SYK criticality and correlated metals

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PHYSICS
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HARVARD

Ordinary metals and quasiparticles

What are quasiparticles ?

- **Quasiparticles are additive excitations:** The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

Ordinary metals and quasiparticles

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\rm eq} \sim \frac{\hbar E_F^3}{U^2 (k_B T)^2} \quad , \quad \text{as } T \to 0,$$

where U is the strength of interactions, and E_F is the Fermi energy.

- Similarly, a quasiparticle model implies a resistivity

$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau} \sim U^2 T^2 \quad \text{with} \;\; \tau \sim \tau_{\rm eq}$$

- These times are much longer than the 'Planckian time' $\hbar/(k_B T)$, which we will find in systems without quasiparticle excitations.

$$\tau \sim \tau_{\rm eq} \gg \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

Remarkable recent observation of 'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ, is

$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau}$$

with a universal scattering rate

$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar} \,,$$

independent of the strength of interactions!

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of *T*-linear resistivity vs Planckian limit in seven materials.

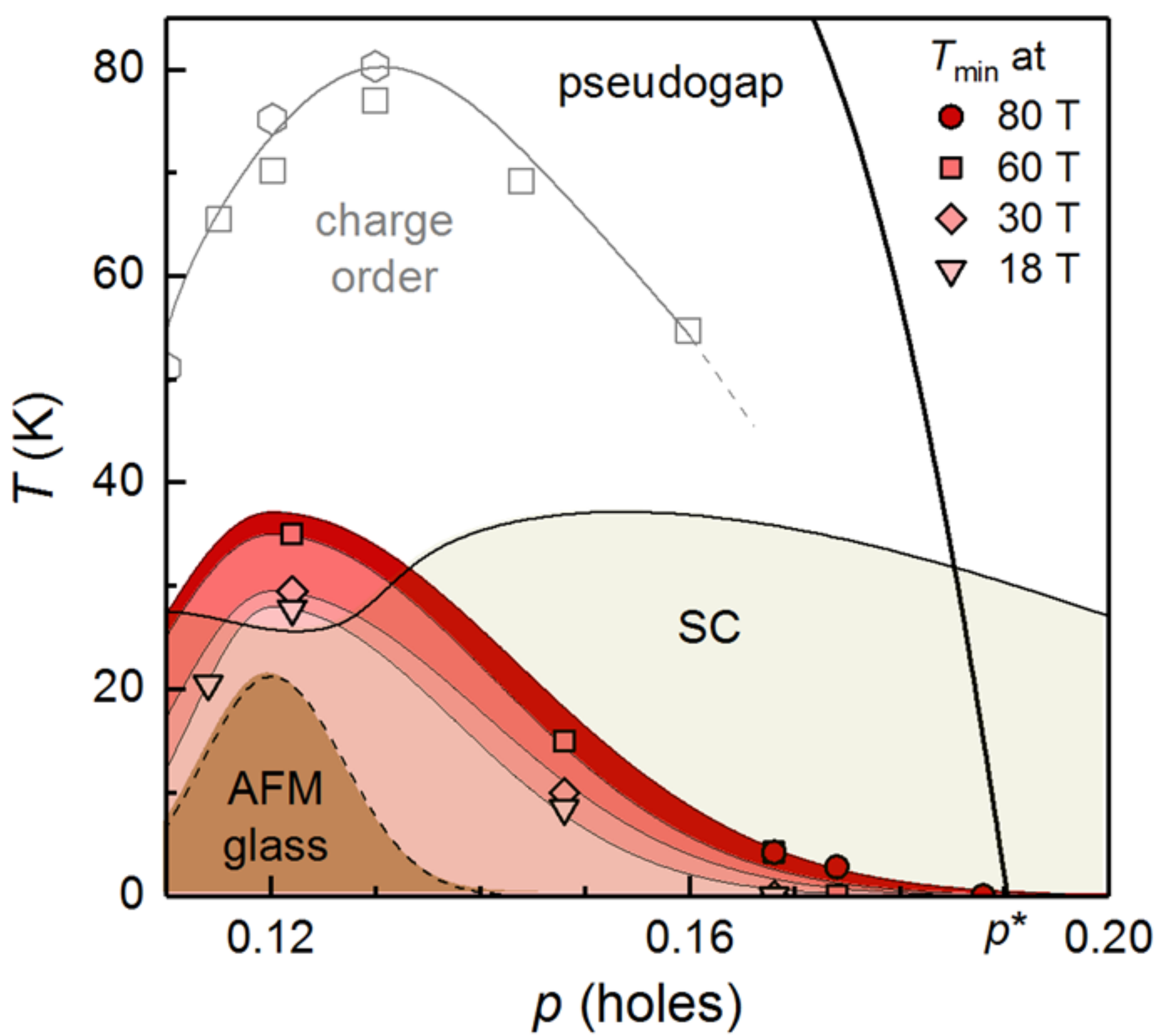
$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Nature Physics doi: 10.1038/s41567-020-0950-5

Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]



Quasi-static magnetism in the pseudogap state of $La_{2-x}Sr_xCuO_4$. Temperature – doping phase diagram representing T_{min}, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of T_{min} in zero-field, the dashed line (brown area) represents the extrapolated $T_{min}(B=0)$. While not exactly equal to the freezing temperature T_f (see Fig. 2), T_{min} is closely tied to T_f and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

Will describe a
series of
increasingly realistic
(partly) solvable
random models of
correlated metals

1. Quantum matter with quasiparticles:
   random matrix model

2. Quantum matter without quasiparticles:
   the complex SYK model

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No "Mottness"

3. Random *J* model (insulator)
   RG analysis and exact exponent

4. Random Hubbard and *t-J* models
   Numerical results

5. Random t-*J* model (metals): *exact exponents*

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A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a
N x *N* random matrix

A simple model of a metal with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

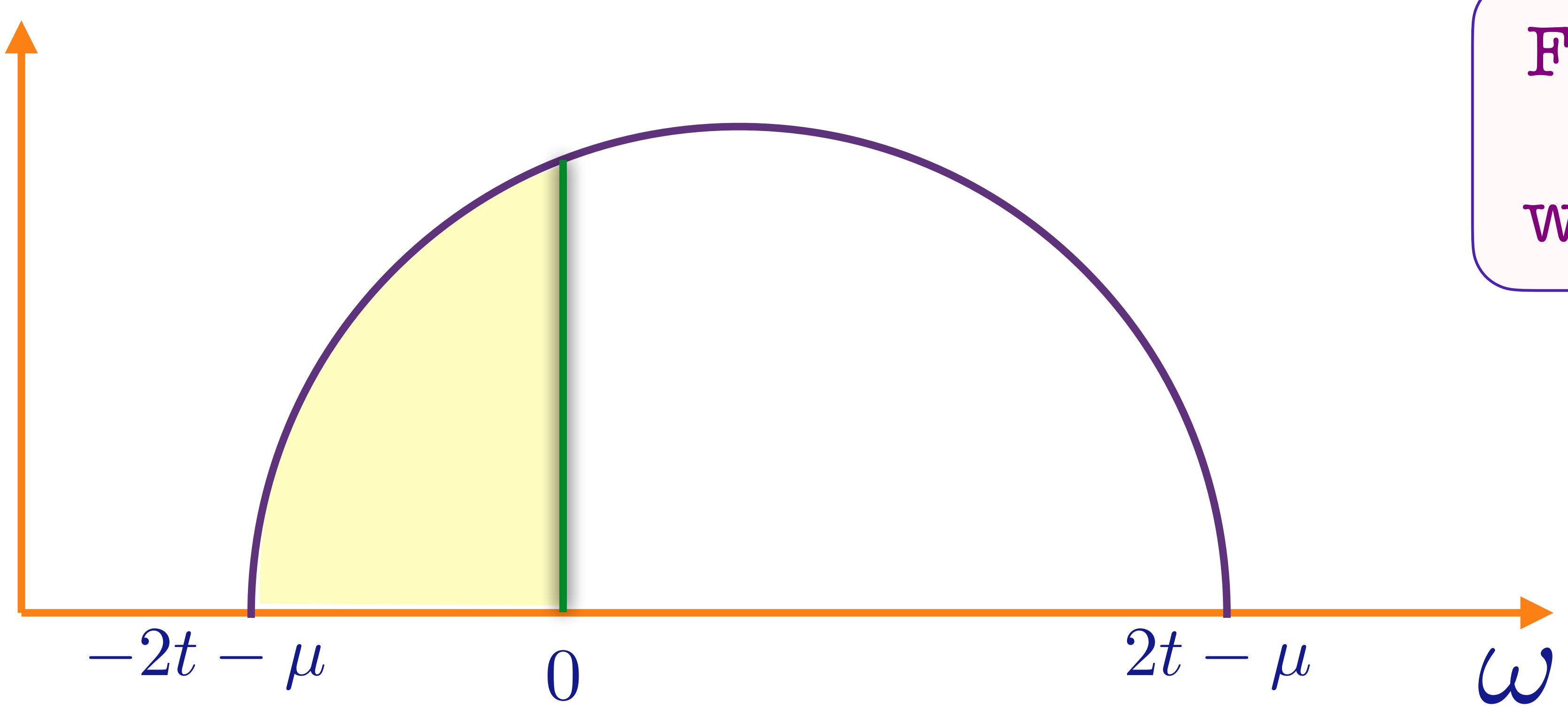
$$G(\tau) \equiv -T_\tau \left\langle c_i(\tau) c_i^\dagger(0) \right\rangle$$

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation: yields $G(\tau) \sim 1/\tau$.

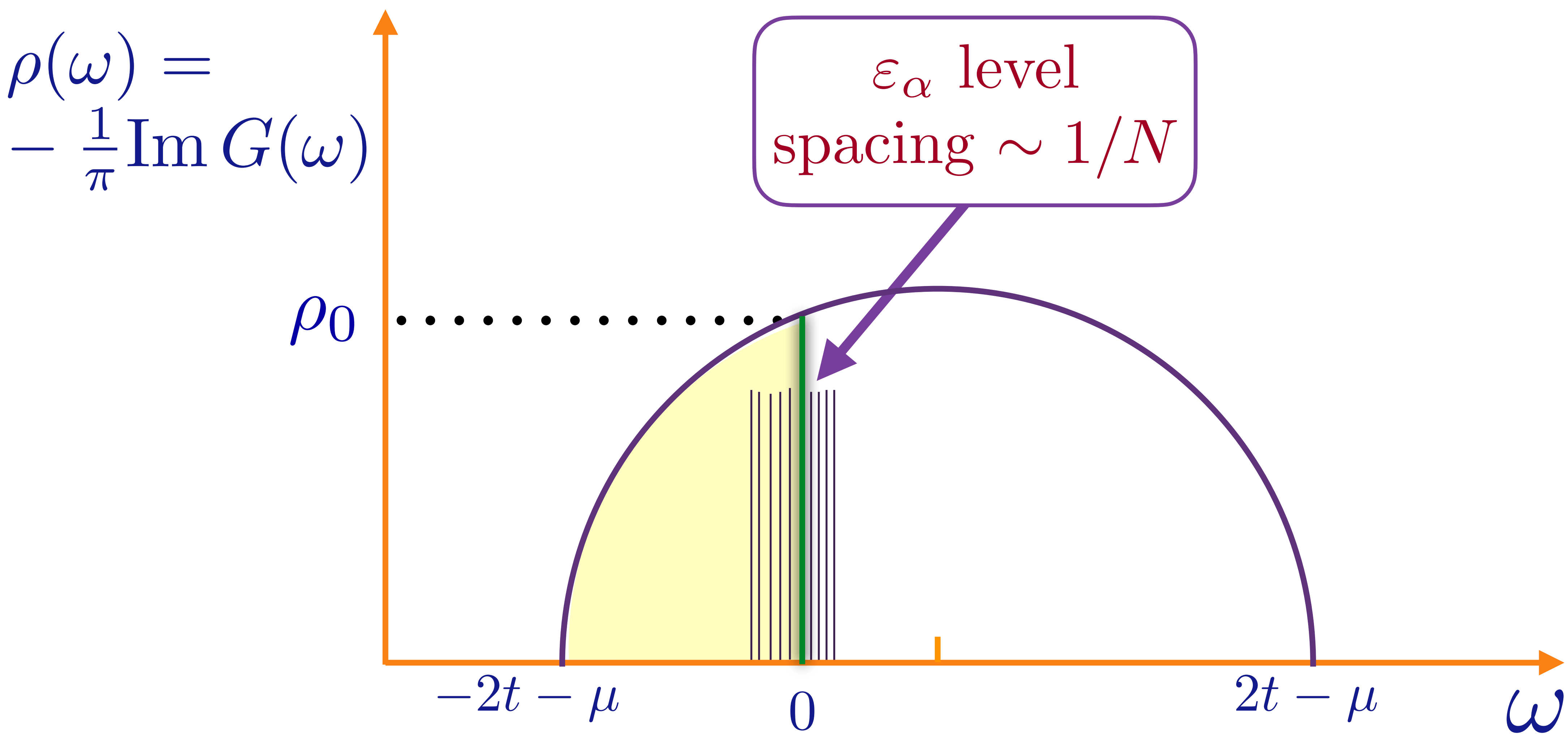
$\rho(\omega) = -\frac{1}{\pi} \text{Im}\, G(\omega)$



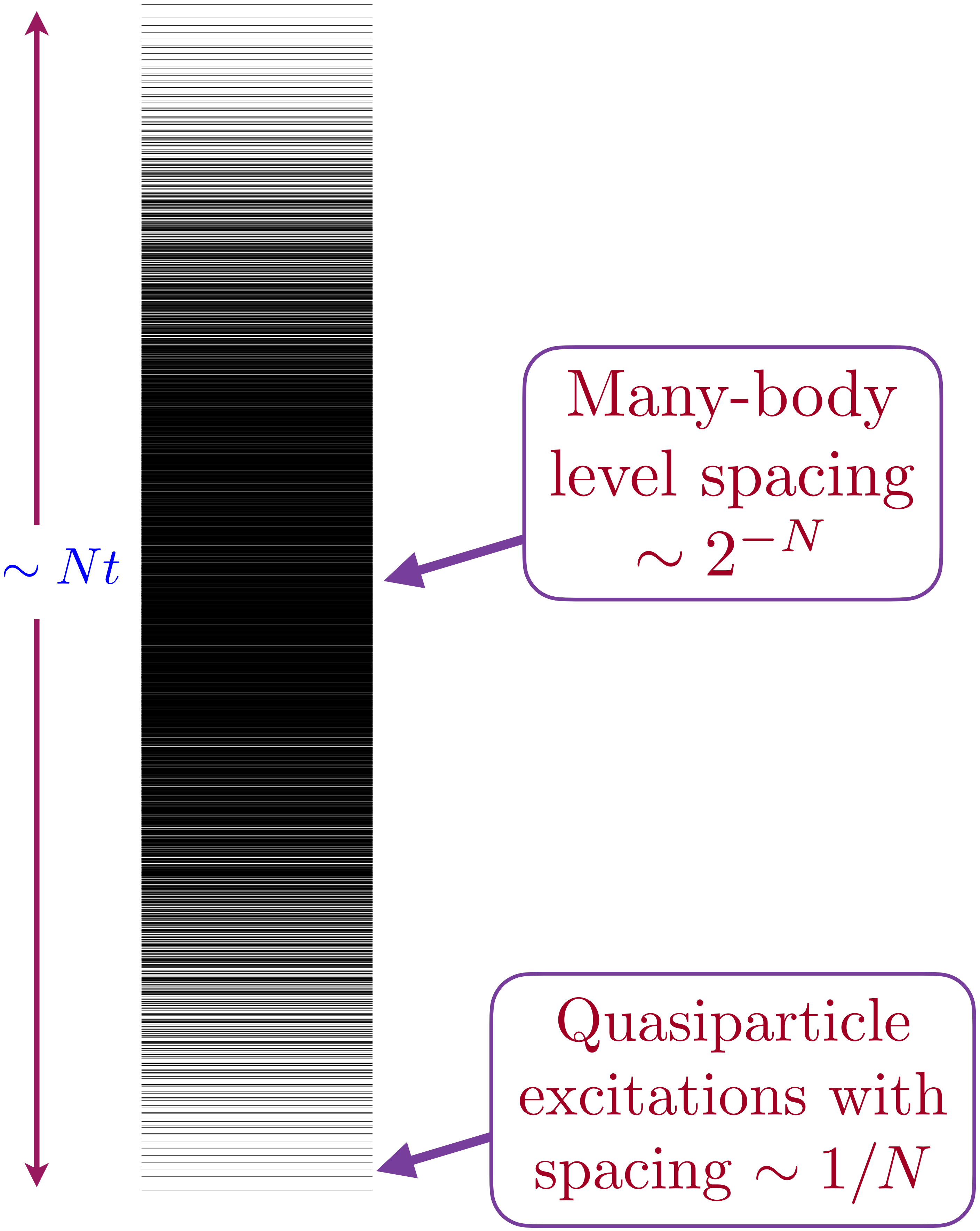
For details: see whiteboard

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The single-particle density of states is $\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$, and $\rho_0 \equiv \rho(\omega = 0)$.



A simple model of a metal with quasiparticles



There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

A simple model of a metal with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$. We compute the lifetime of a quasiparticle, τ_α, in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamitonian with energy ε_α. By Fermi's Golden rule, for ε_α at the Fermi energy

$$\begin{aligned}\frac{1}{\tau_\alpha} &= \pi U^2 \rho_0^3 \int d\varepsilon_\beta d\varepsilon_\gamma d\varepsilon_\delta f(\varepsilon_\beta)(1 - f(\varepsilon_\gamma))(1 - f(\varepsilon_\delta))\delta(\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_\gamma - \varepsilon_\delta) \\ &= \frac{\pi^3 U^2 \rho_0^3}{4} T^2\end{aligned}$$

where ρ_0 is the density of states at the Fermi energy, and $f(\epsilon) = 1/(e^{\epsilon/T} + 1)$ is the Fermi function.

Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

1. Quantum matter with quasiparticles:
random matrix model

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the complex SYK model

No "Mottness"

3. Random *J* model (insulator)
RG analysis and exact exponent

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5. Random t-*J* model (metals): *exact exponents*

The complex SYK model

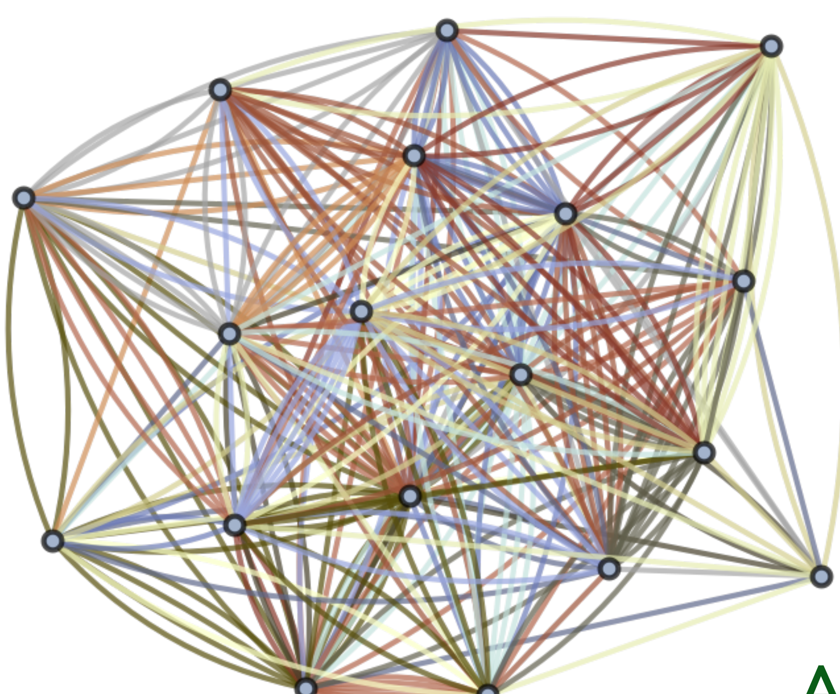
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

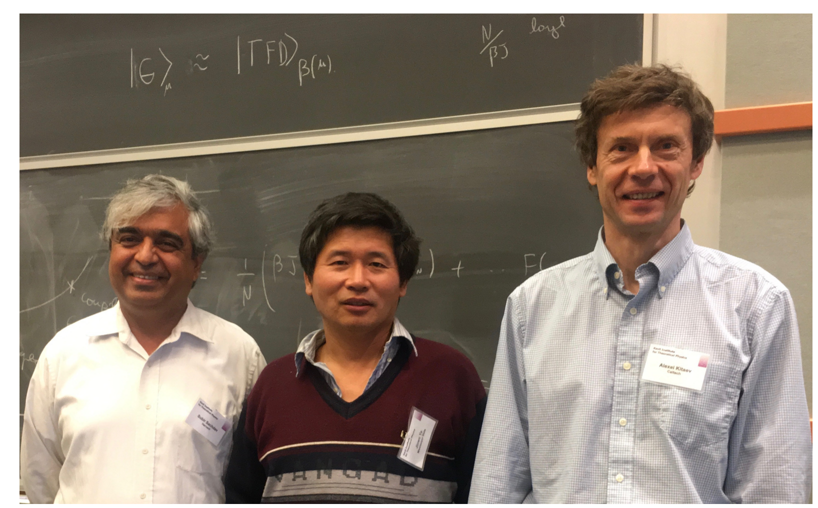
$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta - \mu \sum_\alpha c^\dagger_\alpha c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c^\dagger_\beta + c^\dagger_\beta c_\alpha = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c^\dagger_\alpha c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.





