Electrical and thermal transport near quantum phase transitions in condensed matter, and in dyonic black holes

Sean Hartnoll (KITP)
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Subir Sachdev (Harvard)

Outline

Transport near strongly interacting quantum critical points

1. The superfluid-insulator transition in the boson Hubbard model:
   Hydrodynamic-collisionless crossover of a CFT

2. Exact solutions of CFTs in 1+1 dimensions
   No hydrodynamics

3. Exact solution of a CFT in 2+1 dimensions - Yang-Mills theory with N=8 supersymmetry:
   Black holes in AdS_4

4. General hydrodynamic theory in the presence of a magnetic field, chemical potential and impurities:
   Nernst effect in the cuprate superconductors;
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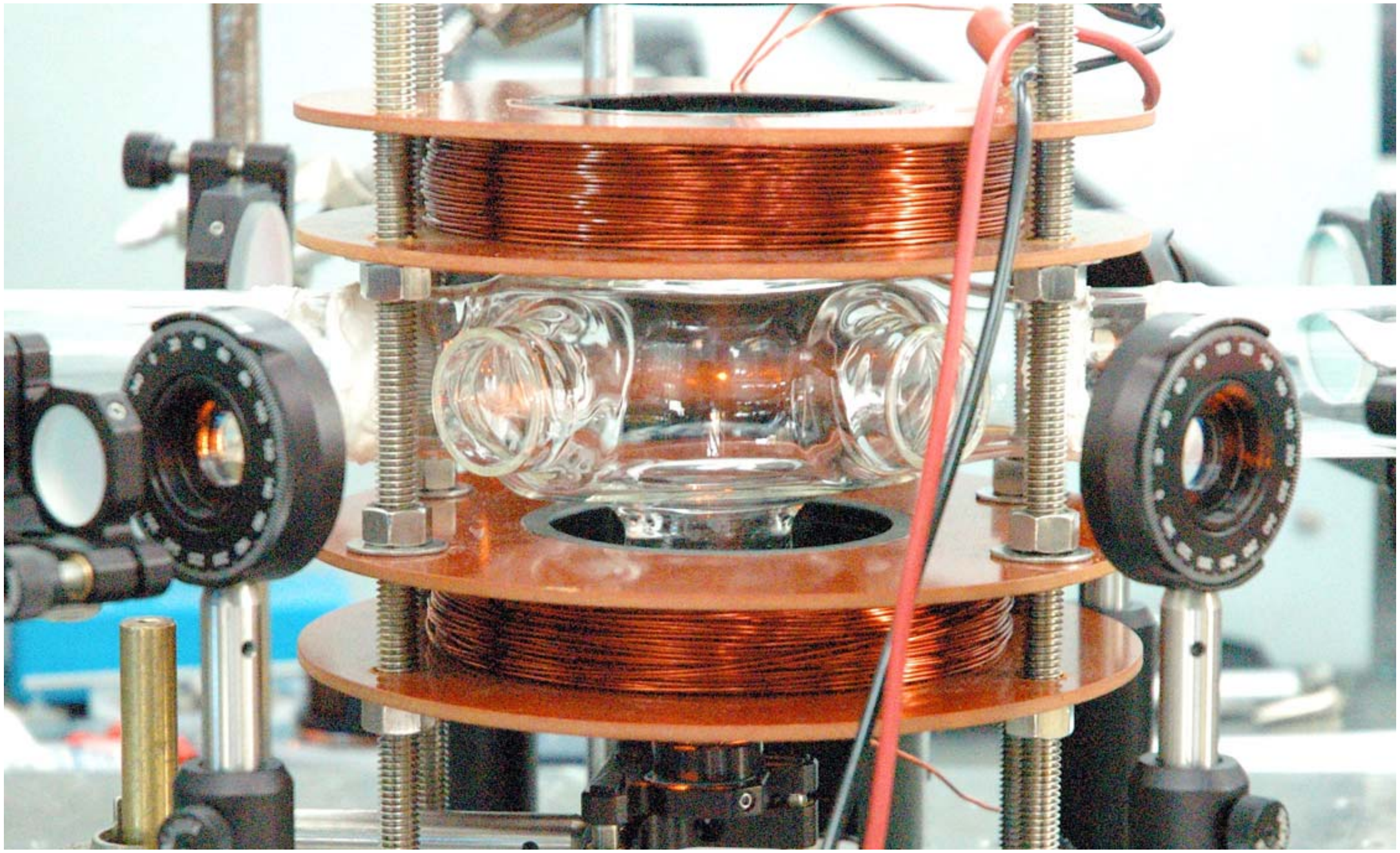
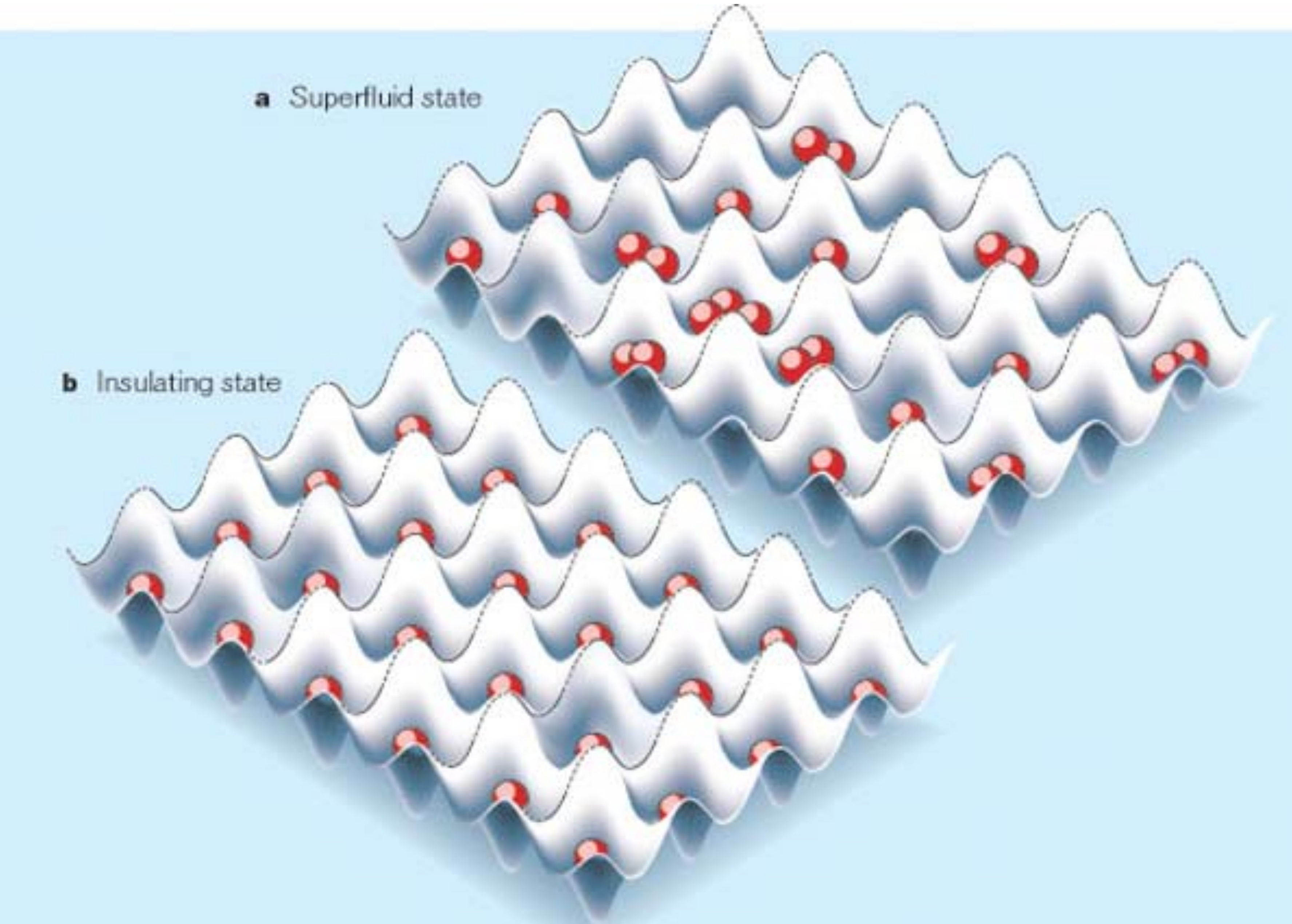


