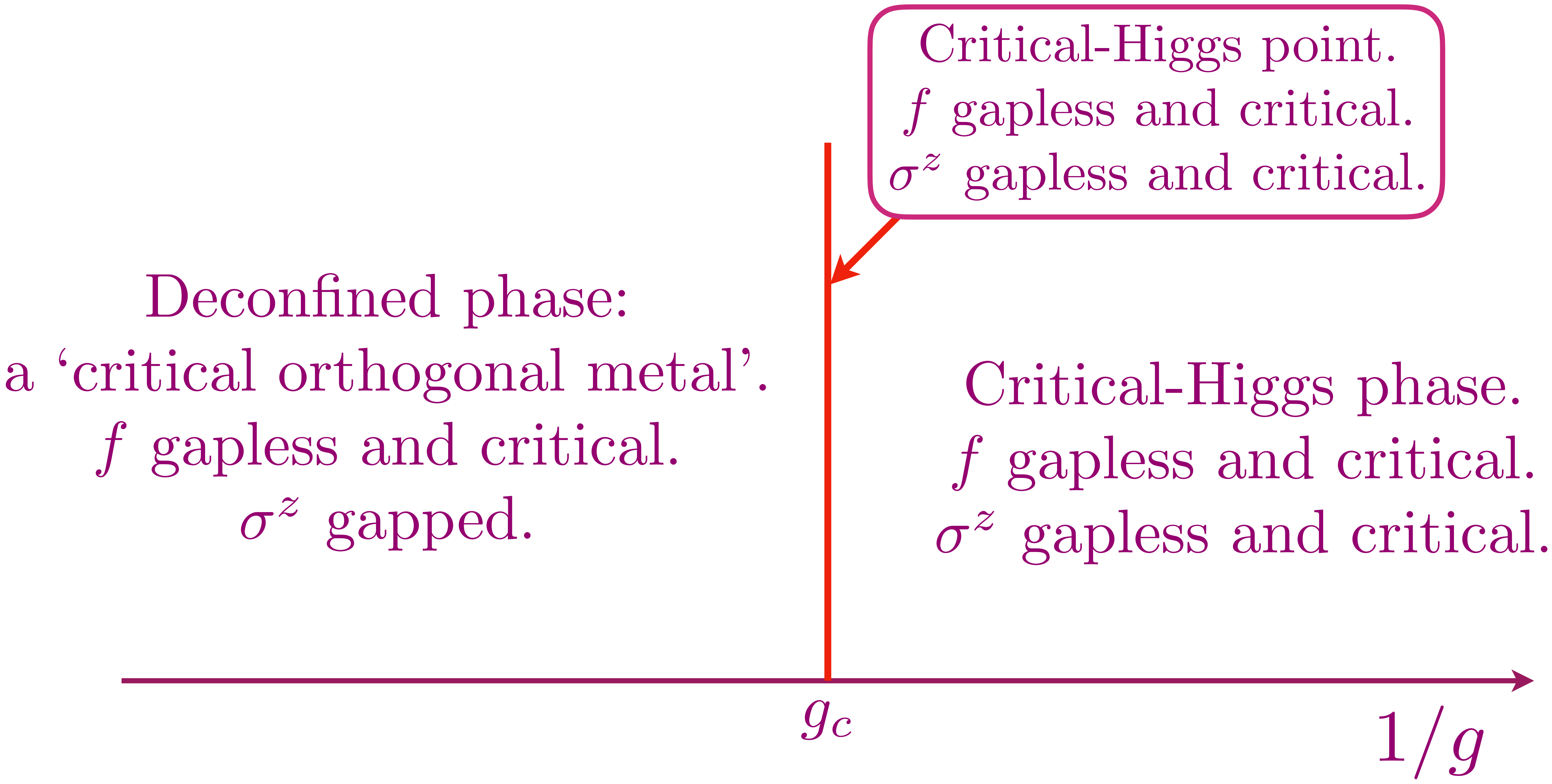
$$H = \sum_i \left(-\mu f_{i\alpha}^\dagger f_{i\alpha} - g \sigma_i^x \right) + \frac{1}{\sqrt{NM}} \sum_{i,j} t_{ij} \sigma_i^z \sigma_j^z f_{i\alpha}^\dagger f_{j\alpha} + \frac{1}{\sqrt{NM}} \sum_{i,j} J_{ij} f_{i\alpha}^\dagger f_{i\beta} f_{j\beta}^\dagger f_{j\alpha}$$

where t_{ij} and J_{ij} are random numbers.

What is the fate of the $\mathbb{Z}_2$ gauge theory as a function of the coupling g?

A solvable model

There is no confining/Higgs phase, but there is a phase transition between a deconfined phase and a "critical-Higgs" phase.