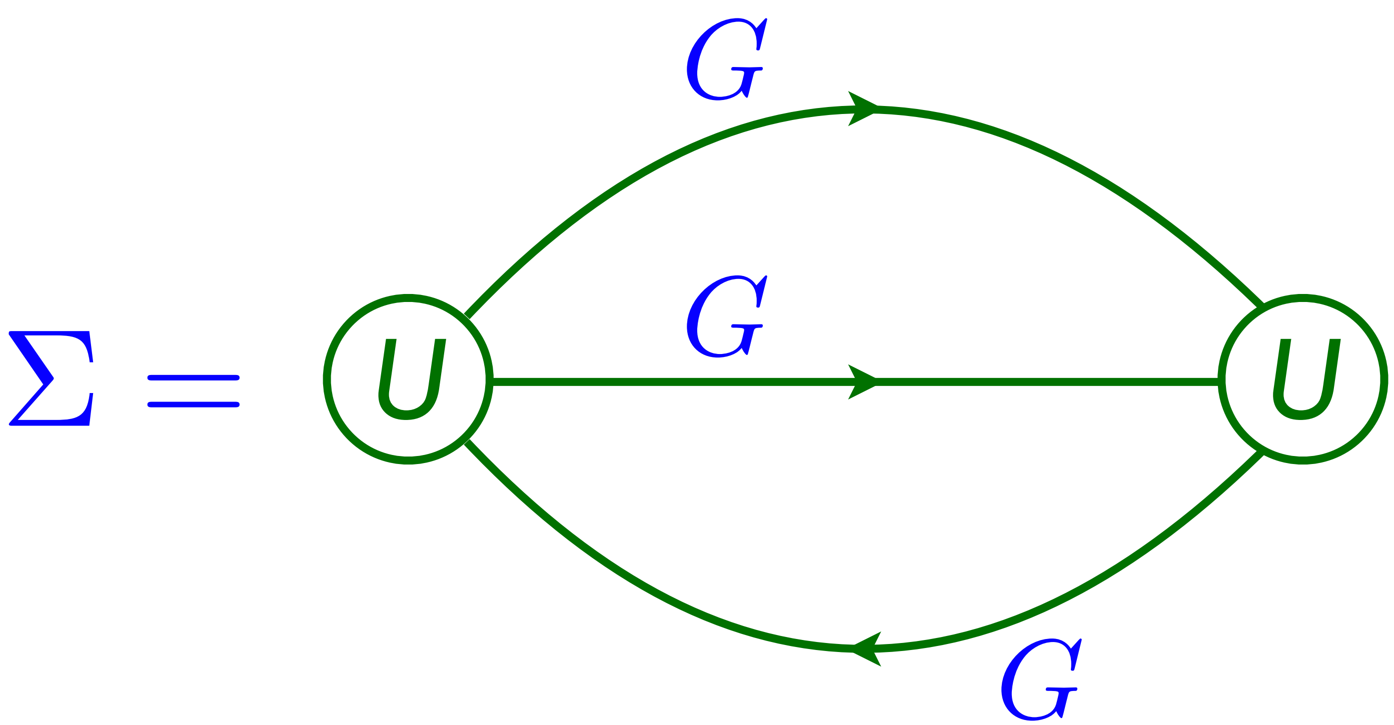
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$



S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

The complex SYK model

The large N limit is given by the sum of "melon" Feynman graphs

For long times $\tau > 0$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = \frac{A}{\sqrt{\tau}}$$

$$\left\langle c_\alpha^\dagger(\tau) c_\alpha(0) \right\rangle = e^{-2\pi\mathcal{E}} \frac{A}{\sqrt{\tau}}$$

For details: see whiteboard and Sarosi, Sec. 4.4

The parameter $\mathcal{E} = \mathbb{C}\left(\epsilon/U\right)$ determines the particle-hole asymmetry, and has a universal "Luttinger" relation to $\mathcal{Q}$.

In a Fermi liquid,

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = \left\langle c_\alpha^\dagger(\tau) c_\alpha(0) \right\rangle = \widetilde{A}/\tau$$

The complex SYK model

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

The complex SYK model

$$G(i\omega) = \frac{1}{\cancel{i\omega} + \cancel{\mu} - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \cancel{\mu} - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

At frequencies $\ll U$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations. The singular part of the self-energy and the Green's function obey

$$\int_0^\beta d\tau_2 \, \Sigma_{\text{sing}}(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\Sigma_{\text{sing}}(\tau_1, \tau_2) = -U^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

A. Kitaev, 2015

S. Sachdev, PRX **5,** 041025 (2015)

The complex SYK model

$$\int_0^\beta d\tau_2 \, \Sigma_(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\Sigma(\tau_1, \tau_2) = -U^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

These equations are invariant under

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{G}(\sigma_1, \sigma_2)$$

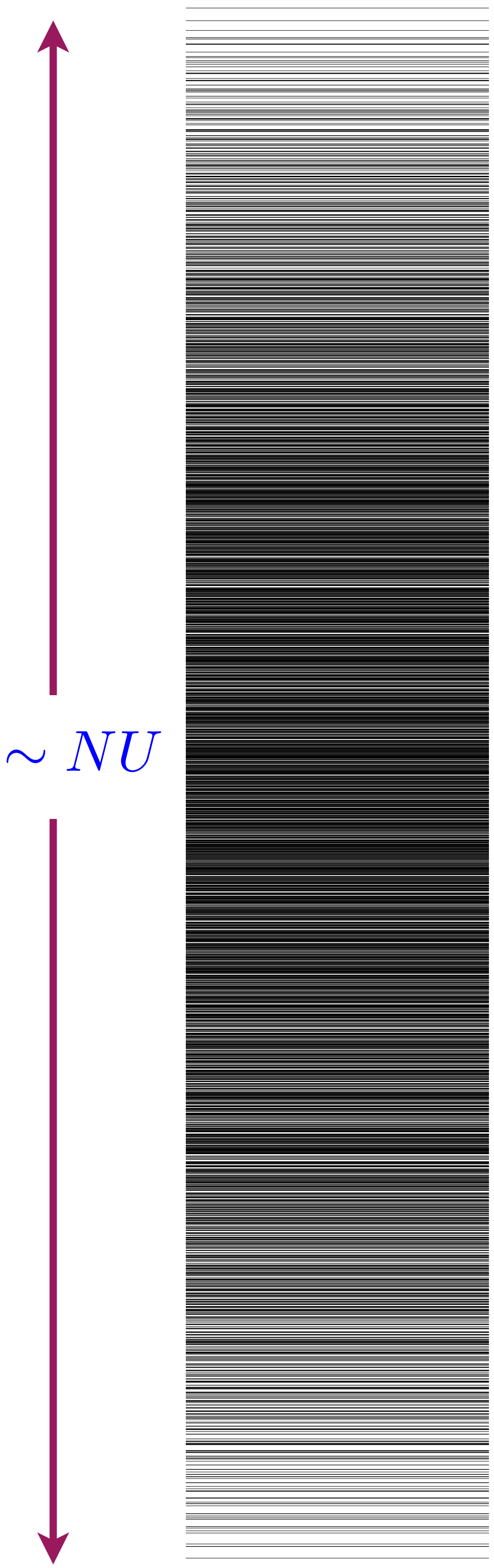
$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.
By using $f(\sigma) = \tan(\pi T \sigma)/(\pi T)$ and
$g(\sigma) = e^{-2\pi \mathcal{E} T \sigma}$, we can now obtain
the $T > 0$ solution from the $T = 0$ solution.

A. Kitaev, 2015

S. Sachdev, PRX **5,** 041025 (2015)

The complex SYK model



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

There are 2^N many body levels with energy E. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$, where $s_0 < \ln 2$ is determined by integrating

$$\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}\,.$$

At $\mathcal{Q} = 1/2$,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where G is Catalan's constant.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

SYK criticality

Key properties

1. There is a quantum critical state, without quasiparticle excitations, for a range of charge densities around $\mathcal{Q} = 1/2$.

2. There is a non-zero extensive entropy as $T \to 0$

$$\lim_{T \to 0} \lim_{N \to \infty} \frac{S}{N} = \mathcal{S}_0(\mathcal{Q}) \neq 0$$

   This entropy is *not* due to an exponentially large ground degeneracy. Instead, it reflects an exponentially small many-body level spacing $\sim e^{-N\mathcal{S}_0}$ down to the ground state.

3. Thermal equilibration in a 'Planckian time' $\sim \hbar/(k_B T)$

SYK criticality

Key properties

4. The leading low temperature behavior of many observables is controlled by a time reparameterization soft mode. A Schwarzian action for this soft mode is implied by an emergent SL(2,R) symmetry. Specifically, the entropy is $S(T)/N = \mathcal{S}_0(\mathcal{Q}) + \gamma T$, where γ propotional to the co-efficient of the Schwarzian.

5. Maximal quantum Lyapunov exponent for the out-of-time-order correlator (OTOC):

$$\left\langle c_a^\dagger(t) c_b(0) c_a(t) c_b^\dagger(0) \right\rangle = C_0 + C_1 \left(\frac{e^{\lambda t}}{N} \right) + \ldots$$

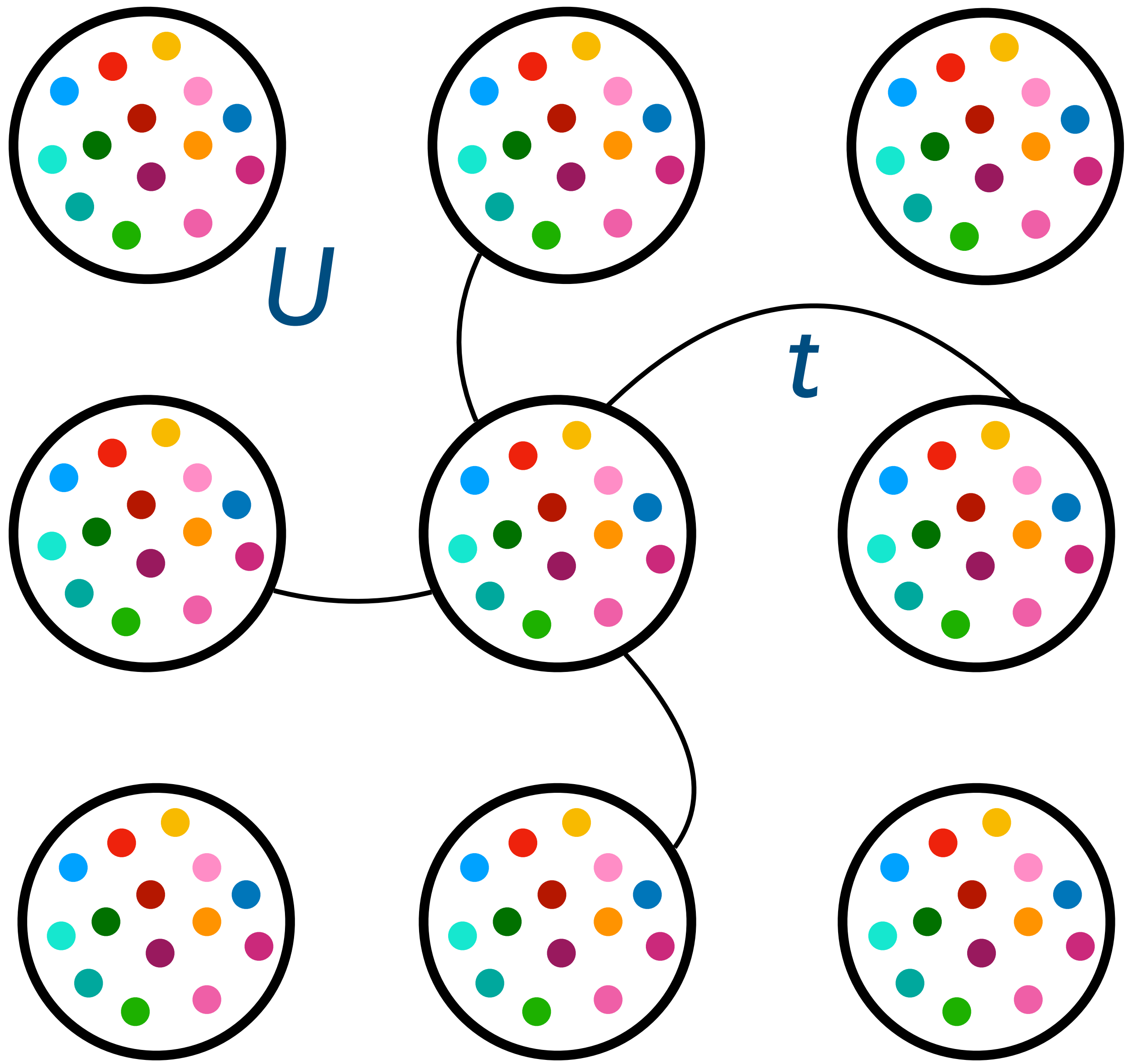
   with $\lambda = 2\pi k_B T/\hbar$.

6. For spinful fermions, spin correlations decay as

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim 1/|\tau|$$

A lattice SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{i} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c_{i\alpha}^\dagger c_{j\alpha}$$



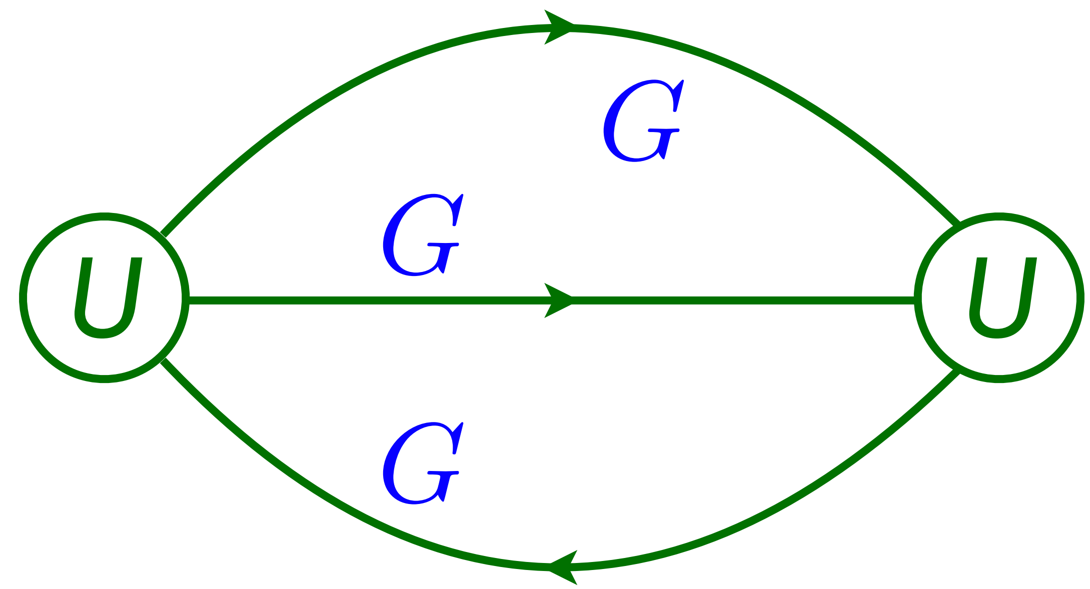
Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017); Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

A lattice SYK model

The large N limit is still given by the sum of "melon" diagrams.

$$G(k, i\omega) = \frac{1}{i\omega - \epsilon_k - \Sigma(k, i\omega)} \qquad \Sigma =$$

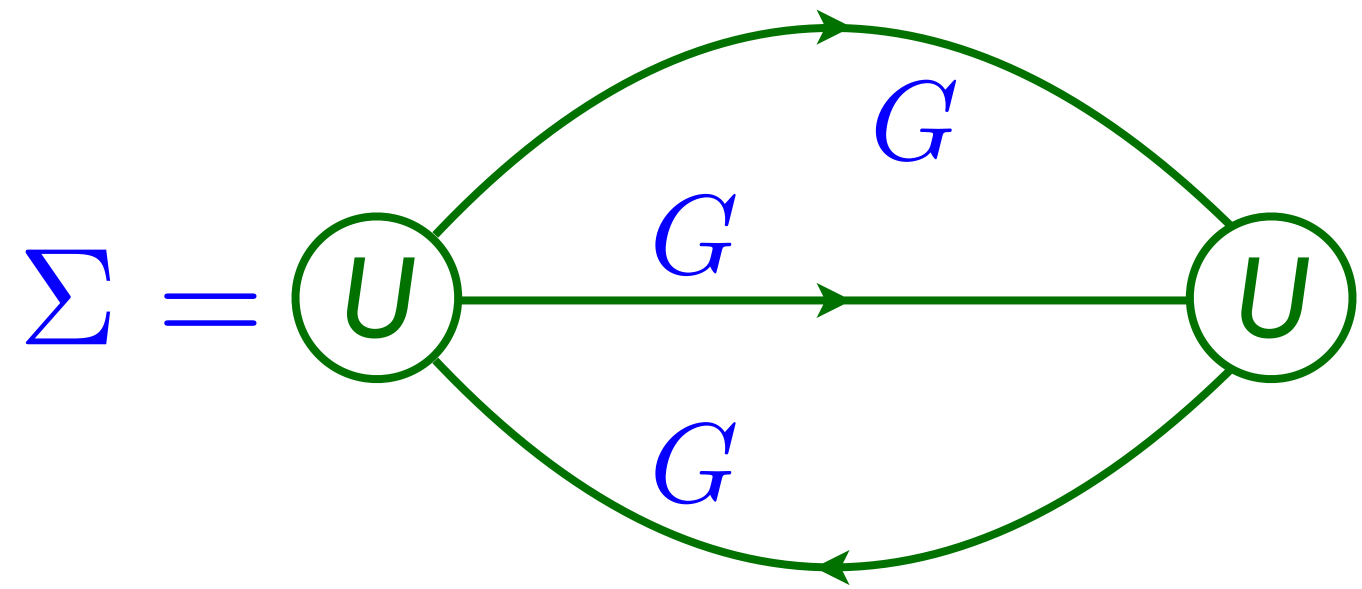


At energy scales $T \ll t^2/U$, the contribution from Σ only modifies the dispersion, and we obtain a solution is similar to the disordered metal with quasiparticles.

A lattice SYK model

The large N limit is still given by the sum of "melon" diagrams.

$$G(k, i\omega) = \frac{1}{i\omega - \epsilon_k - \Sigma(k, i\omega)}$$



For $t^2/U \ll k_B T \ll U$ we obtain an 'incoherent metal' with no Fermi surface or quasiparticles with

$$G(\mathbf{k}, \omega) = G_{\rm SYK}(\epsilon, \hbar\omega/(k_B T))$$

independent of $\mathbf{k}$. In this regime, there is a linear-in-T resistivity but only with bad metal behavior with $\rho > h/e^2$, and co-efficient dependent upon U:

$$\rho \sim \frac{h}{e^2} \frac{k_B T}{t^2/U}$$

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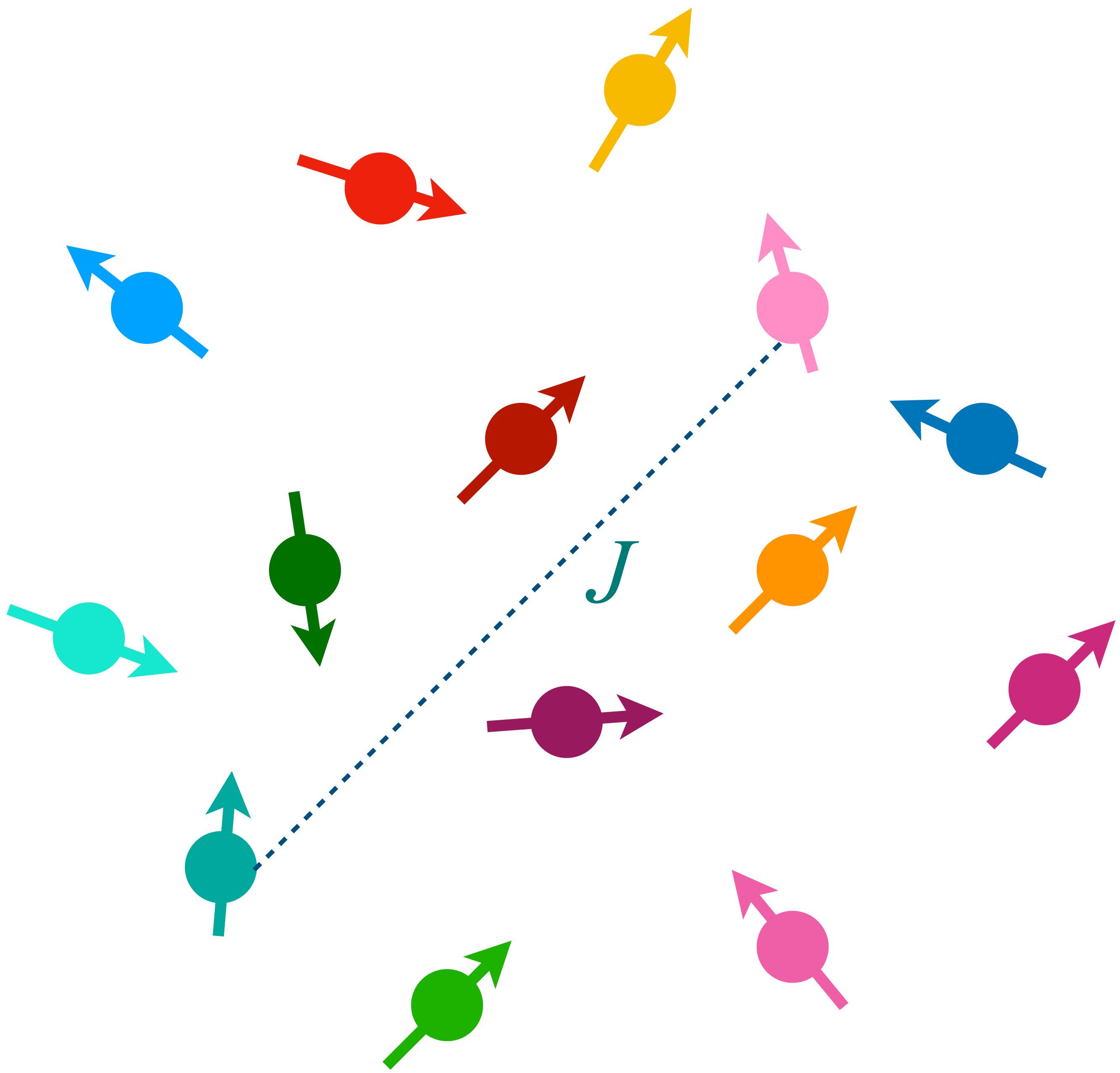
Random J model (insulator)