Trap for ultracold ^{87}Rb atoms



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Boson Hubbard model

Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

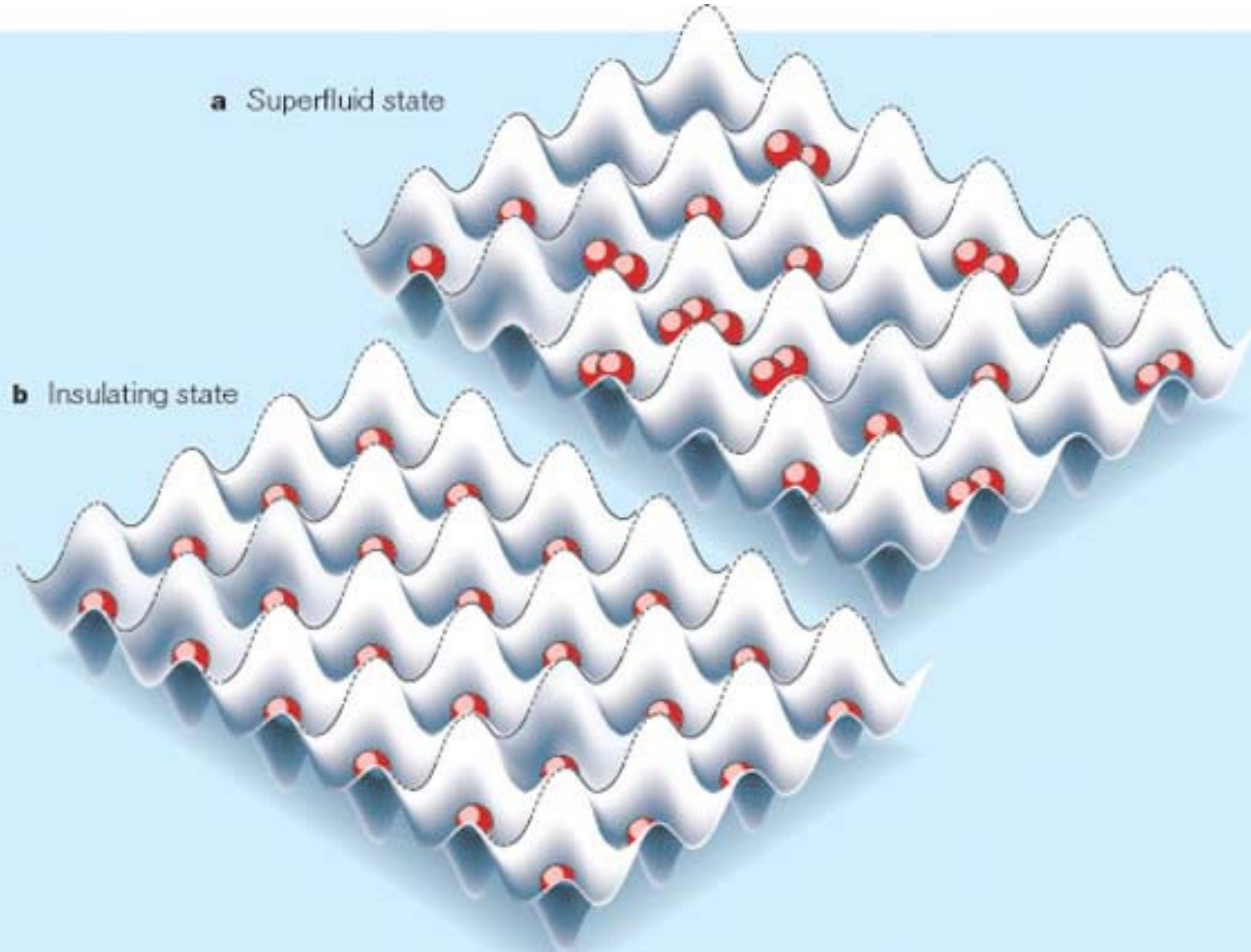
$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2} \sum_j n_j (n_j - 1) + \cdots$$

$$n_j \equiv b_j^\dagger b_j$$

M.PA. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher *Phys. Rev. B* **40**, 546 (1989).

For small U/t, superfluid

For large U/t, insulator



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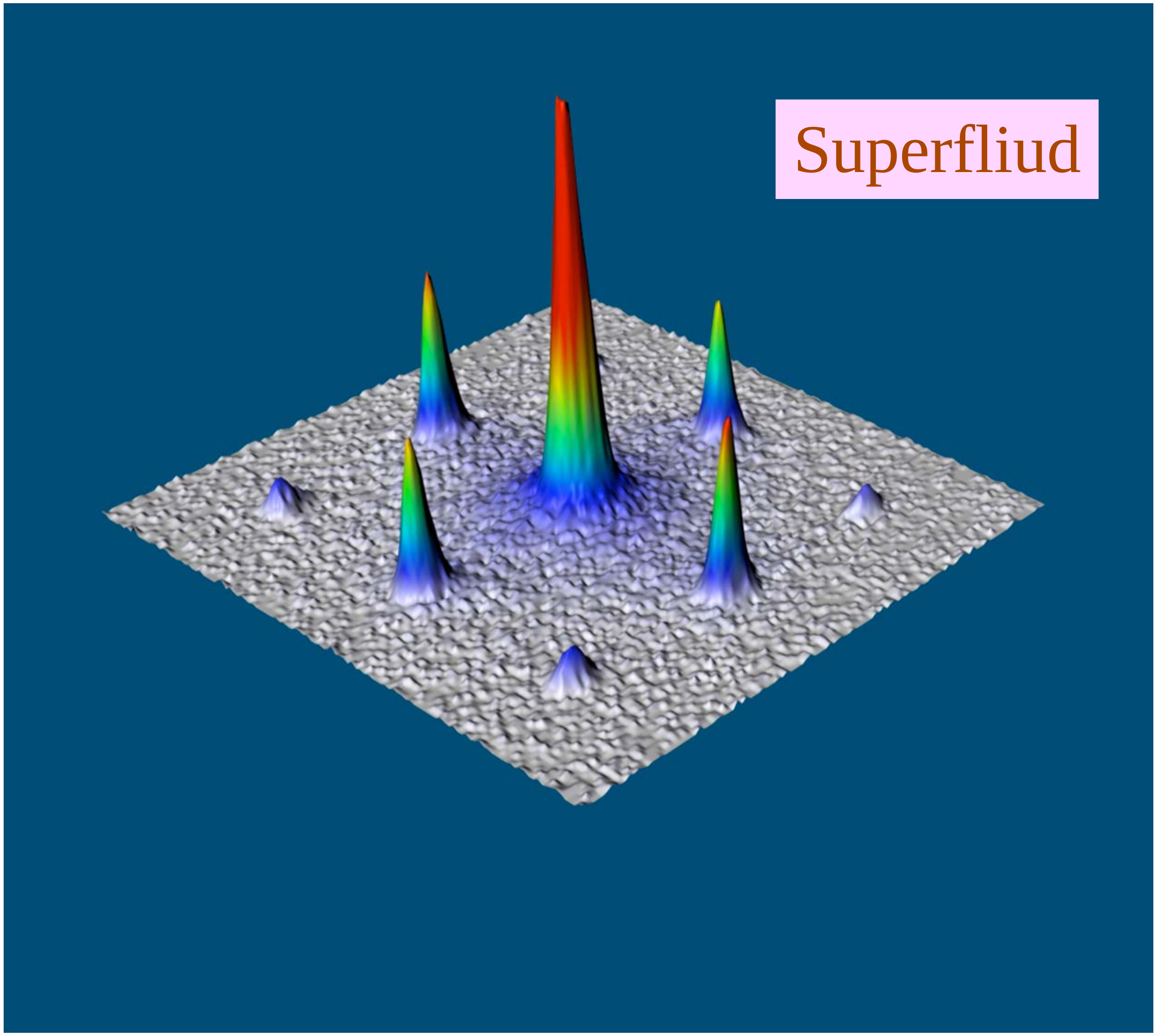
Velocity distribution of ^{87}Rb atoms