(Wenbo Fu, Yingfei Gu, SS, G. Tarnopolsky)

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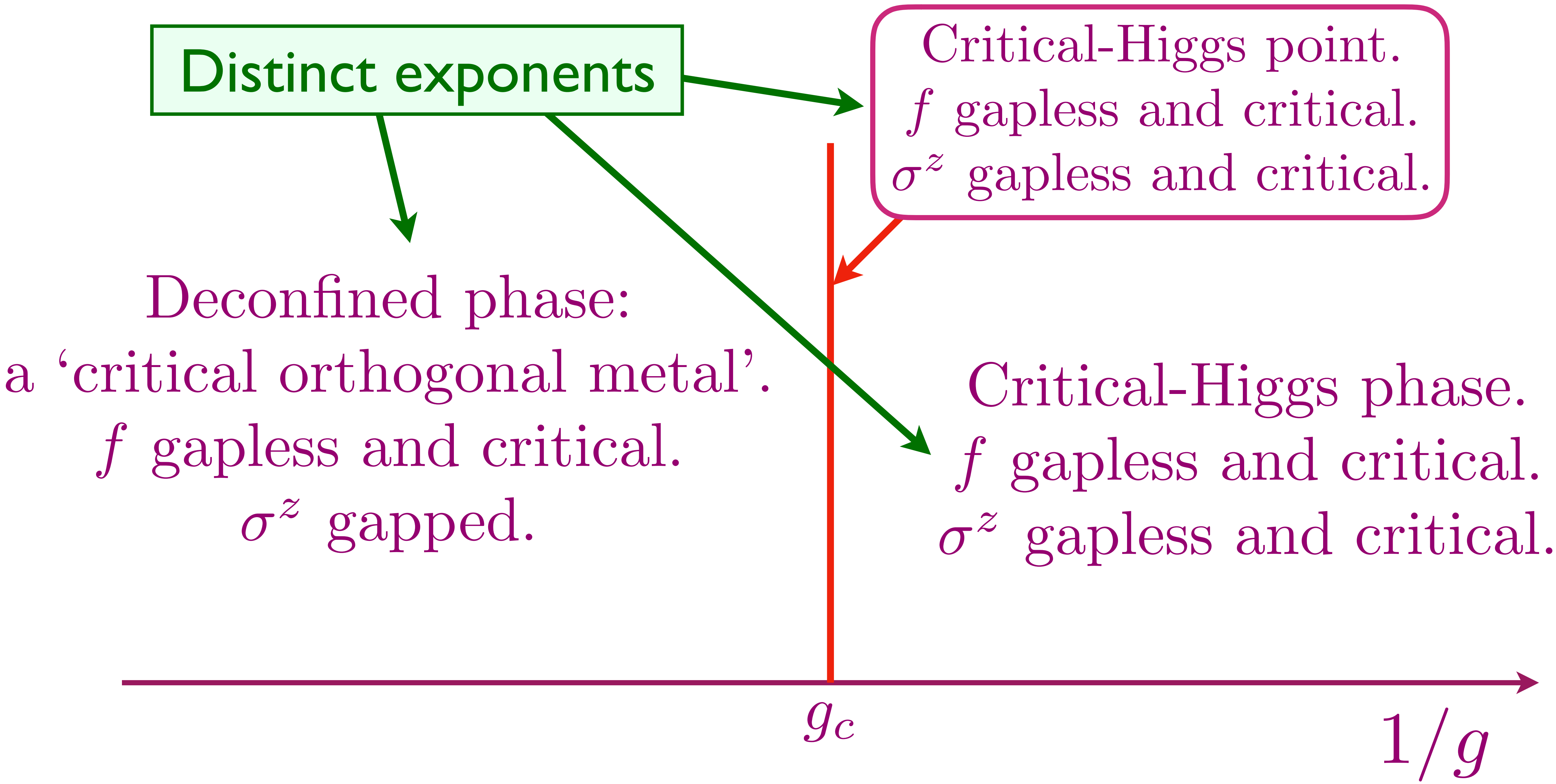

(Wenbo Fu, Yingfei Gu, SS, G. Tarnopolsky)

Thoughts on field theories of ultra-quantum metals

- Breakdown of quasiparticles requires strong coupling to a low energy collective mode

- In all known cases, we can write down the singular processes in terms of a continuum field theory of the fermions near the Fermi surface coupled to the collective mode.

- In all known cases, the continuum critical theory has a conserved total (pseudo-) momentum, $\vec{P}$, which commutes with the Hamiltonian. This momentum may not be equal to the crystal momentum of the underlying lattice model.

Thoughts on field theories of ultra-quantum metals

- As long as $\chi_{\vec{J},\vec{P}} \neq 0$ (where $\vec{J}$ is the electrical current) the d.c. resistivity of the critical theory is exactly zero. This is the case even though the electron self energy can be highly singular and there are no fermionic quasiparticles (many well-known papers on non-Fermi liquid transport ignore this point.)

- We need to include additional (dangerously) irrelevant umklapp corrections to obtain a non-zero resistivity. Because these additional corrections are irrelevant, it is difficult to see how they can induce a linear-in-T resistivity.

Thoughts on field theories of ultra-quantum metals

Theories of metallic states without quasiparticles
in the presence of disorder

- Well-known perturbative theory of disordered metals has 2 classes of known fixed points, the insulator at strong disorder, and the metal at weak disorder. The latter state has long-lived, extended quasiparticle excitations (which are not plane waves).

- **Needed: a metallic fixed point at intermediate disorder and strong interactions without quasiparticle excitations.** Although disorder is present, it largely self-averages at long scales.

- SYK models