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$



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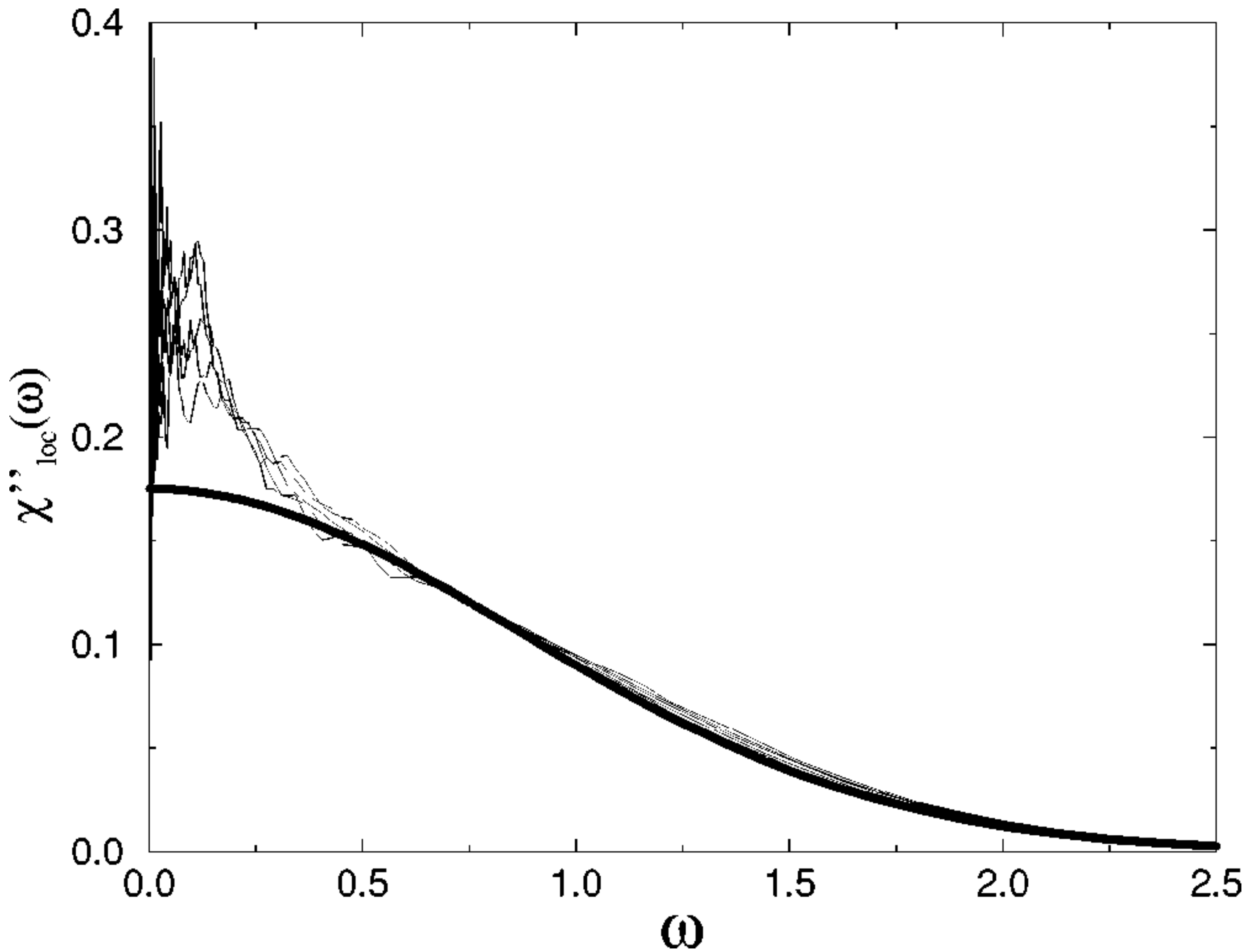
Numerical studies for SU(2) spin-1/2 show spin-glass order!

Spin-glass order parameter:

$$q = \lim_{t\to\infty} \overline{\left\langle \vec{S}_i(t)\vec{S}_i(0)\right\rangle}\,.$$

Exact diagonalization results for $\chi''_{\rm loc}(\omega)$
Analytic continuation of

$$\chi_{\rm loc}(i\omega_n) = \int_0^\beta d\tau e^{i\omega_n\tau}\overline{\left\langle \vec{S}_i(\tau)\vec{S}_i(0)\right\rangle}$$



L. Arrachea and M. J. Rozenberg, PRB **65**, 224430 (2002)

Random J model (insulator)

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{i\beta}, \quad \sum_\alpha f_{i\alpha}^\dagger f_{i\alpha} = 1$$

$$J_{ij} \text{ random}, \overline{J_{ij}} = 0, \overline{J_{ij}^2} = J^2$$

Random J model (insulator)

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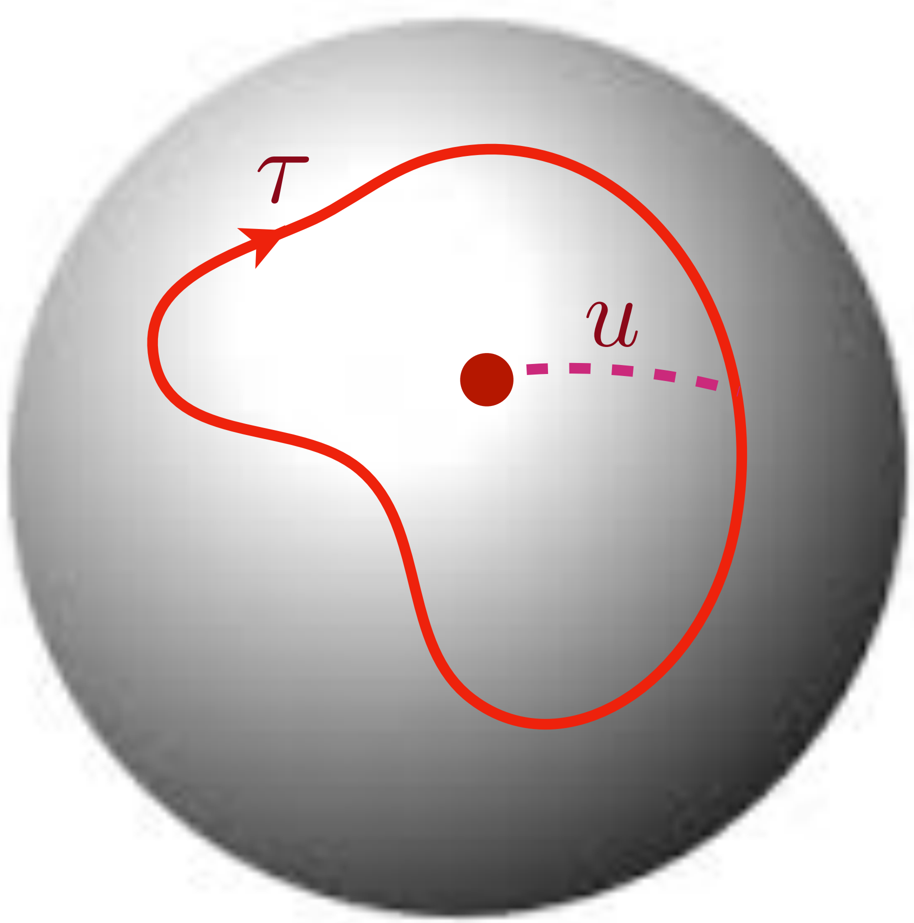
Random J model (insulator)

$$\mathcal{Z} = \int \mathcal{D}\vec{S}(\tau)\delta(\vec{S}^2 - 1)e^{-\mathcal{S}_B - \mathcal{S}_J}$$

$$\mathcal{S}_B = \frac{i}{2}\int_0^1 du \int d\tau \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial \tau} \times \frac{\partial \vec{S}}{\partial u} \right)$$

$$\mathcal{S}_J = -\frac{J^2}{2}\int d\tau d\tau' Q(\tau - \tau')\vec{S}(\tau) \cdot \vec{S}(\tau') \,.$$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

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$$\mathcal{S}_B = \frac{i}{2}\int_0^1 du \int d\tau \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial \tau} \times \frac{\partial \vec{S}}{\partial u} \right)$$

$$\mathcal{S}_J = -\frac{J^2}{2}\int d\tau d\tau' Q(\tau - \tau')\vec{S}(\tau) \cdot \vec{S}(\tau') \,.$$

From this action we compute

$$\overline{Q}(\tau - \tau') = \frac{1}{3}\left\langle \vec{S}(\tau) \cdot \vec{S}(\tau') \right\rangle_{\mathcal{Z}}$$

and then impose the self-consistency condition

$$Q(\tau) = \overline{Q}(\tau).$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Random *J* model (insulator): large *M*

Express the spin operator in terms of fermions $\vec{S} = (1/2) f^\dagger_\alpha \vec{\sigma}_{\alpha\beta} f_\beta$, and let $\alpha = 1 \ldots M$. The fermions obey the constraint

$$\sum_{\alpha=1}^{M} f^\dagger_\alpha f_\alpha = \frac{M}{2}$$

In the large M limit we obtain for the fermion Green's function G and self energy Σ (same as the SYK equations)

$$G(i\omega) = \frac{1}{i\omega - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

The solution is

$$G(\tau) \sim \frac{\mathrm{sgn}(\tau)}{\sqrt{\tau}} \quad , \quad \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Random J model (insulator):RG

We assume a power-law decay

$$Q(\tau) \sim \frac{\gamma^2}{|\tau|^\alpha}.$$

Ignore the self-consistency condition for now. We decouple the $\vec{S}(\tau)\cdot\vec{S}(0)$ interaction by introducing a bosonic (ϕ_a, $a = 1\ldots3$) bath. Then the problem reduces to the Hamiltonian

$$H_{\rm imp} \;=\; \gamma S_a\,\phi_a(0) + \frac{1}{2}\int d^dx\,\left[\pi_a^2 + (\partial_x\phi_a)^2\right]$$

where π_a is canonically conjugate to the field ϕ_a, and $\phi_a(0) \equiv \phi_a(x=0)$. We identify $Q(\tau)$ with temporal correlator of $\phi_a(0)$, and then we need $\alpha = d - 1$.

M. Vojta, C. Buragohain, and S. Sachdev, PRB **61**, 15152 (2000)
S. Sachdev, Physica C **357**, 78 (2001)

Random J model (insulator):RG

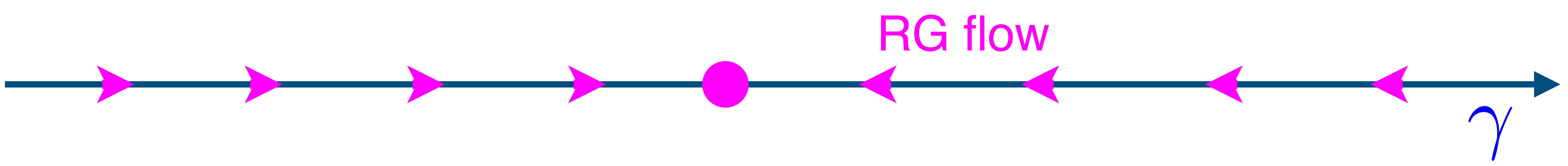
- The β-function of γ can by computed order-by-order in $\epsilon = 2 - \alpha$

$$\frac{d\gamma}{d\ell} = \epsilon \frac{\gamma}{2} - \gamma^3 \,.$$

For details: see whiteboard and Whitsitt, Sec. III

- There is an attractive fixed point at $\gamma = \gamma^* = \mathcal{O}(\sqrt{\epsilon})$.

- Because of the quantized Berry phase (Wess-Zumino-Witten) term, the renormalization of the coupling γ is given only by the wavefunction renormalization. We can then prove that at this fixed point $\overline{Q}(\tau) \sim 1/|\tau|^{2-\alpha}$ to all orders in ϵ.



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- The self-consistency condition therefore yields

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|} .$$

to all orders in ϵ.

M. Vojta, C. Buragohain, and S. Sachdev, PRB **61**, 15152 (2000)
S. Sachdev, Physica C **357**, 78 (2001)

1. Quantum matter with quasiparticles: random matrix model

2. Quantum matter without quasiparticles: the complex SYK model

No "Mottness"

3. Random *J* model (insulator)
   RG analysis and exact exponent

4. Random Hubbard and *t-J* models
   Numerical results

5. Random t-*J* model (metals): *exact exponents*

Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

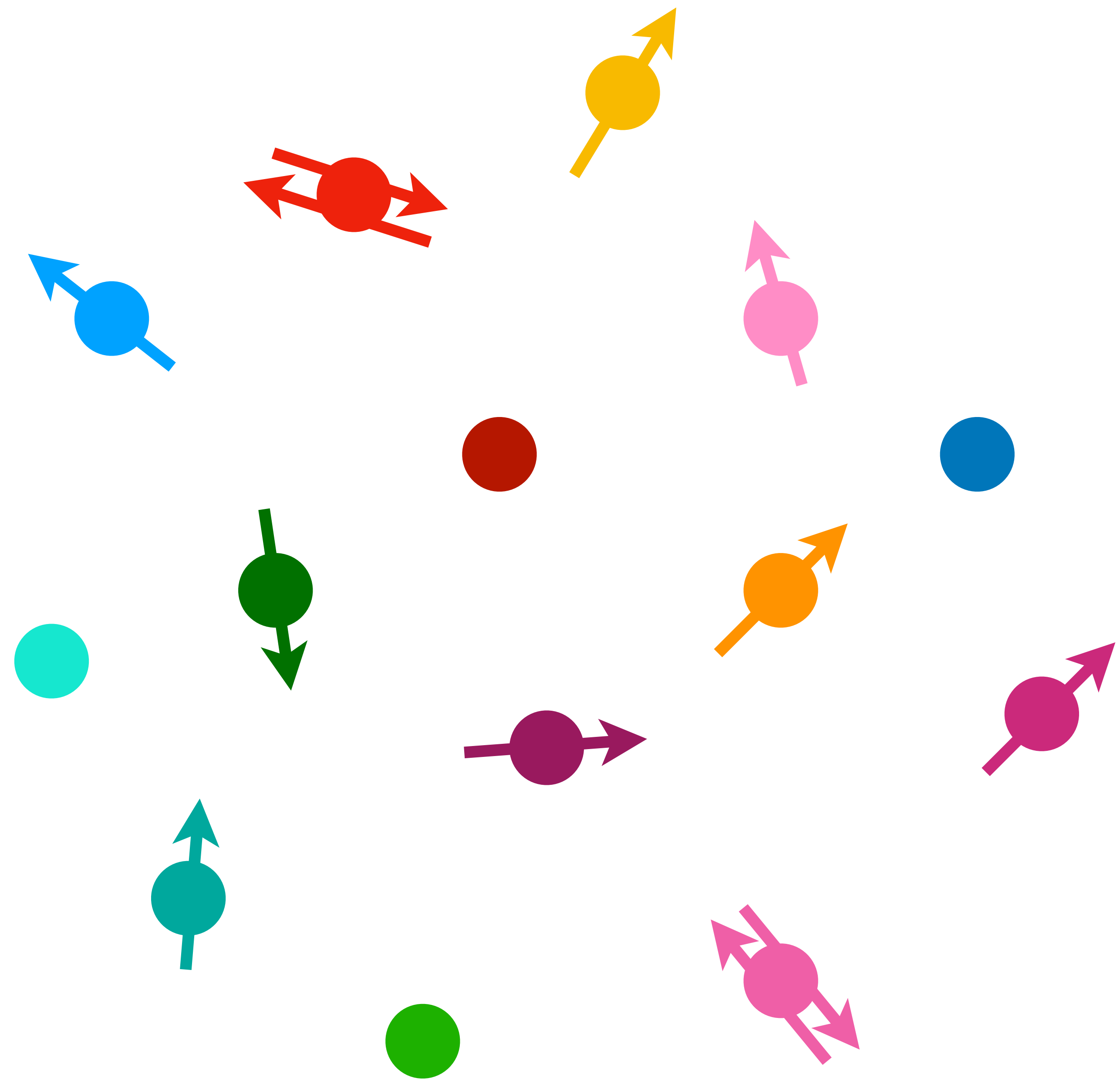
$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^{\dagger} c_{i\alpha},$$

$$J_{ij} \text{ random}, \overline{J_{ij}} = 0, \overline{J_{ij}^2} = J^2$$
$$t_{ij} \text{ random}, \overline{t_{ij}} = 0, \overline{t_{ij}^2} = t^2$$

$$U_H > 0 \text{ non-random}$$

Random *t-J-U_H* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



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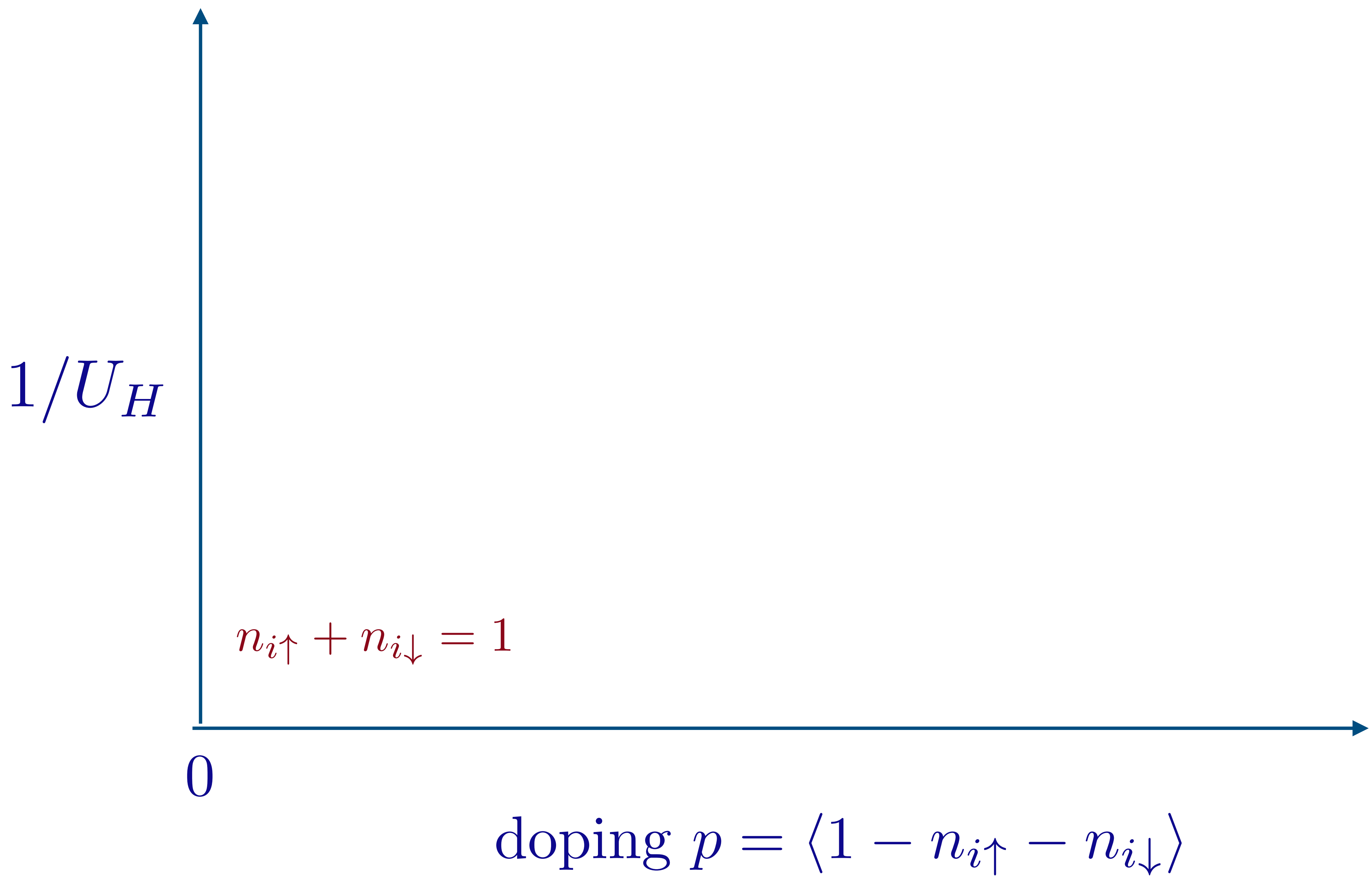
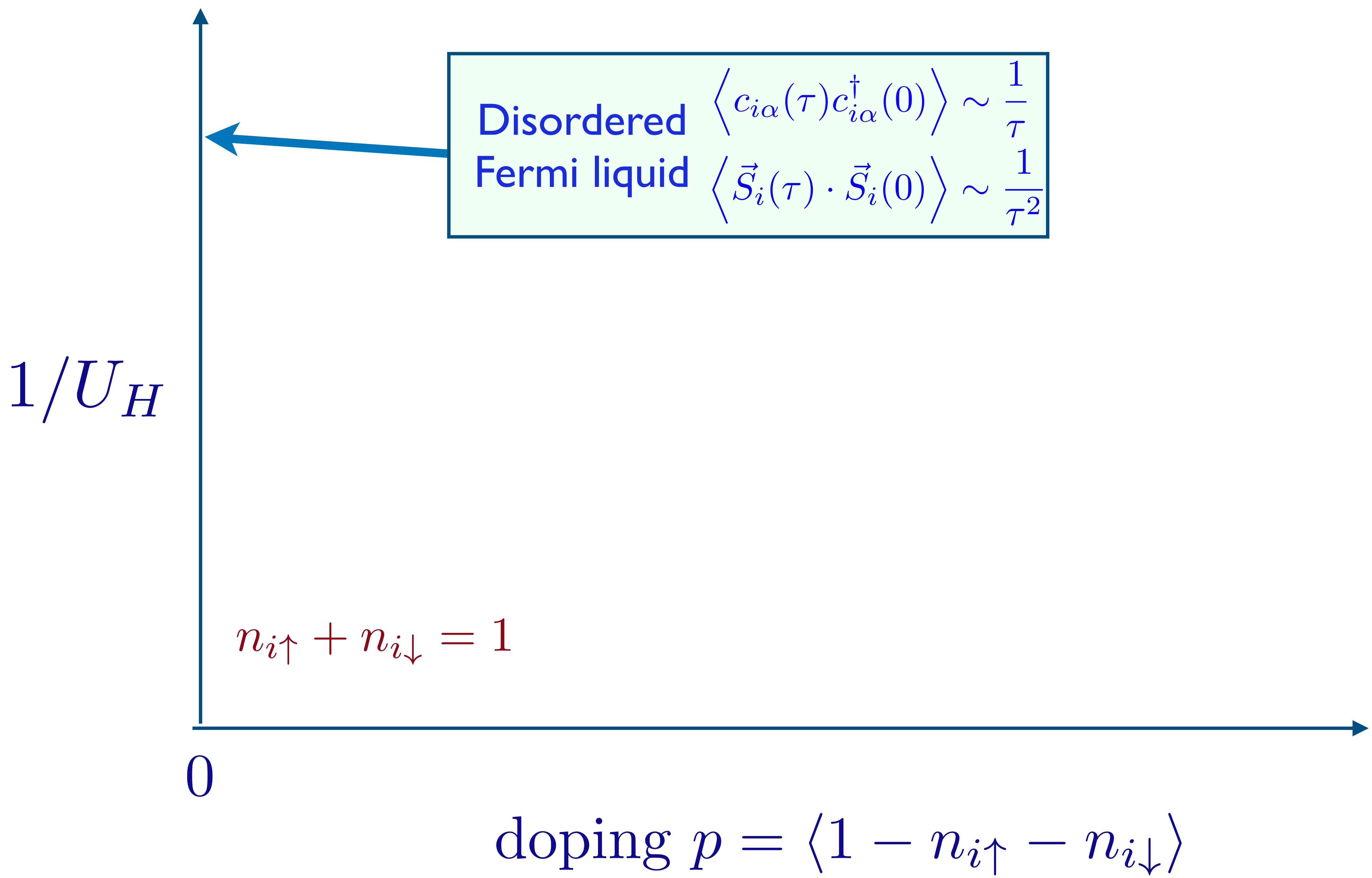