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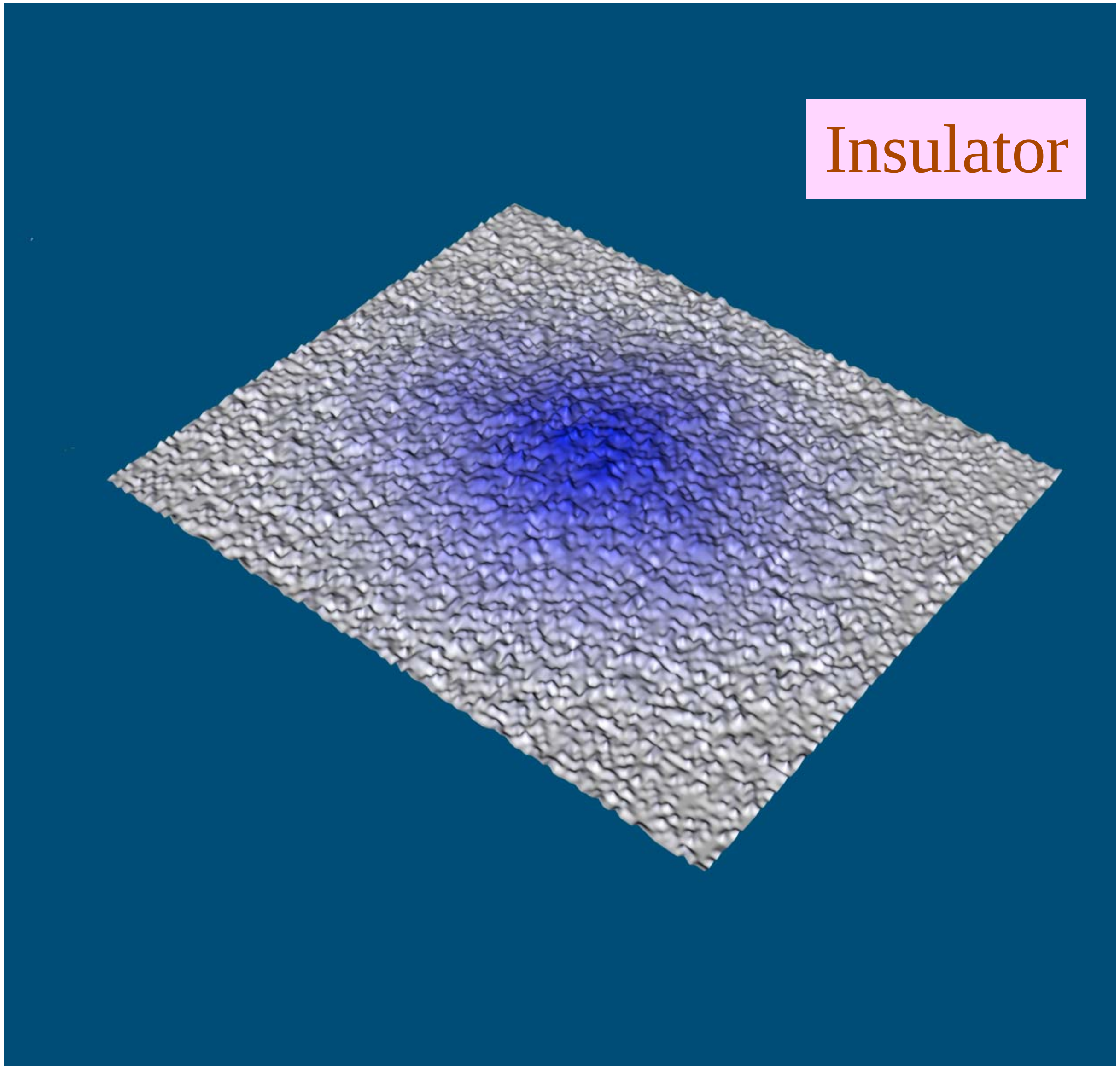