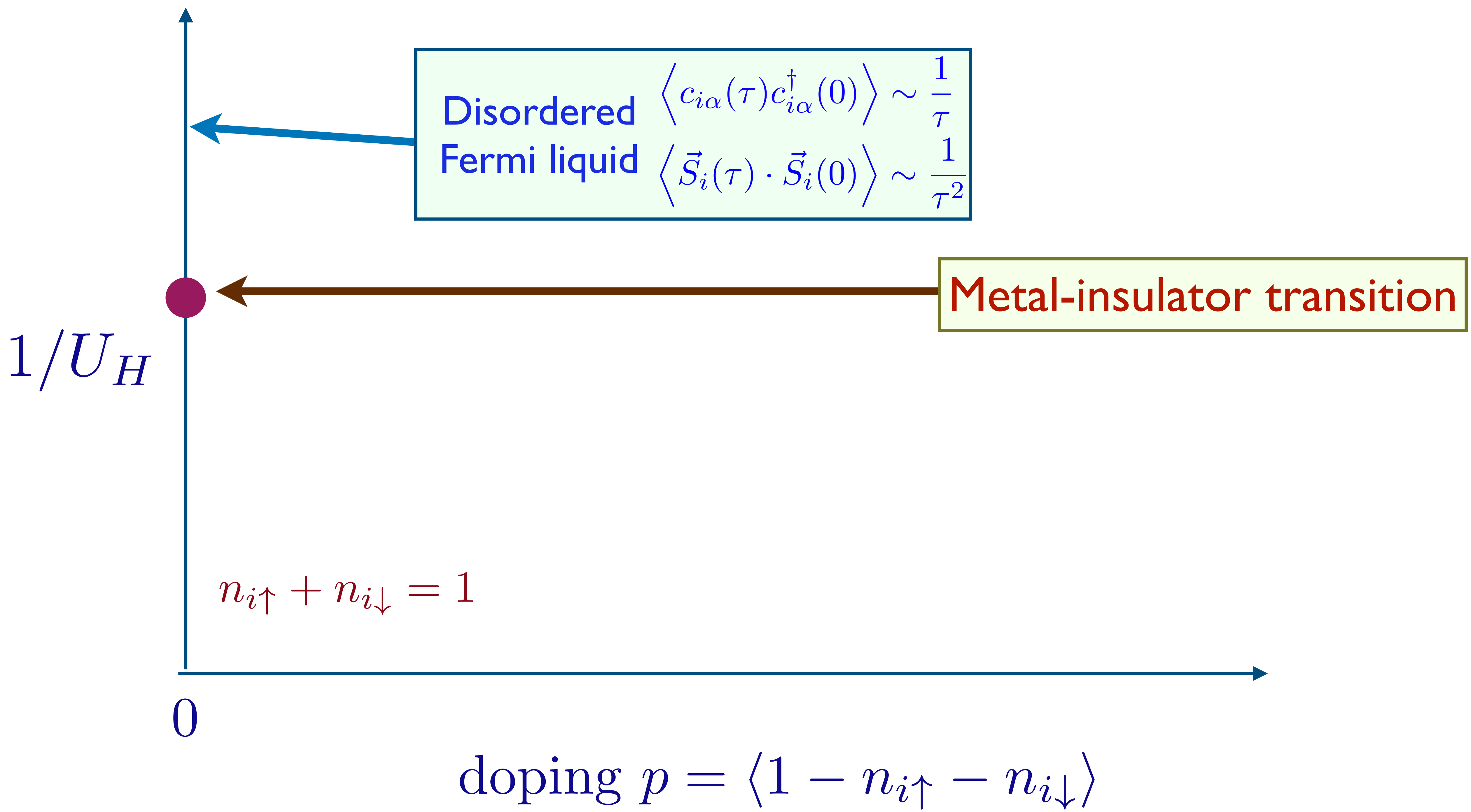
Random *t-J-U_H* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



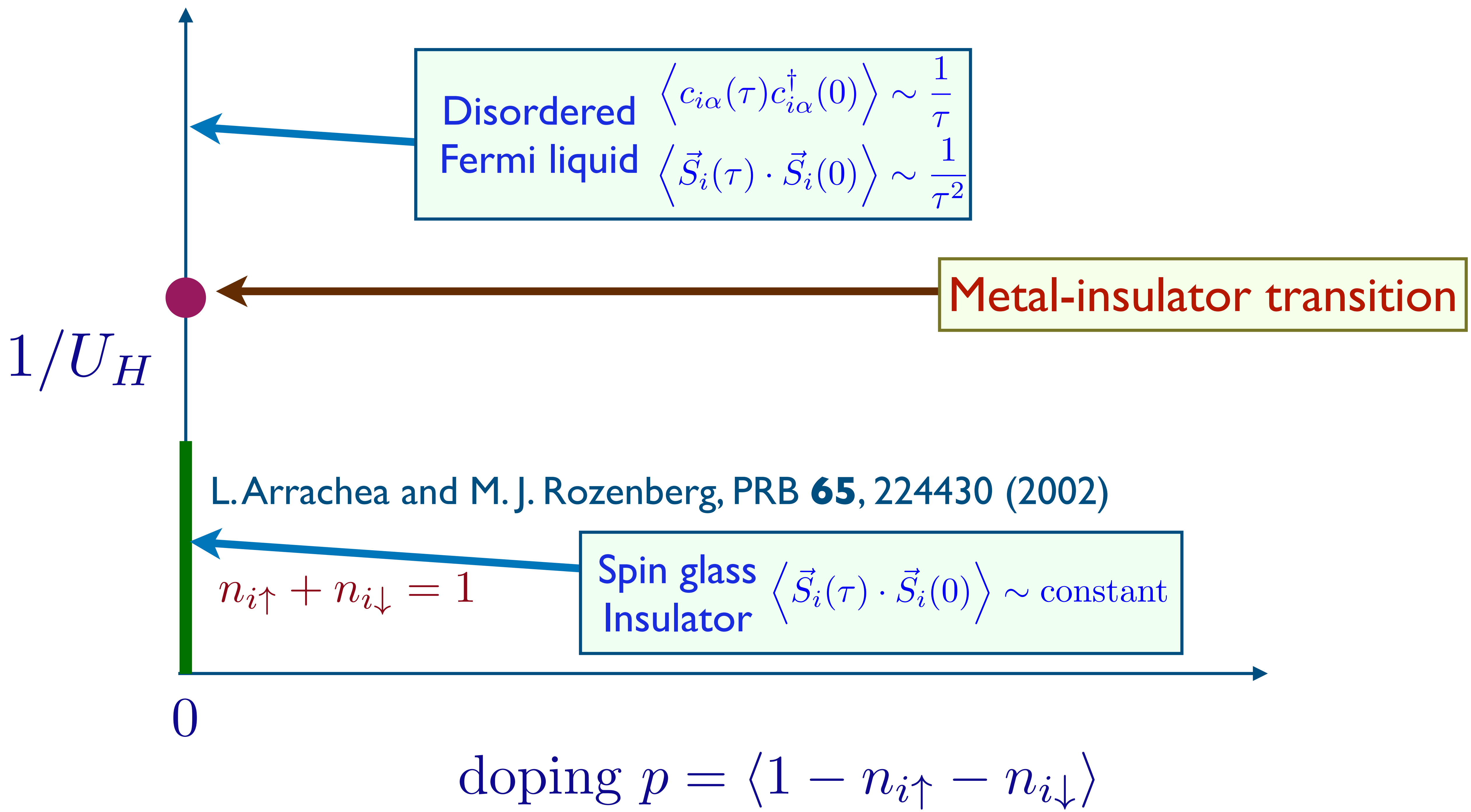
Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \boxed{\frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j} + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



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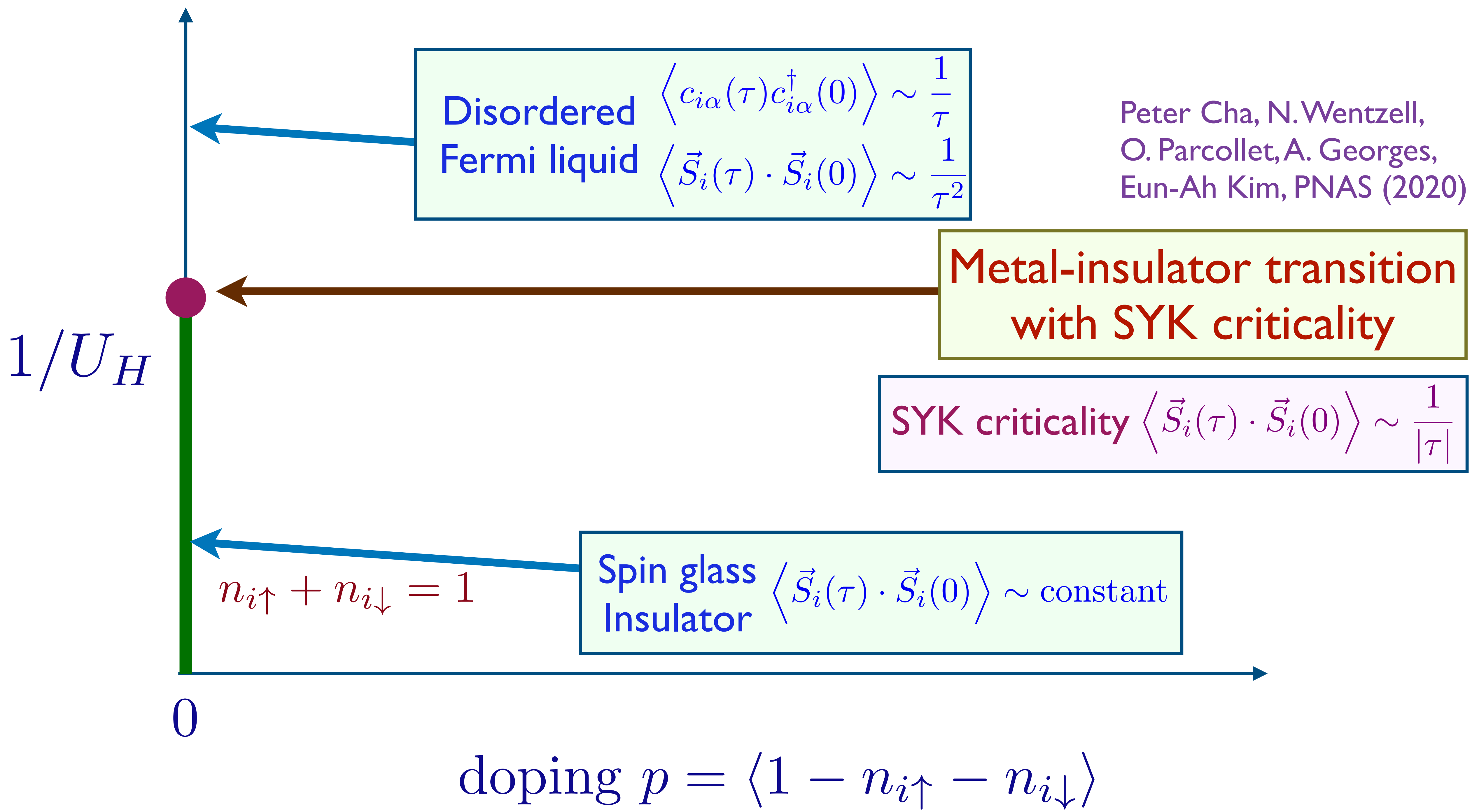
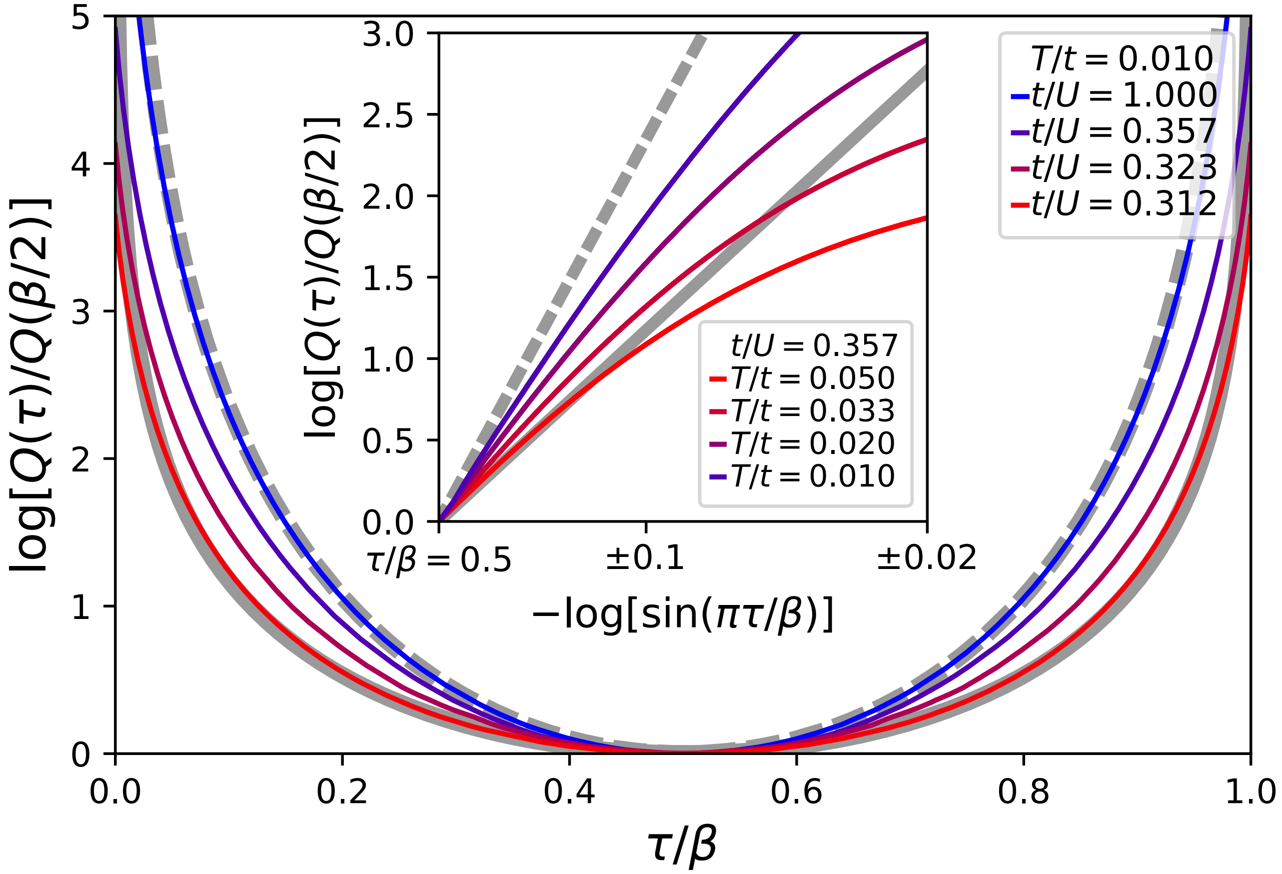
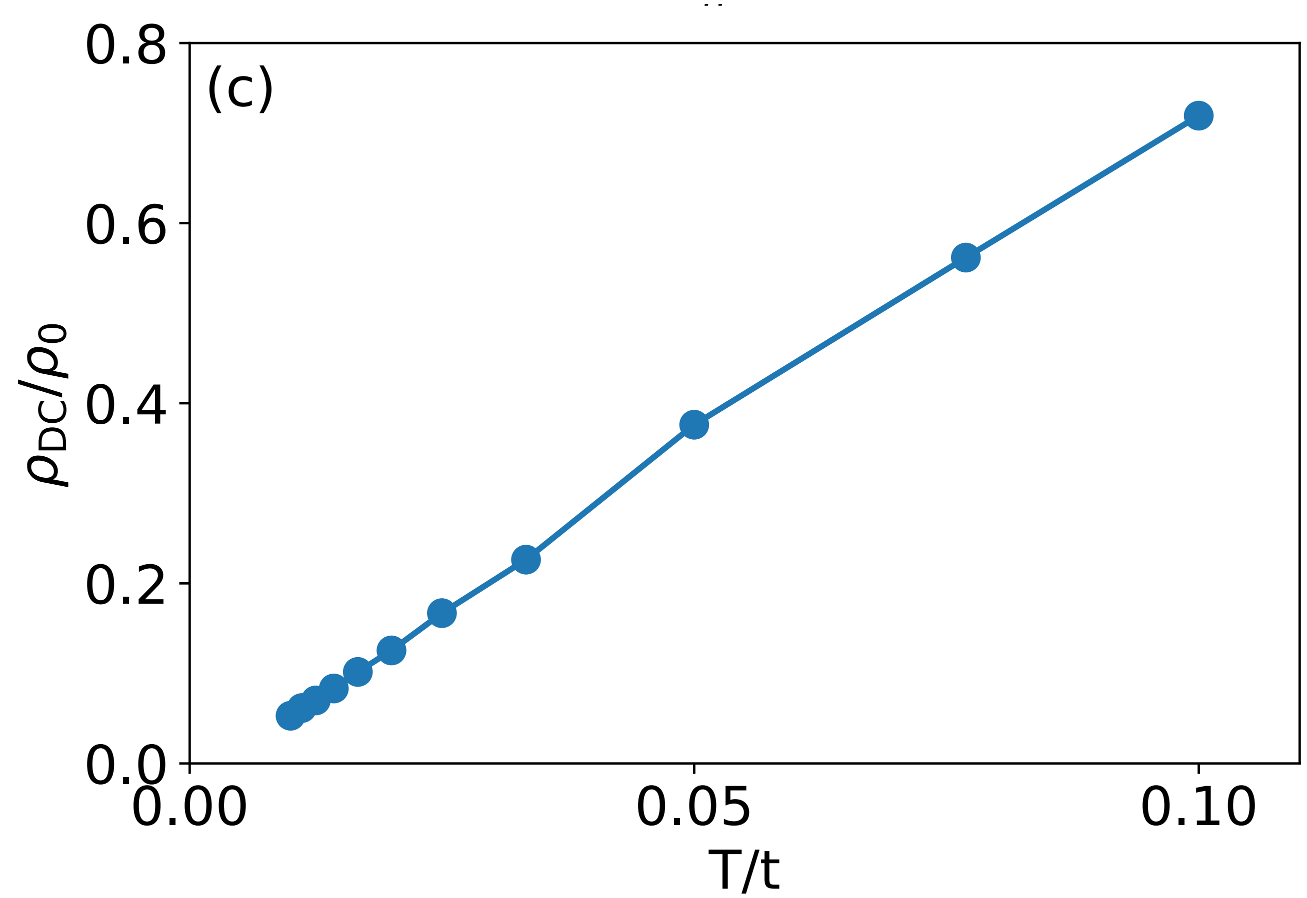
$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



Peter Cha, N. Wentzell, O. Parcollet, A. Georges, Eun-Ah Kim, PNAS (2020)

Linear resistivity and Sachdev-Ye-Kitaev (SYK) spin liquid behavior in a quantum critical metal with spin-1/2 fermions





Critical spin correlations:

$$\left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|}$$

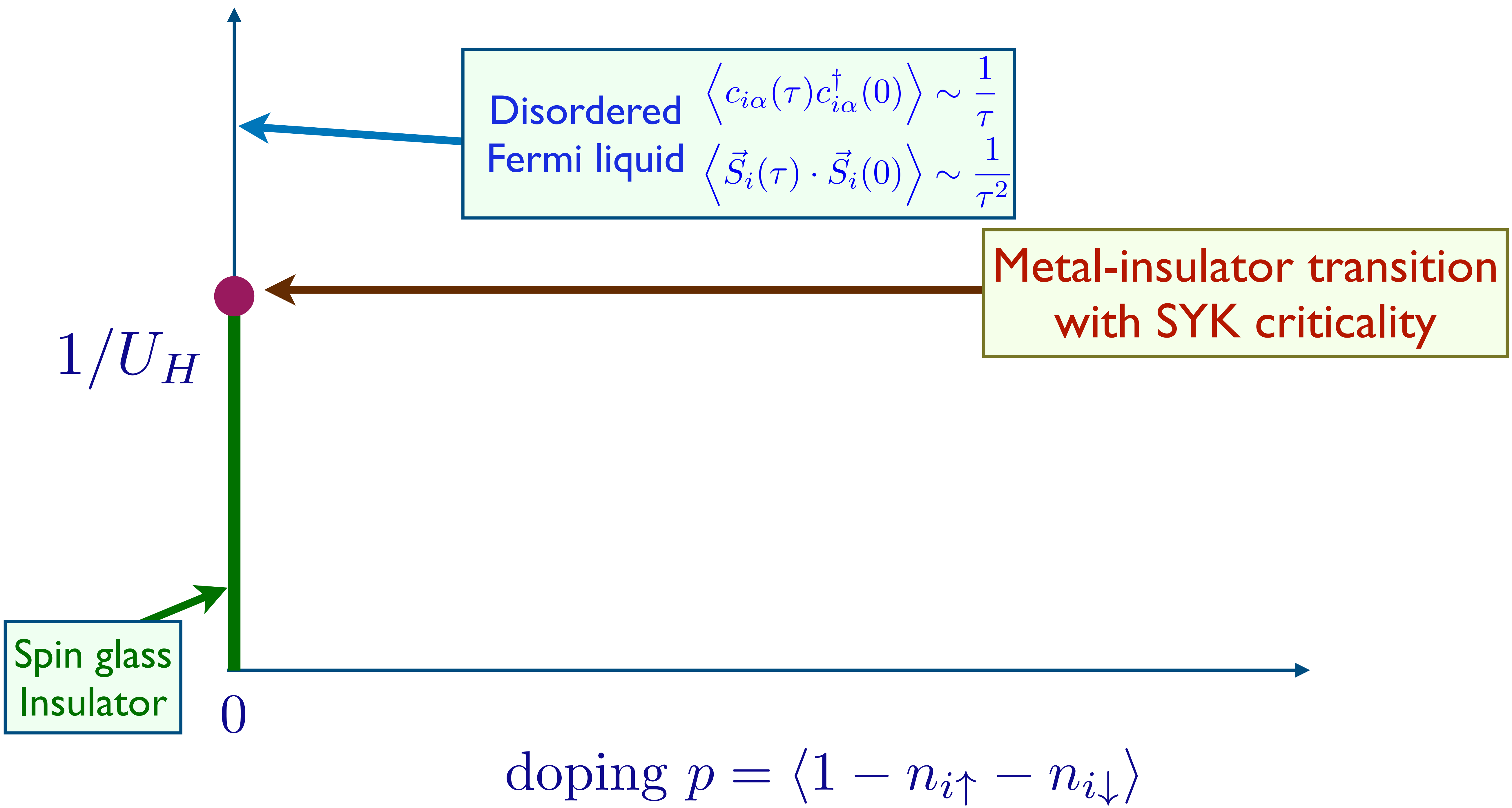
Resisitivity $\rho \sim T$ to the lowest T at the critical point

Onset of insulating gap and spin glass order co-incide.