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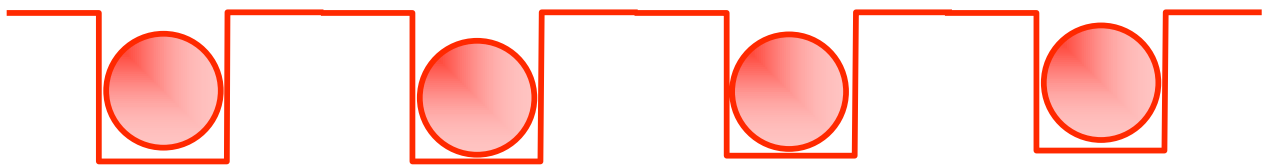
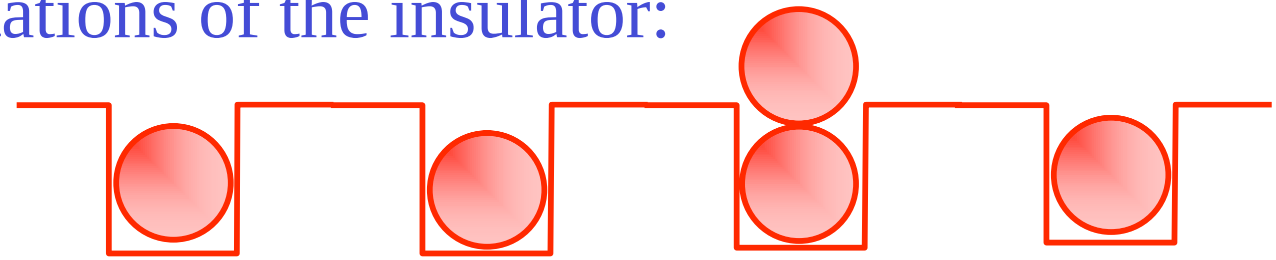
The insulator:



Excitations of the insulator:



Particles ~ $\psi^\dagger$

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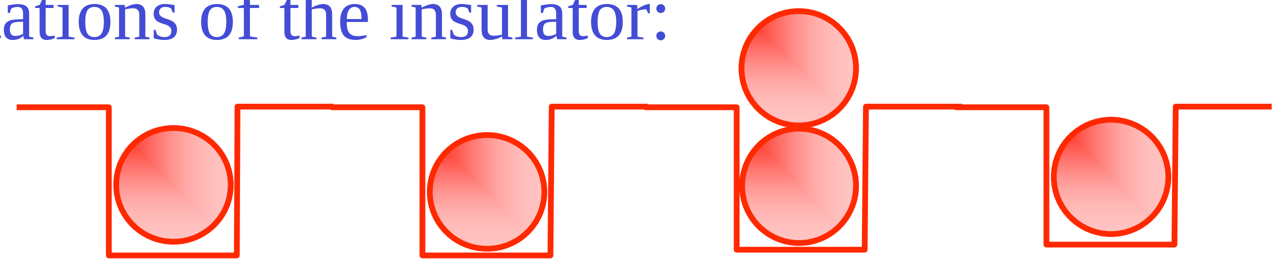
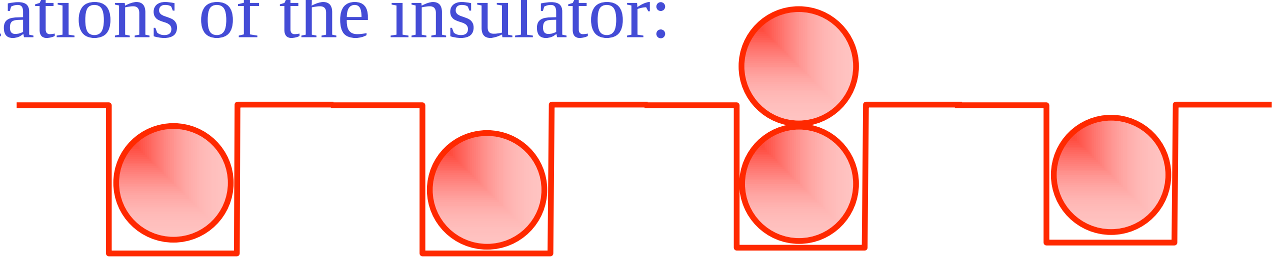


Particles $\sim \psi^{\dagger}$

Holes $\sim \psi$

Excitations of the insulator:



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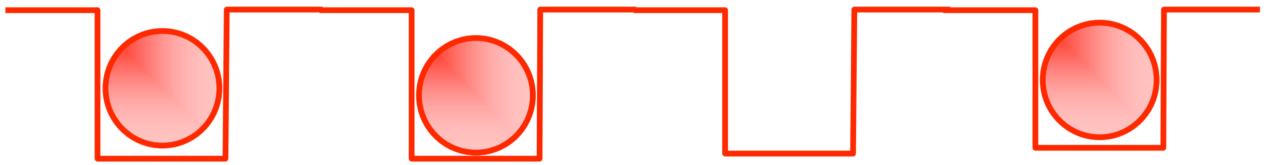
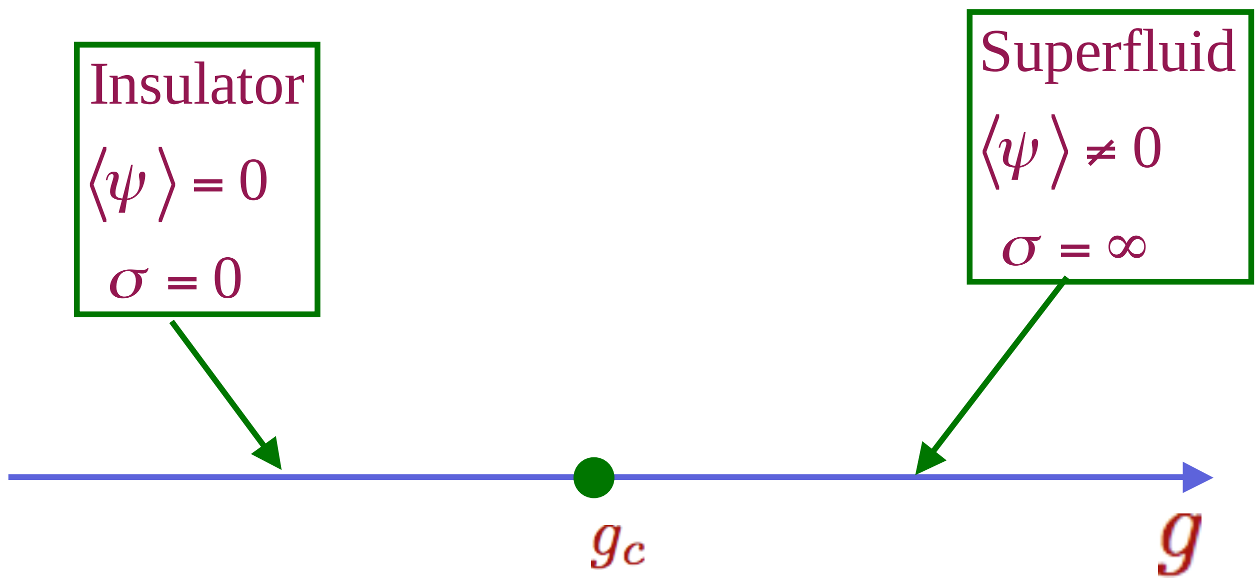