Peter Cha, N. Wentzell, O. Parcollet, A. Georges, Eun-Ah Kim, PNAS (2020)

Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



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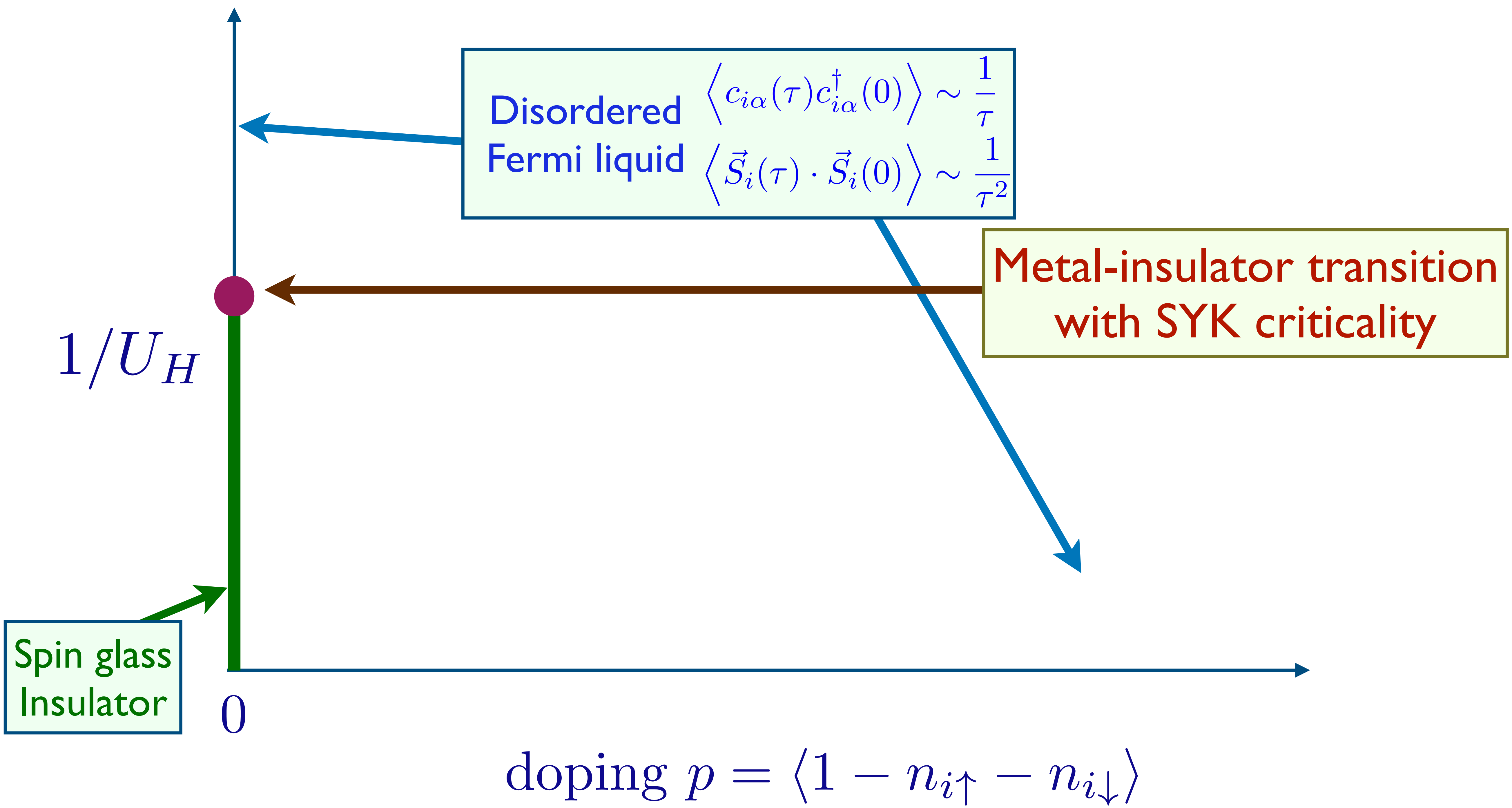

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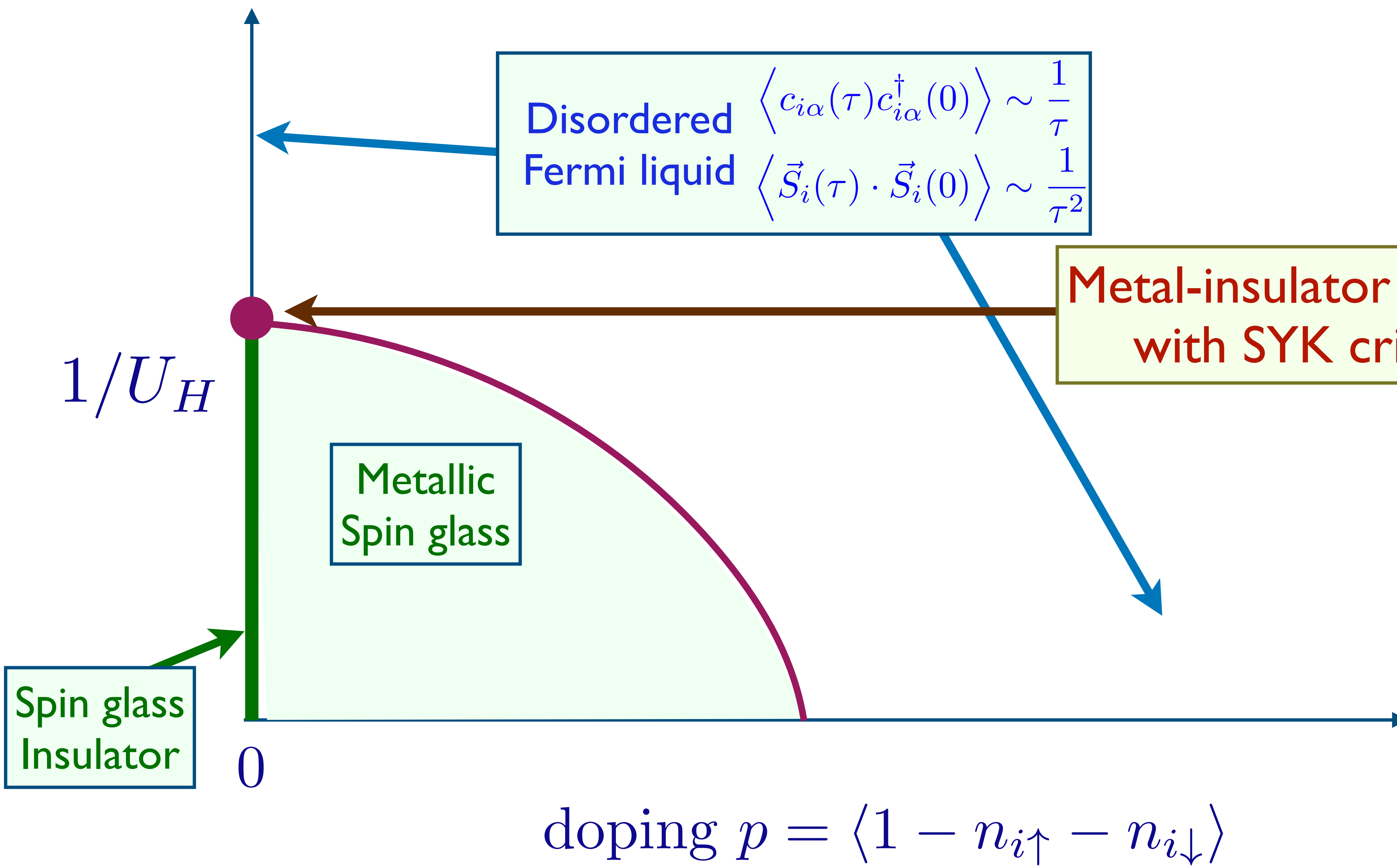

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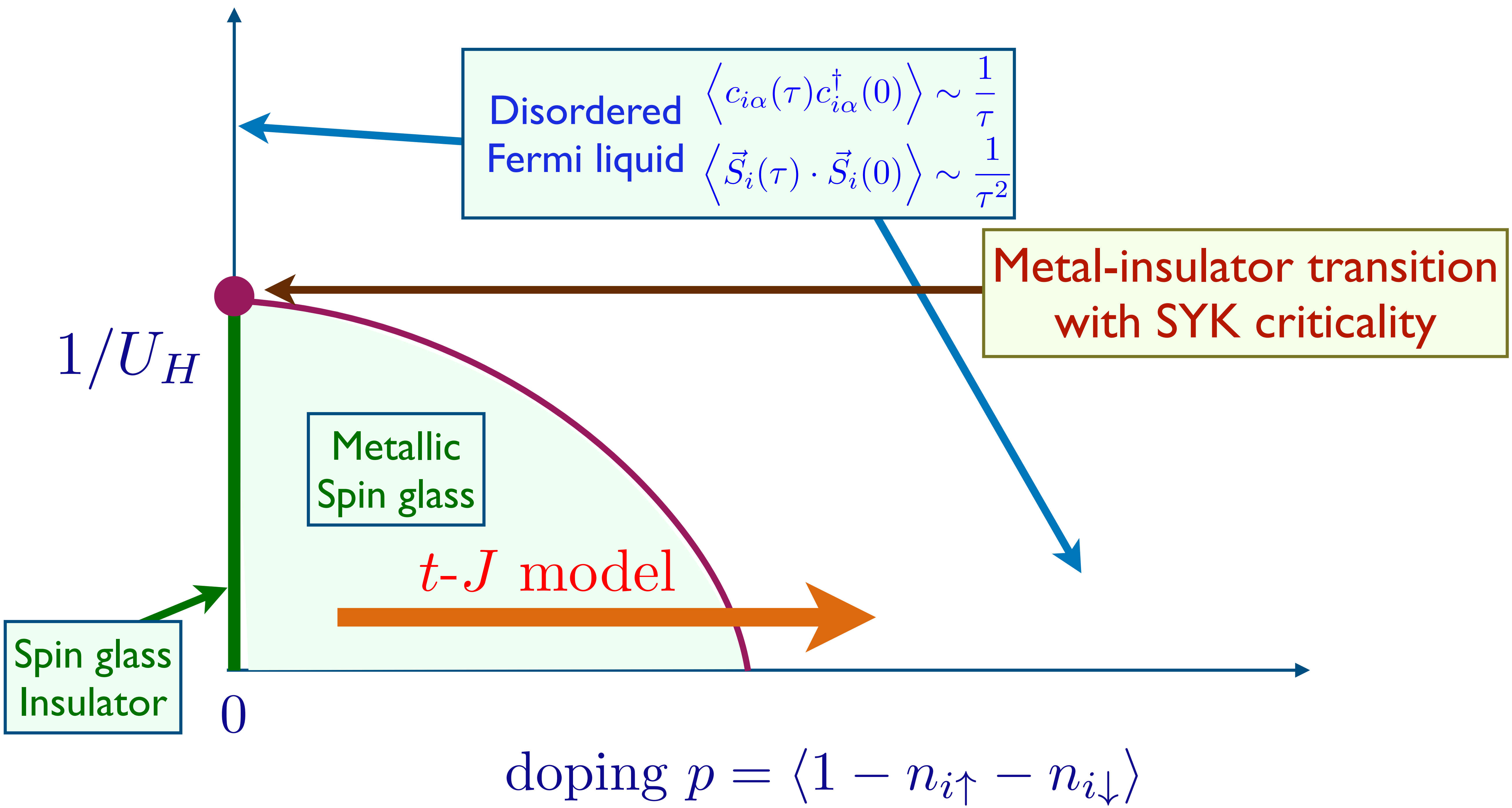


Random *t-J* model $(U_H \to \infty)$

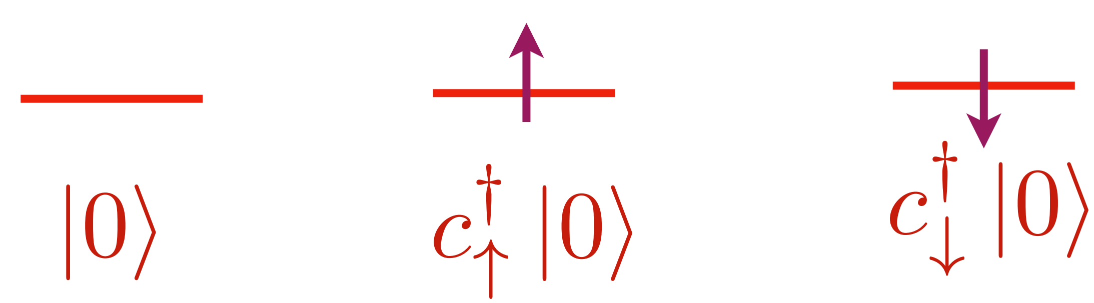
$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^\dagger\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_\alpha c_{i\alpha}^\dagger c_{i\alpha} \leq 1, \quad \frac{1}{N}\sum_{i\alpha} c_{i\alpha}^\dagger c_{i\alpha} = 1 - p$$

$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$
$$t_{ij} \text{ random}, \ \overline{t_{ij}} = 0, \ \overline{t_{ij}^2} = t^2$$



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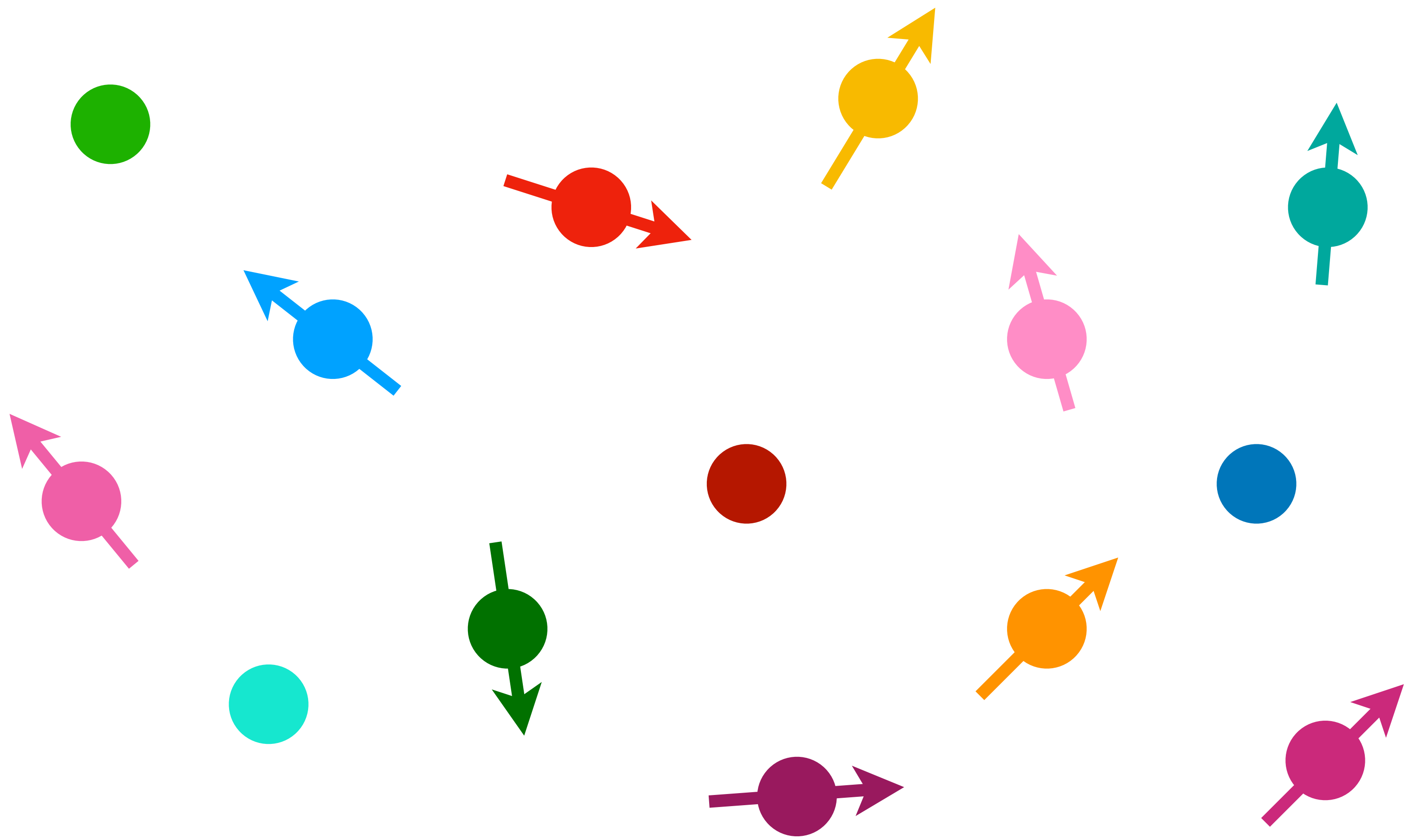
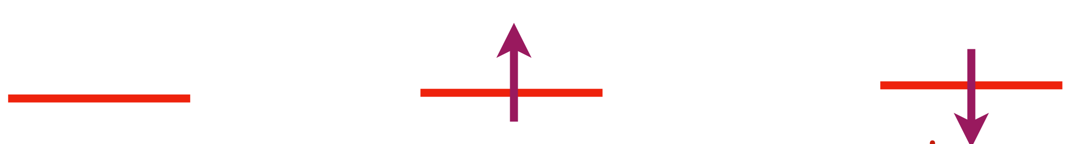
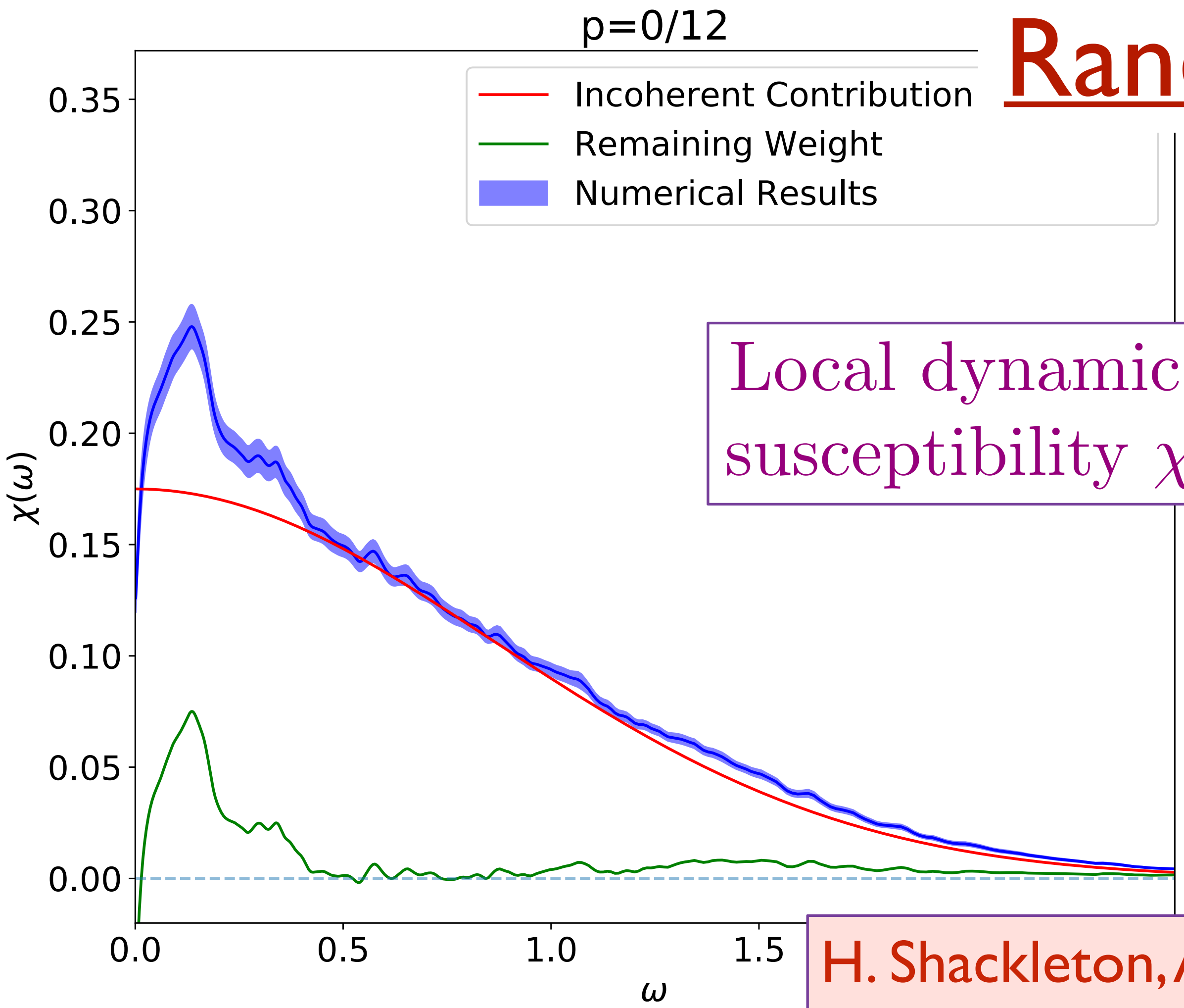


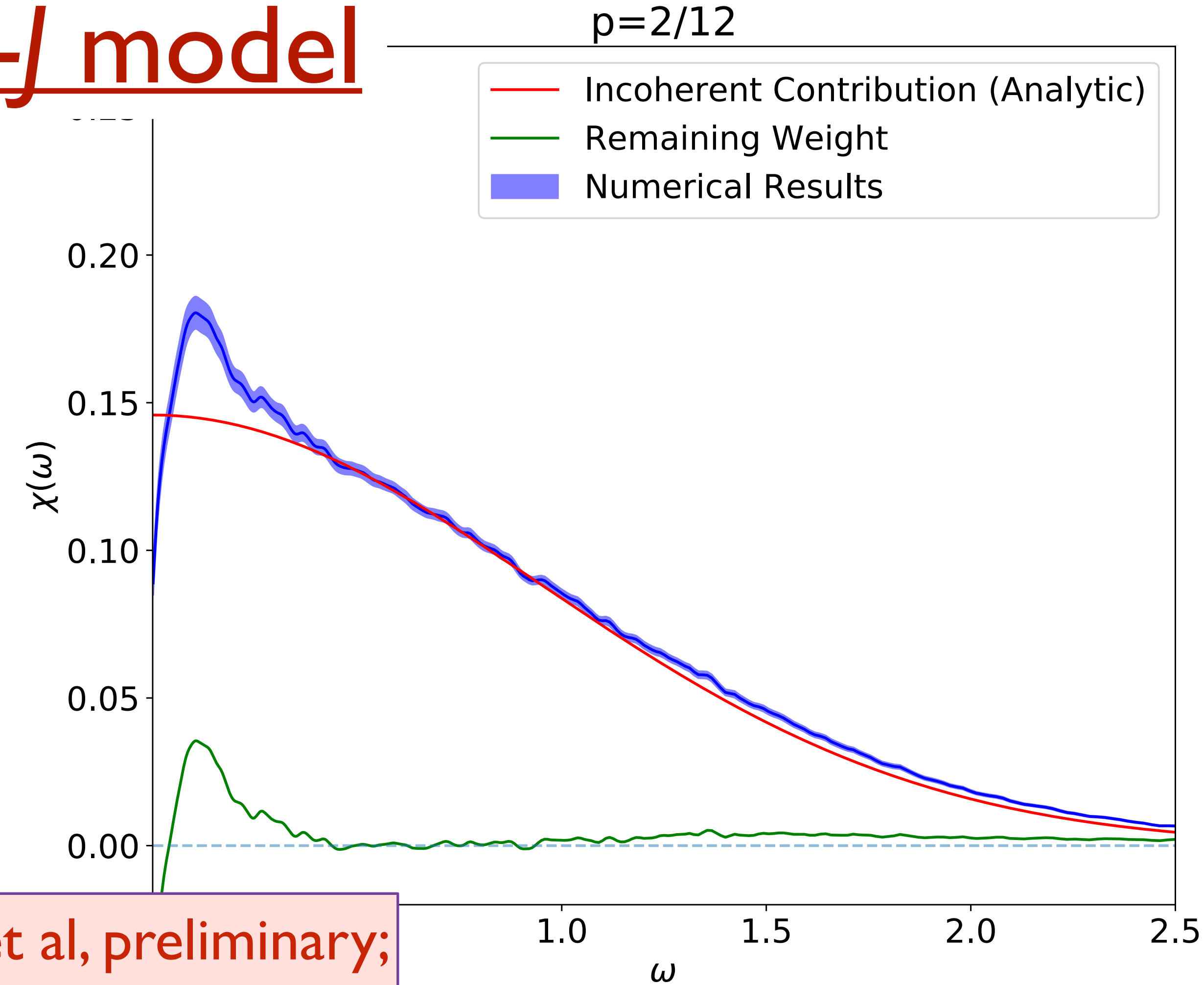


$|0\rangle$ $\quad$ $c_{\uparrow}^{\dagger}\,|0\rangle$ $\quad$ $c_{\downarrow}^{\dagger}\,|0\rangle$

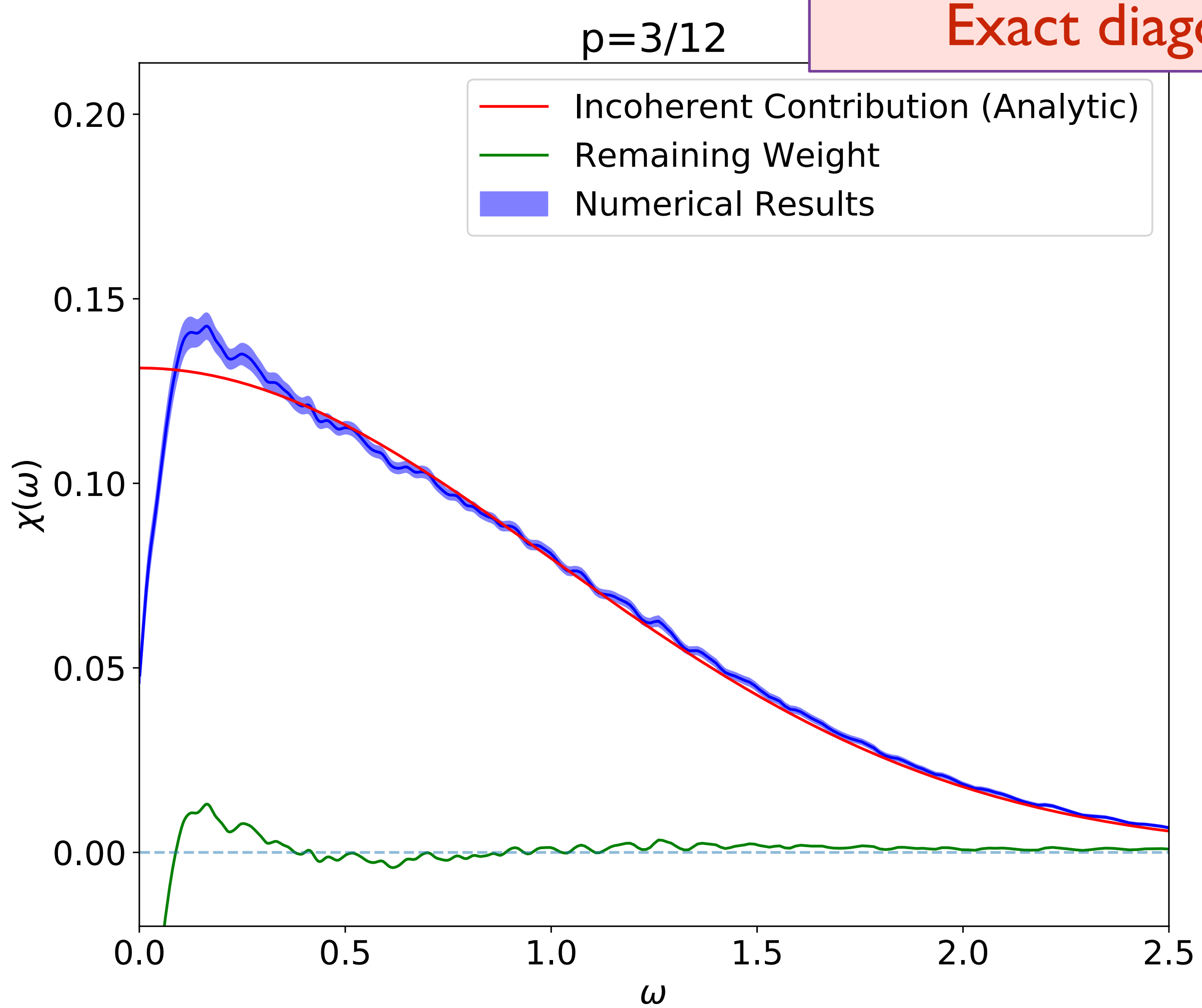
Random *t-J* model

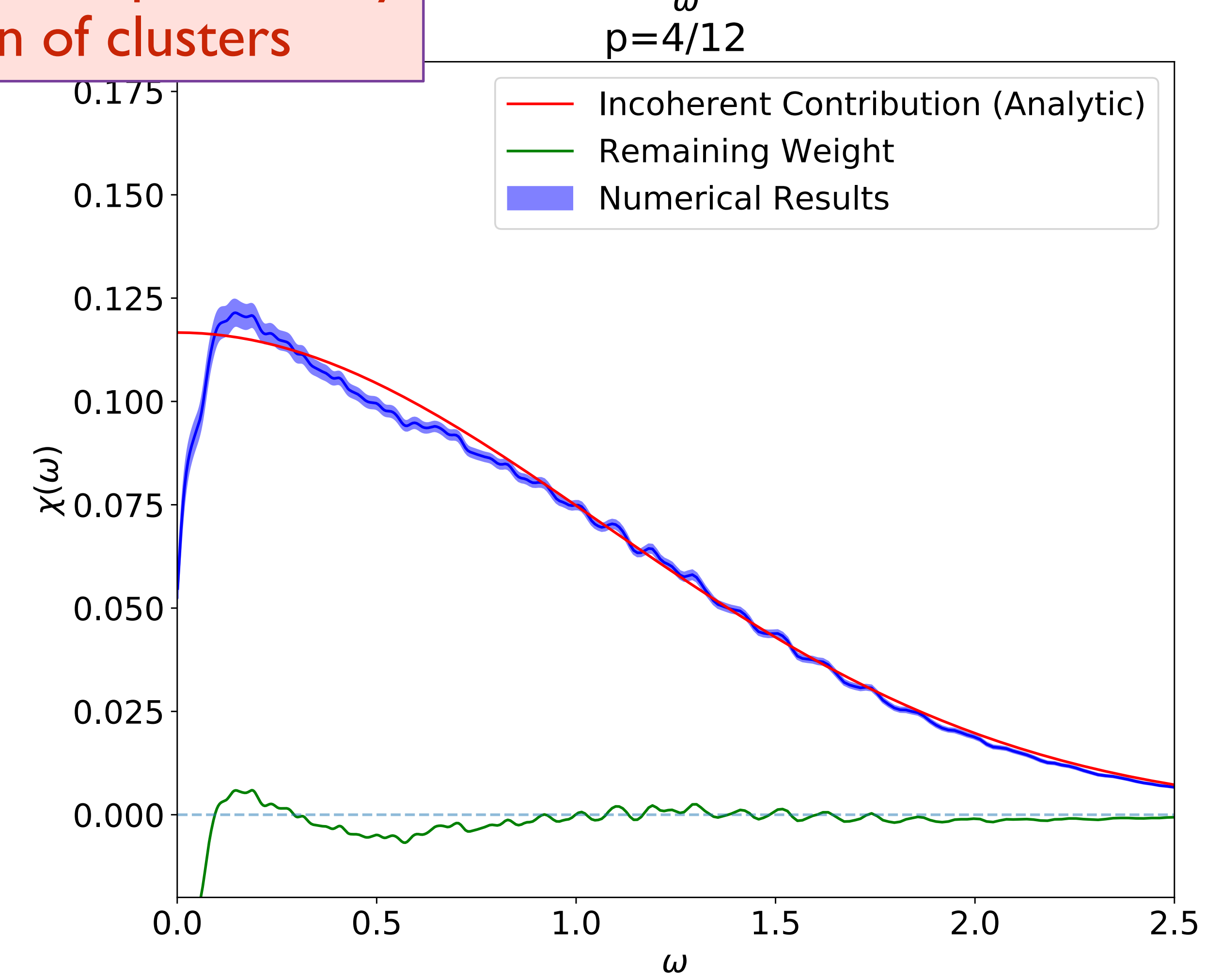
Local dynamic spin susceptibility $\chi''(\omega)$





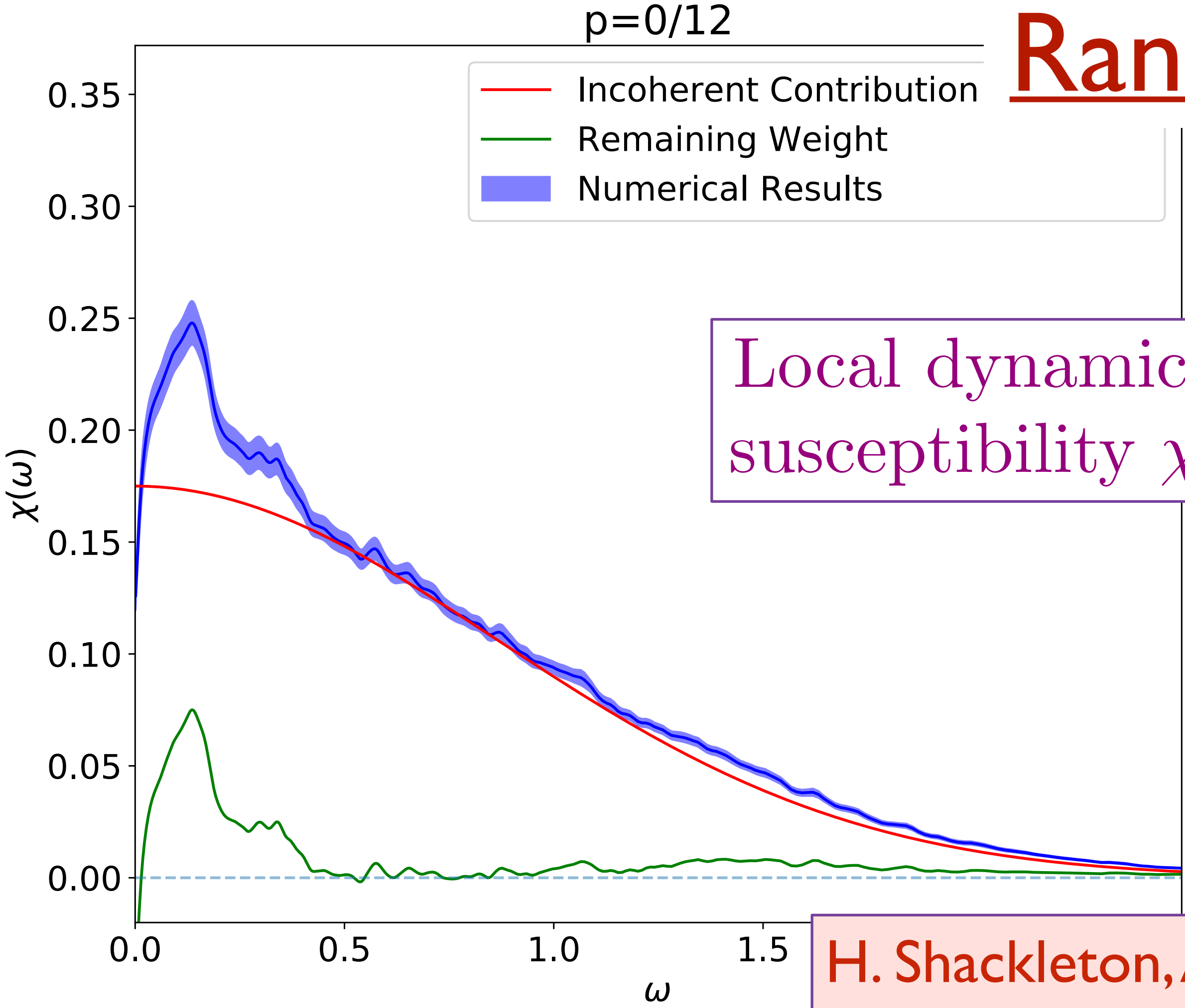
H. Shackleton, A. Wietek et al, preliminary;
Exact diagonalization of clusters