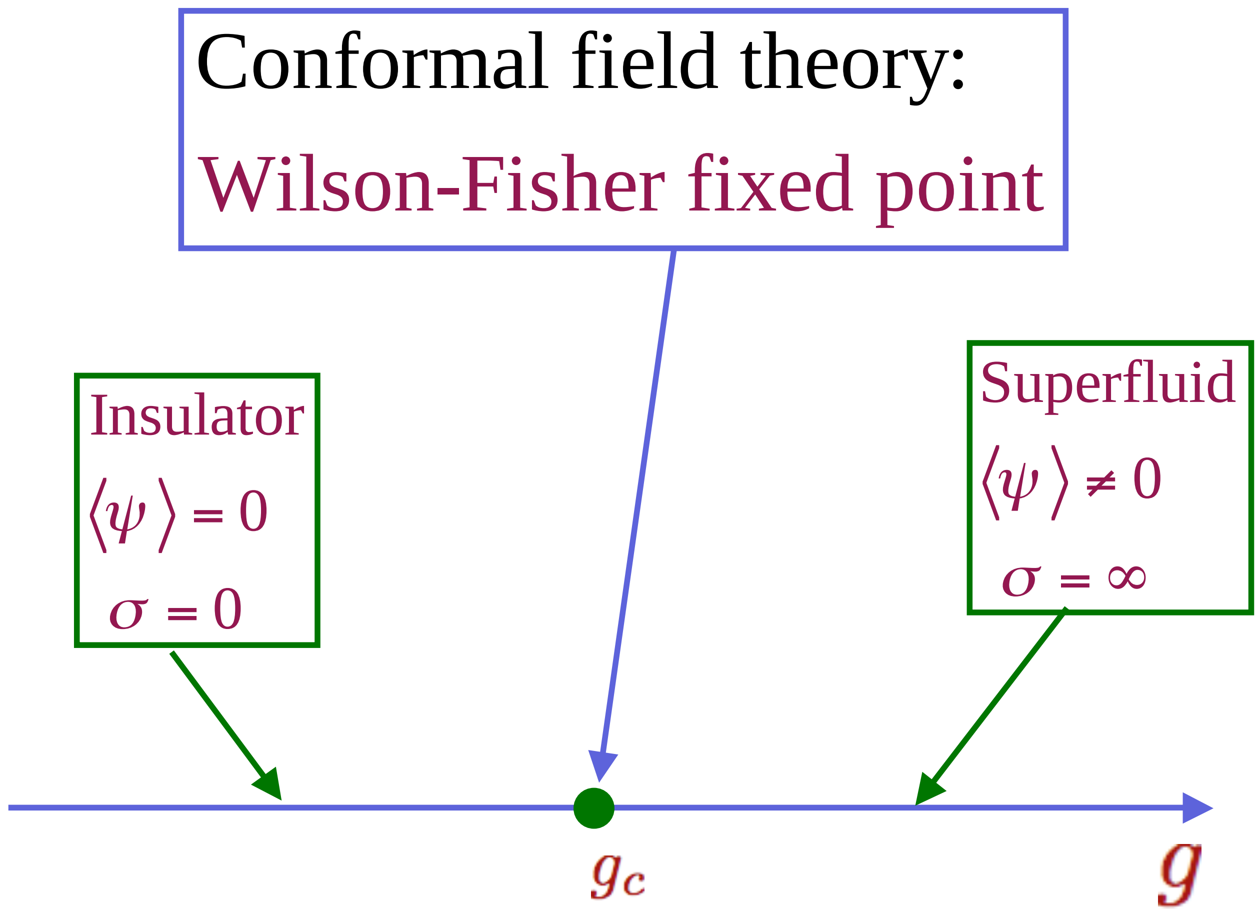
Holes ~ ψ

Density of particles = density of holes →
"relativistic" field theory for ψ

$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

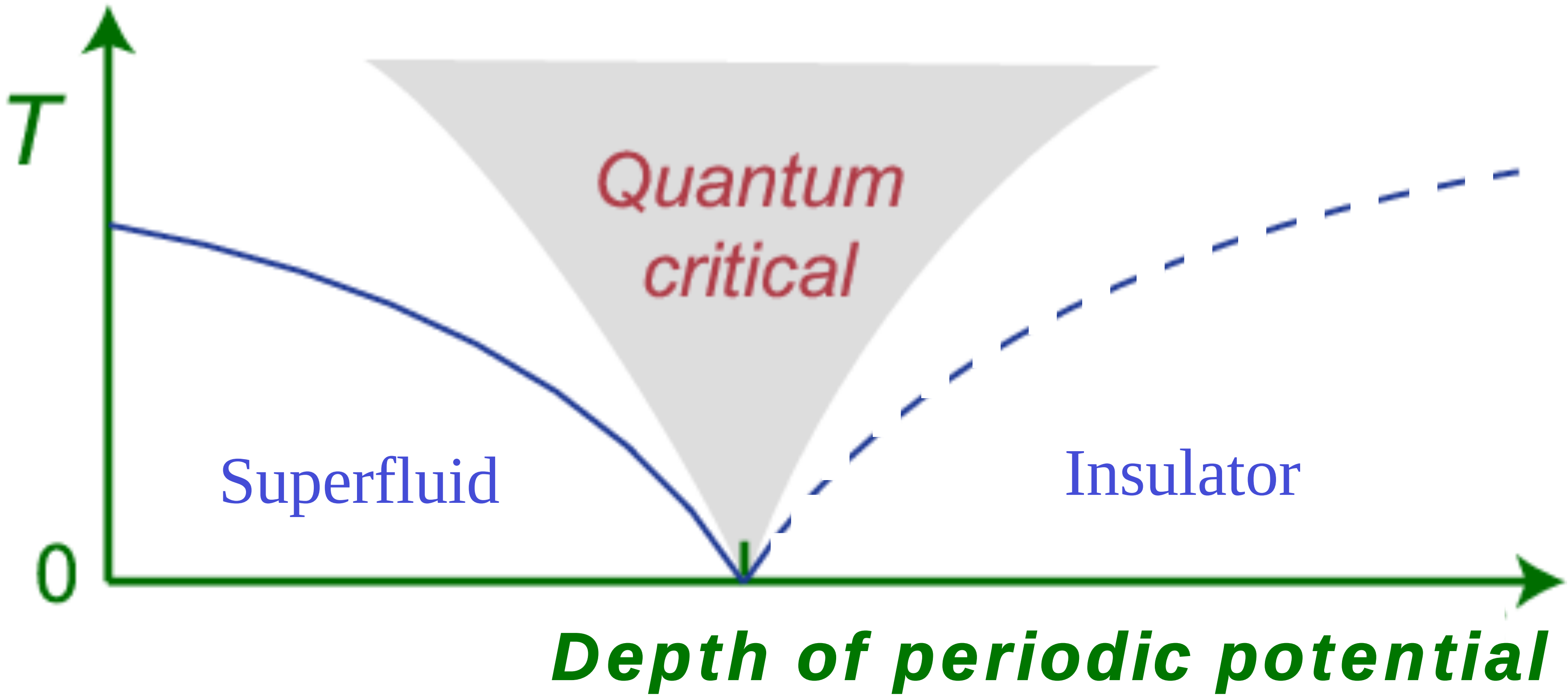
Insulator $\leftrightarrow \langle\psi\rangle = 0$, Superfluid $\leftrightarrow \langle\psi\rangle \neq 0$