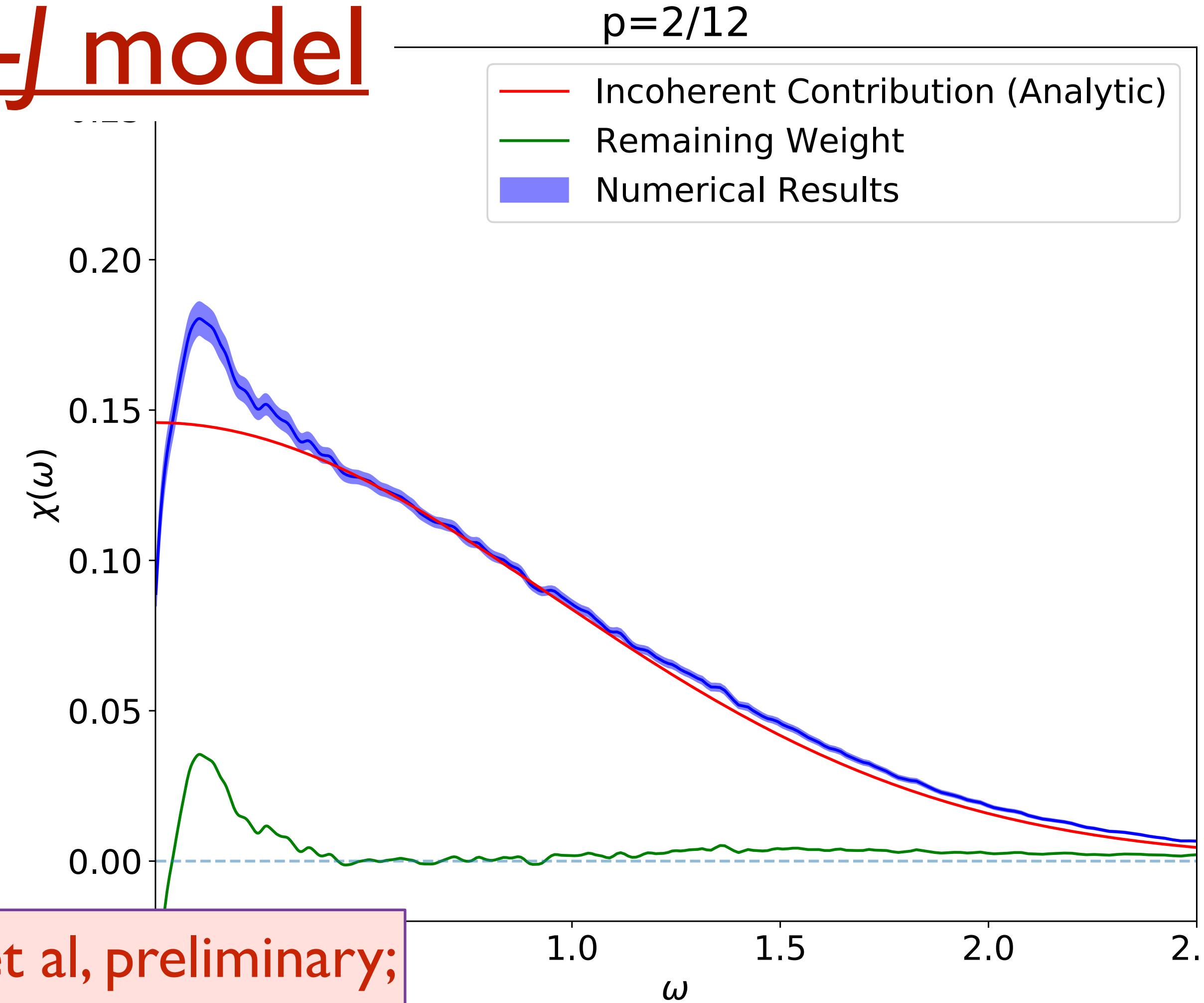




Random *t-J* model

Local dynamic spin susceptibility $\chi''(\omega)$





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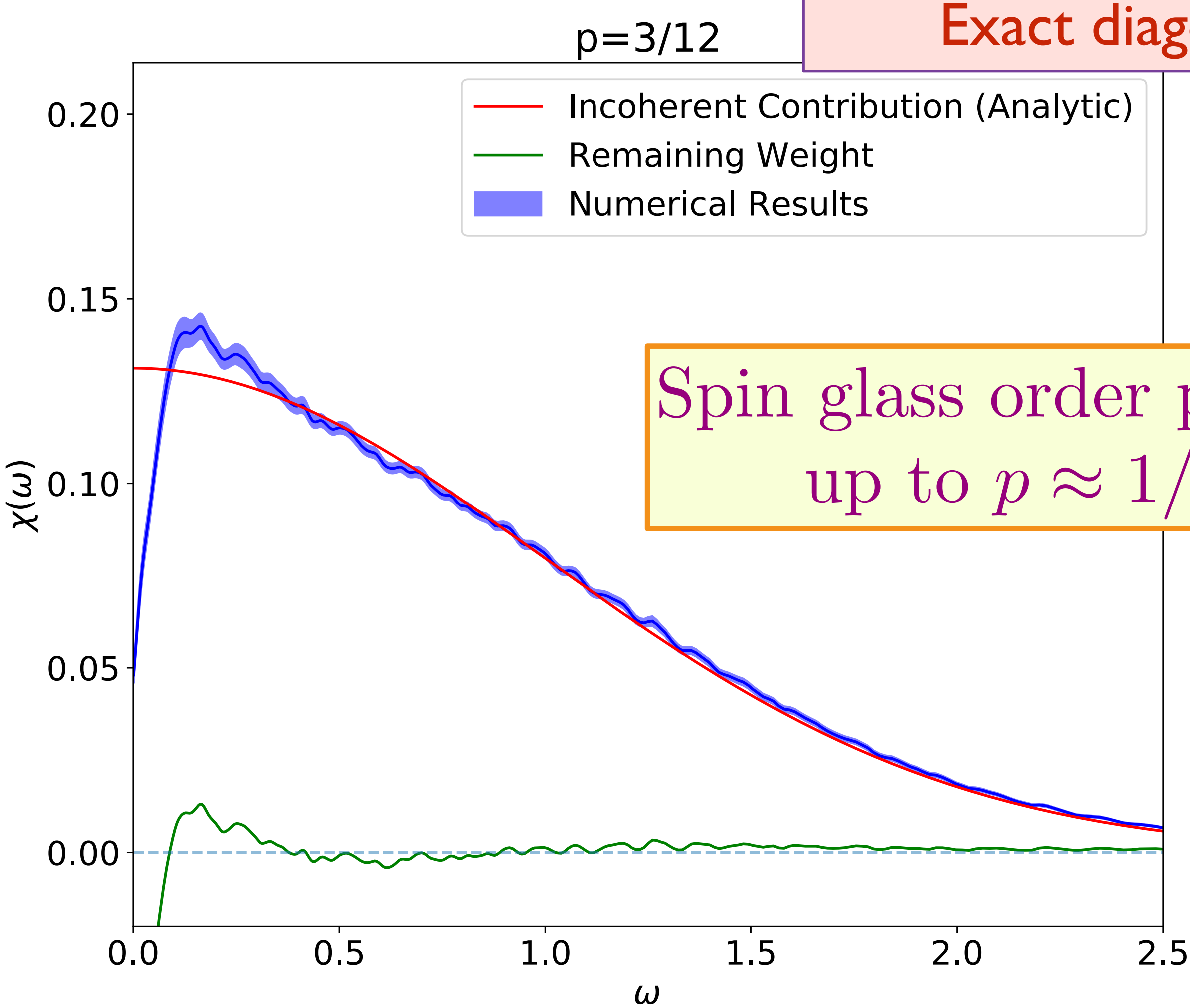
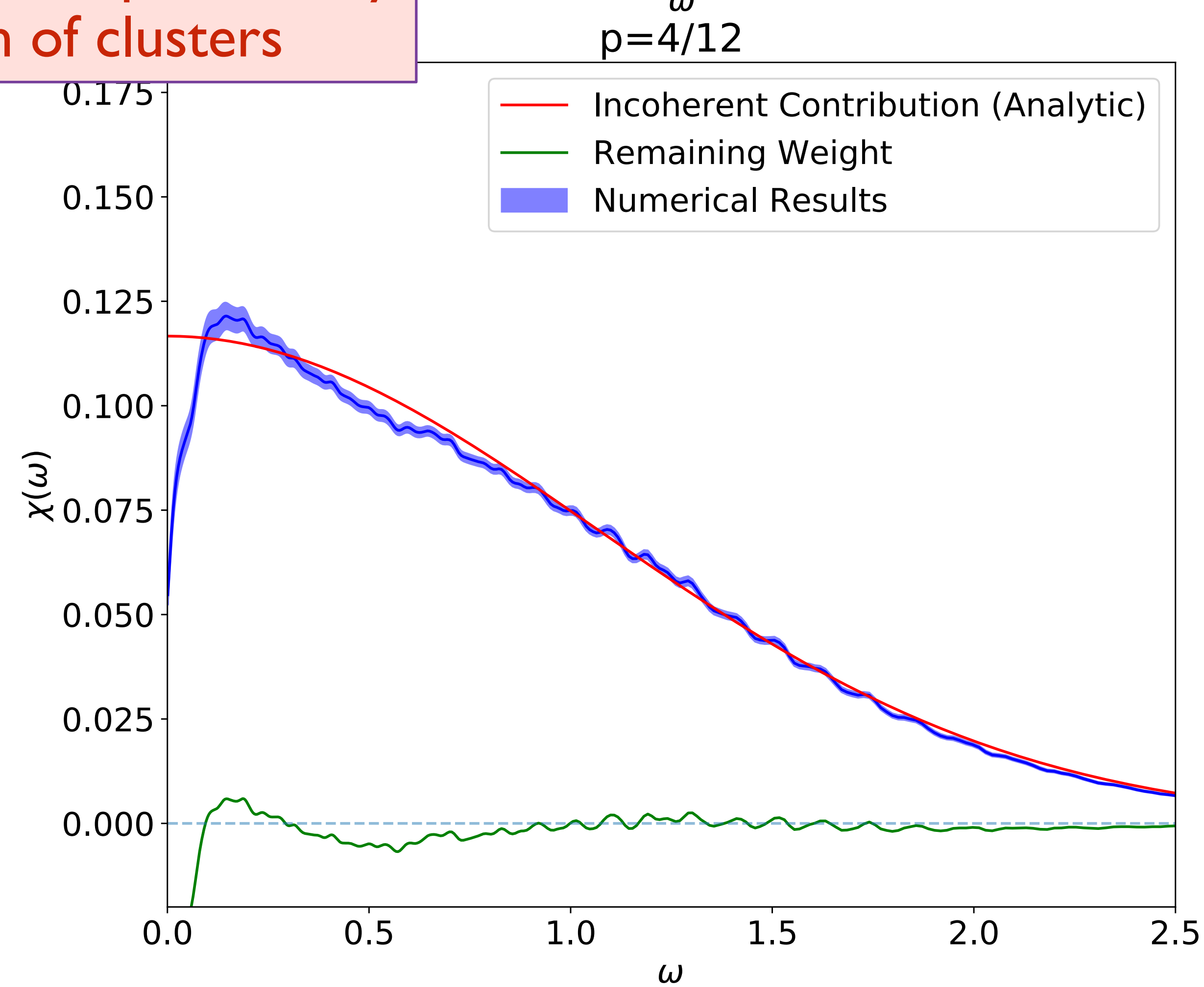
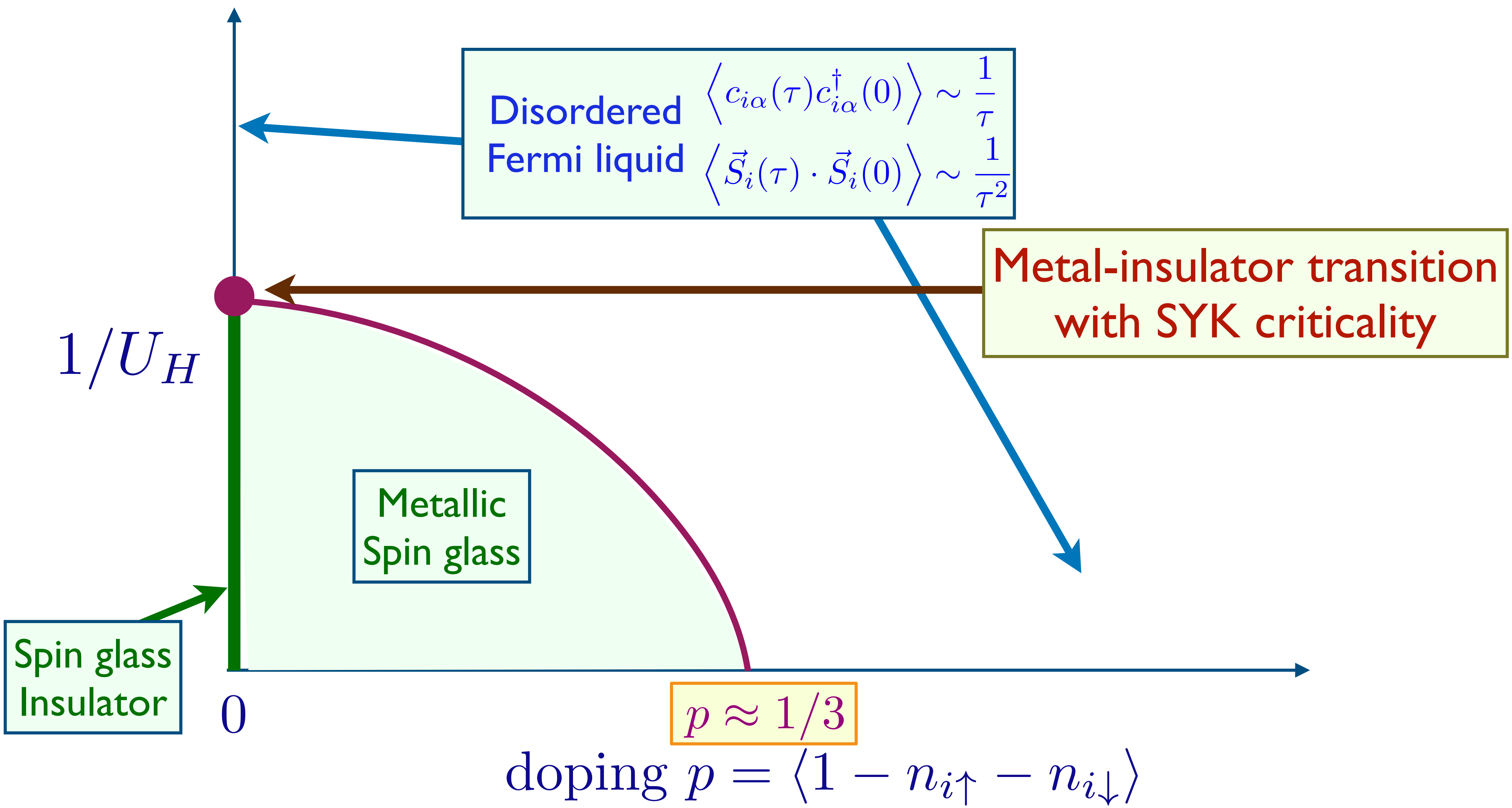




Spin glass order present up to $p \approx 1/3$.

Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



1. Quantum matter with quasiparticles: random matrix model

2. Quantum matter without quasiparticles: the complex SYK model

No "Mottness"

3. Random *J* model (insulator)
   RG analysis and exact exponent

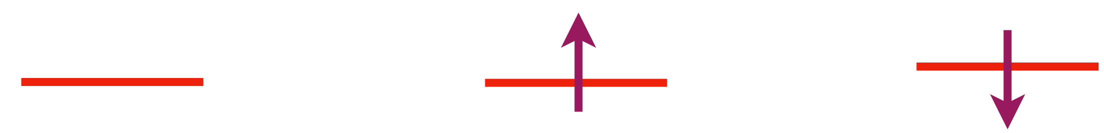
4. Random Hubbard and *t*-*J* models
   Numerical results

5. Random t-*J* model (metals): *exact exponents*

Random t-J model (metal)

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):



$$\begin{aligned} c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \end{aligned}$$

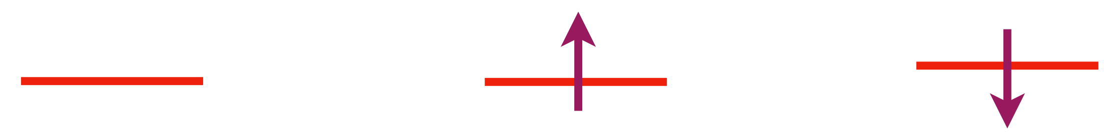
U(1) gauge invariance, $\quad b \to b e^{i\phi}\,, \quad f_\alpha \to f_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

Random t-J model (metal)

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a fermion $\mathfrak{f}$ (the holon) and a boson $\mathfrak{b}_\alpha$ (the spinon):



$\mathfrak{f}^\dagger \left|v\right\rangle$ $\qquad$ $\mathfrak{b}_\uparrow^\dagger \left|v\right\rangle$ $\qquad$ $\mathfrak{b}_\downarrow^\dagger \left|v\right\rangle$

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_\alpha^\dagger \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} &= 1 \end{aligned}$$

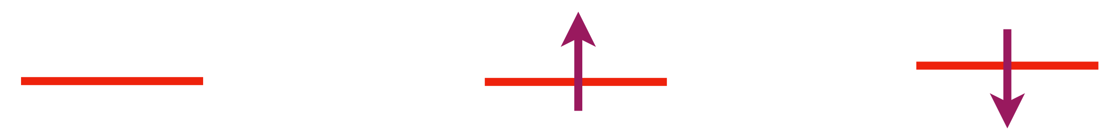
U(1) gauge invariance, $\qquad \mathfrak{f} \to \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

Random t-J model (metal)

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a fermion $\mathfrak{f}$ (the holon) and a boson $\mathfrak{b}_\alpha$ (the spinon):



$$\mathfrak{f}^\dagger \left|v\right\rangle \qquad \mathfrak{b}_\uparrow^\dagger \left|v\right\rangle \qquad \mathfrak{b}_\downarrow^\dagger \left|v\right\rangle$$

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_\alpha^\dagger \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} &= 1 \end{aligned}$$

$\mathrm{SU}(1|2) \equiv \mathrm{SU}(2|1)$

U(1) gauge invariance, $\quad \mathfrak{f} \to \mathfrak{f} e^{i\phi}, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

Random t-J model (metal)

$$\mathcal{Z} = \int \mathcal{D}\mathcal{P}(\tau) e^{-\mathcal{S}_B - \mathcal{S}_{tJ}}$$

$$\mathcal{S}_B = i \int_0^1 du \int d\tau \, \mathrm{Tr}\left(\mathcal{P} \partial_\tau \mathcal{P} \partial_u \mathcal{P}\right)$$

$$\mathcal{S}_{tJ} = \int d\tau d\tau' \, \mathrm{Tr}\left(\mathcal{P}(\tau) \mathcal{Q}(\tau - \tau') \mathcal{P}(\tau')\right) + \int d\tau \mathrm{Tr}\left(s_0 \, \mathcal{P}(\tau)\right) .$$

Path integral over a superspin $\mathcal{P}(\tau)$ with a self-consistent self-interaction $\mathcal{Q}(\tau)$ and a 'Zeeman superfield' s_0.

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Random t-J model (metal)

$$\mathcal{Z} = \int \mathcal{D}f_\alpha(\tau)\mathcal{D}b(\tau)\mathcal{D}\lambda(\tau)e^{-\mathcal{S}_B - \mathcal{S}_{tJ}}$$

$$\mathcal{S}_B = \int d\tau \left[f_\alpha^\dagger(\tau)\left(\frac{\partial}{\partial\tau} + i\lambda\right) f_\alpha(\tau) + b^\dagger(\tau)\left(\frac{\partial}{\partial\tau} + i\lambda\right) b(\tau) - i\lambda \right]$$

$$\mathcal{S}_{tJ} = \int d\tau\, s_0 f_\alpha^\dagger(\tau) f_\alpha(\tau) + t^2 \int d\tau d\tau' R(\tau - \tau') c_\alpha^\dagger(\tau) c_\alpha(\tau') - \frac{J^2}{2}\int d\tau d\tau' Q(\tau - \tau')\vec{S}(\tau)\cdot\vec{S}(\tau')\,.$$

From this action we determined the correlators

SU(1|2) theory

$$\overline{R}(\tau - \tau') = -\left\langle c_\alpha(\tau) c_\alpha^\dagger(\tau')\right\rangle_{\mathcal{Z}}$$

$$\overline{Q}(\tau - \tau') = \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(\tau')\right\rangle_{\mathcal{Z}}$$

and finally impose the self-consistency conditions

$$R(\tau) = \overline{R}(\tau) \quad , \quad Q(\tau) = \overline{Q}(\tau).$$

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Random t-J model (metal)

$$\mathcal{Z} = \int \mathcal{D}\mathfrak{b}_\alpha(\tau)\mathcal{D}\mathfrak{f}(\tau)\mathcal{D}\lambda(\tau)e^{-\mathcal{S}_B-\mathcal{S}_{tJ}}$$

$$\mathcal{S}_B = \int d\tau \left[\mathfrak{b}^\dagger_\alpha(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)\mathfrak{b}_\alpha(\tau)+\mathfrak{f}^\dagger(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)\mathfrak{f}(\tau)-i\lambda\right]$$

$$\mathcal{S}_{tJ} = \int d\tau\, s_0\mathfrak{b}^\dagger_\alpha(\tau)\mathfrak{b}_\alpha(\tau)+t^2\int d\tau d\tau' R(\tau-\tau')c^\dagger_\alpha(\tau)c_\alpha(\tau') - \frac{J^2}{2}\int d\tau d\tau' Q(\tau-\tau')\vec{S}(\tau)\cdot\vec{S}(\tau').$$

SU(2|1) theory

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$$\overline{R}(\tau-\tau') = -\left\langle c_\alpha(\tau)c^\dagger_\alpha(\tau')\right\rangle_{\mathcal{Z}}$$

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- The RG analysis is very similar to that for the J model, except that the SU(2) spin is replaced by a SU(1|2) $\cong$ SU(2|1) superspin.

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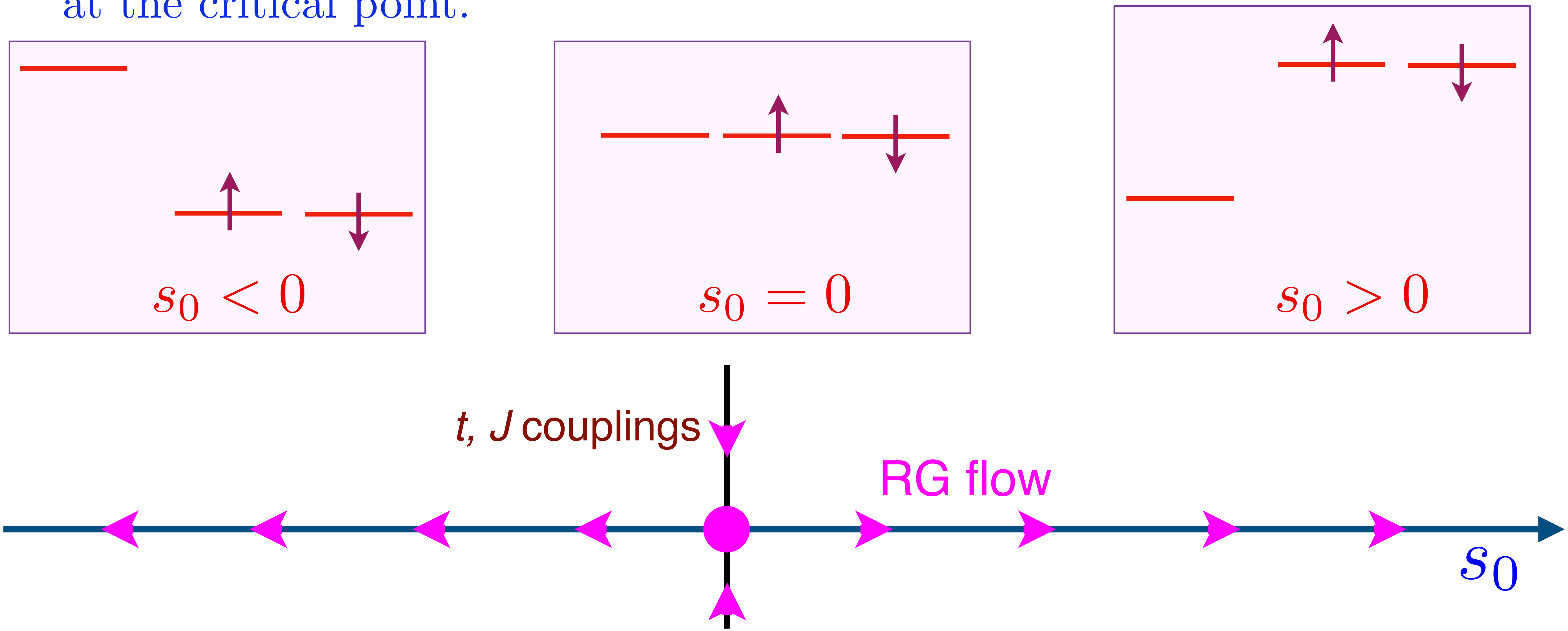

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$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|} \quad , \quad \left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \frac{1}{\tau} \,.$$

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Square of spinon correlator

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|} \quad , \quad \left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \frac{1}{\tau} .$$

Product of spinon and holon correlators

- These exponents do not have a quasiparticle interpretation. However, they can be understood (in a large M limit of a model with SU(M) symmetry) by *fractionalization* of the electron into a spinon and holon, each of which decay as $1/\sqrt{\tau}$.

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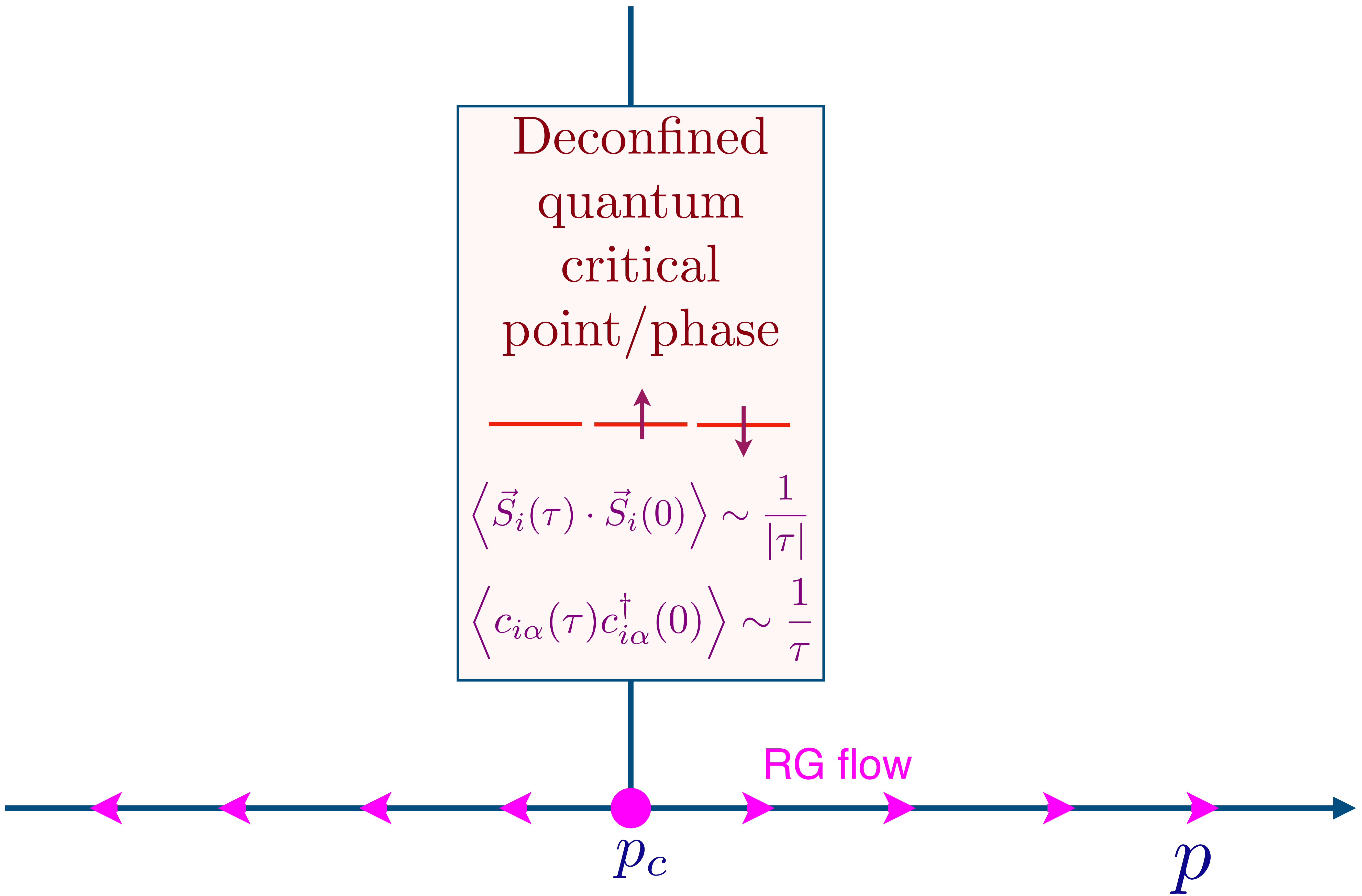
t-J phase diagram: RG using *either* SU(2|1) or SU(1|2)



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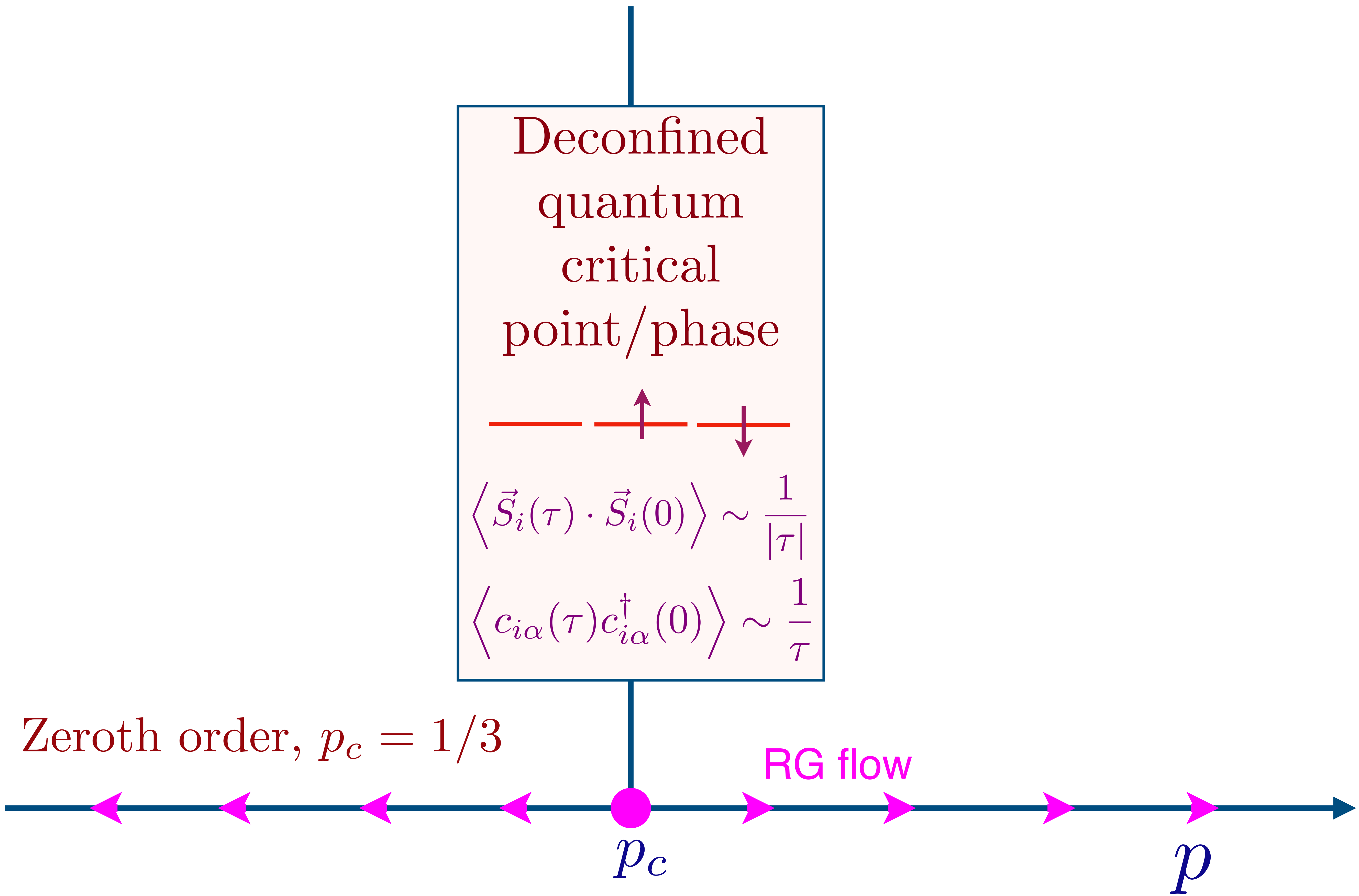

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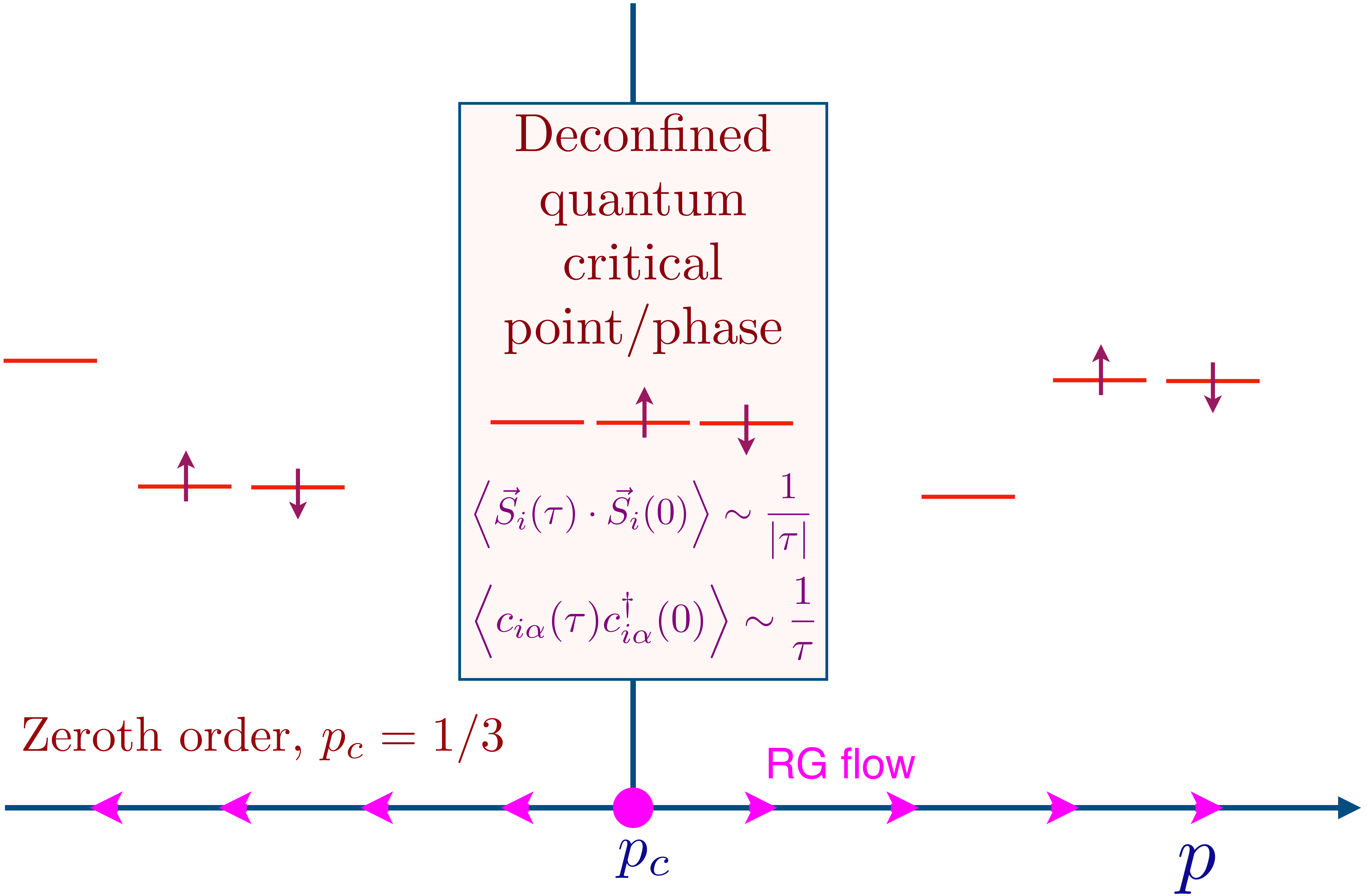

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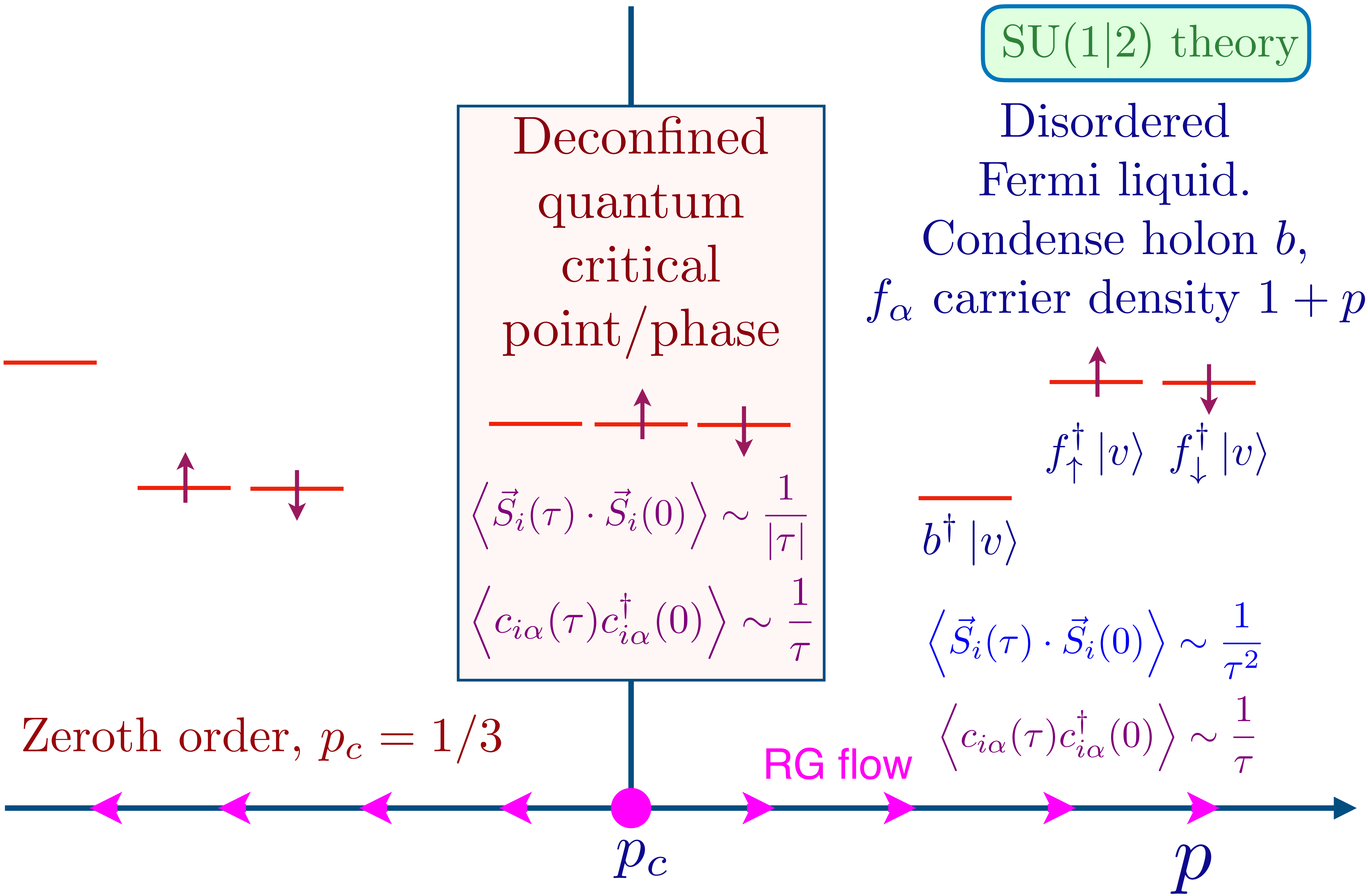

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t-J phase diagram: RG using *either* SU(2|1) or SU(1|2)

SU(2|1) theory

Metallic spin glass.
Condense spinon $\mathfrak{b}_\alpha$,
$\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger |v\rangle$

$\mathfrak{b}^\dagger_\uparrow |v\rangle$ $\mathfrak{b}^\dagger_\downarrow |v\rangle$

$$\left\langle \vec{S}_i(\tau)\cdot\vec{S}_i(0)\right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau)c^\dagger_{i\alpha}(0)\right\rangle \sim \frac{1}{\tau}$$

Deconfined quantum critical point/phase

$$\left\langle \vec{S}_i(\tau)\cdot\vec{S}_i(0)\right\rangle \sim \frac{1}{|\tau|}$$

$$\left\langle c_{i\alpha}(\tau)c^\dagger_{i\alpha}(0)\right\rangle \sim \frac{1}{\tau}$$

SU(1|2) theory

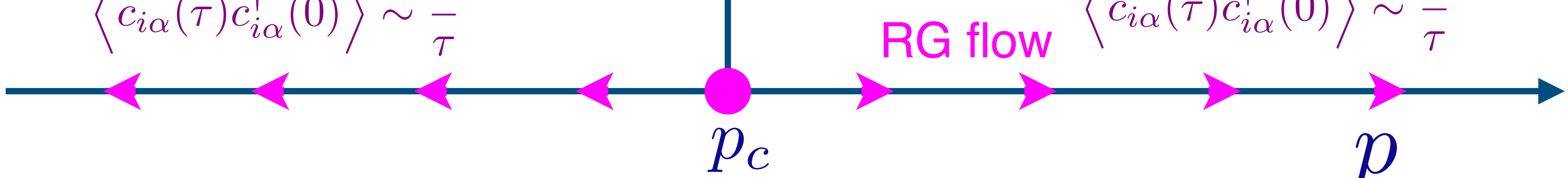
Disordered Fermi liquid.
Condense holon b,
f_α carrier density $1+p$

$f^\dagger_\uparrow |v\rangle$ $f^\dagger_\downarrow |v\rangle$

$b^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau)\cdot\vec{S}_i(0)\right\rangle \sim \frac{1}{\tau^2}$$

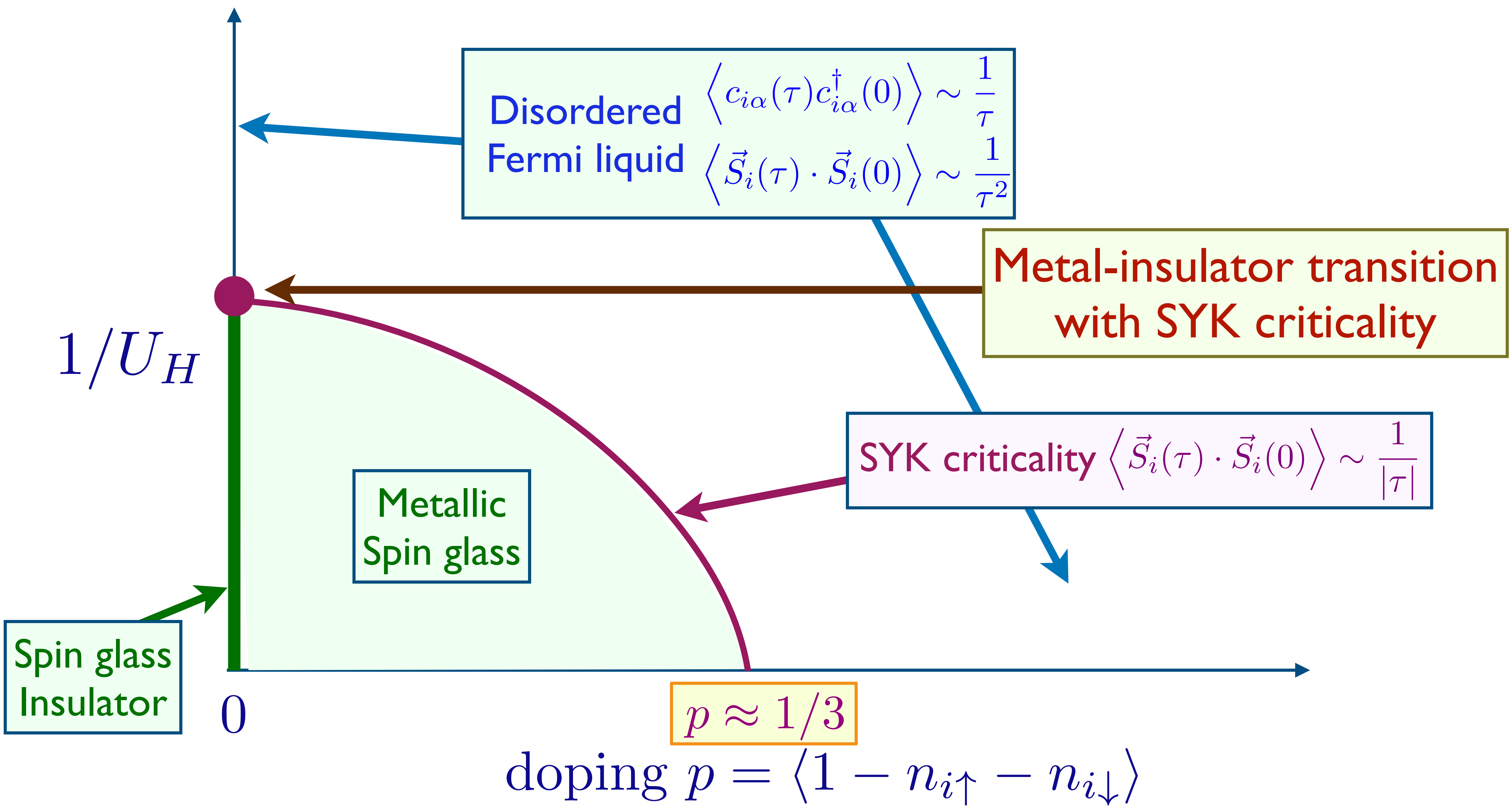
$$\left\langle c_{i\alpha}(\tau)c^\dagger_{i\alpha}(0)\right\rangle \sim \frac{1}{\tau}$$



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



At the critical point/phase of the t-J model, the Fermi liquid-like behavior of the electron Green's function

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^{\dagger}(0) \right\rangle \sim \frac{1}{\tau}$$

leads to a non-zero *residual resitivity*, $\rho(0) \neq 0$.

However, the critical state is *not* a Fermi liquid, as indicated by the slow decay of the spin correlations

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

Moreover, in a Fermi liquid, we expect $\rho(T) - \rho(0) \sim T^2$, which also does not hold here.

Time reparameterization soft mode

The dominant corrections to the SL(2,R) invariant critical Green's function can arise from the time reparameterization soft mode, and these take the form

$$\left\langle c_{i\alpha}(\tau) c^\dagger_{i\alpha}(0) \right\rangle \sim \frac{\pi T}{\sin(\pi T \tau)} \left(1 + \alpha_G \frac{T}{J} \Phi_{\text{non-conformal}}(T\tau) \right)$$

where $\Phi_{\text{non-conformal}}(T\tau)$ is a computable (in the large M limit) scaling function, and α_G is universally proportional to the co-efficient α_S of the Schwarizan action for the time reparameterization mode.

J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)
A. Kitaev and J. Suh, JHEP 183 (2018)
Haoyu Guo, Yingfei Guo, S. Sachdev, Annals of Physics **418**, 168202 (2020)

Time reparameterization soft mode

Computing the resistivity from this Green's function via the Kubo formula, we find

$$\rho(T) = \rho(0)\left(1 + 8\alpha_G \frac{T}{J} + \ldots\right)$$

Haoyu Guo, Yingfei Guo, S. Sachdev, Annals of Physics **418**, 168202 (2020)

Summary: Presented a series of increasingly realistic (partly) solvable random models of correlated metals

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4. No 'Mottness' in SYK-island models.

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6. Random t-J model: transition at a doping density p with many attractive features:

   - Critical correlations $G(\tau) \sim 1/\tau$ and $\chi(\tau) \sim 1/|\tau|$.
   - Can be interpreted in terms of fractionalization with spinon and holon correlators $\sim 1/\sqrt{\tau}$ (deconfined criticality).
   - Linear-in-T resistivity down to $T = 0$ at the critical point from the time reparameterization soft mode.
   - Carrier density p for $p < p_c$, and $1 + p$ for $p > p_c$.