Insulator
$\langle \psi \rangle = 0$
$\sigma = 0$
Superfluid
$\langle \psi \rangle \neq 0$
$\sigma = \infty$
g_c
g



Conformal field theory:
Wilson-Fisher fixed point
Insulator
$\langle \psi \rangle = 0$
$\sigma = 0$
Superfluid
$\langle \psi \rangle \neq 0$
$\sigma = \infty$
g_c
g

Non-zero temperature phase diagram



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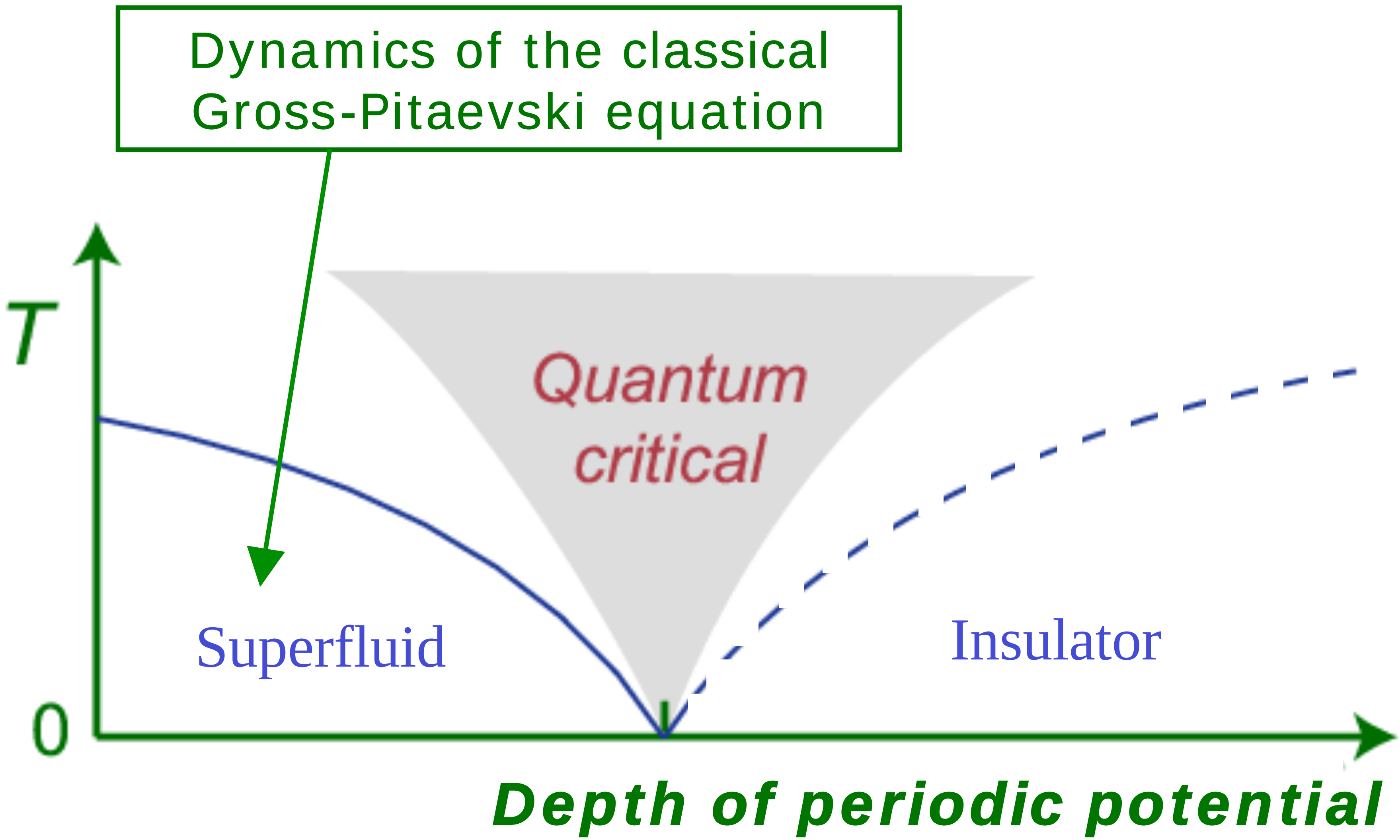

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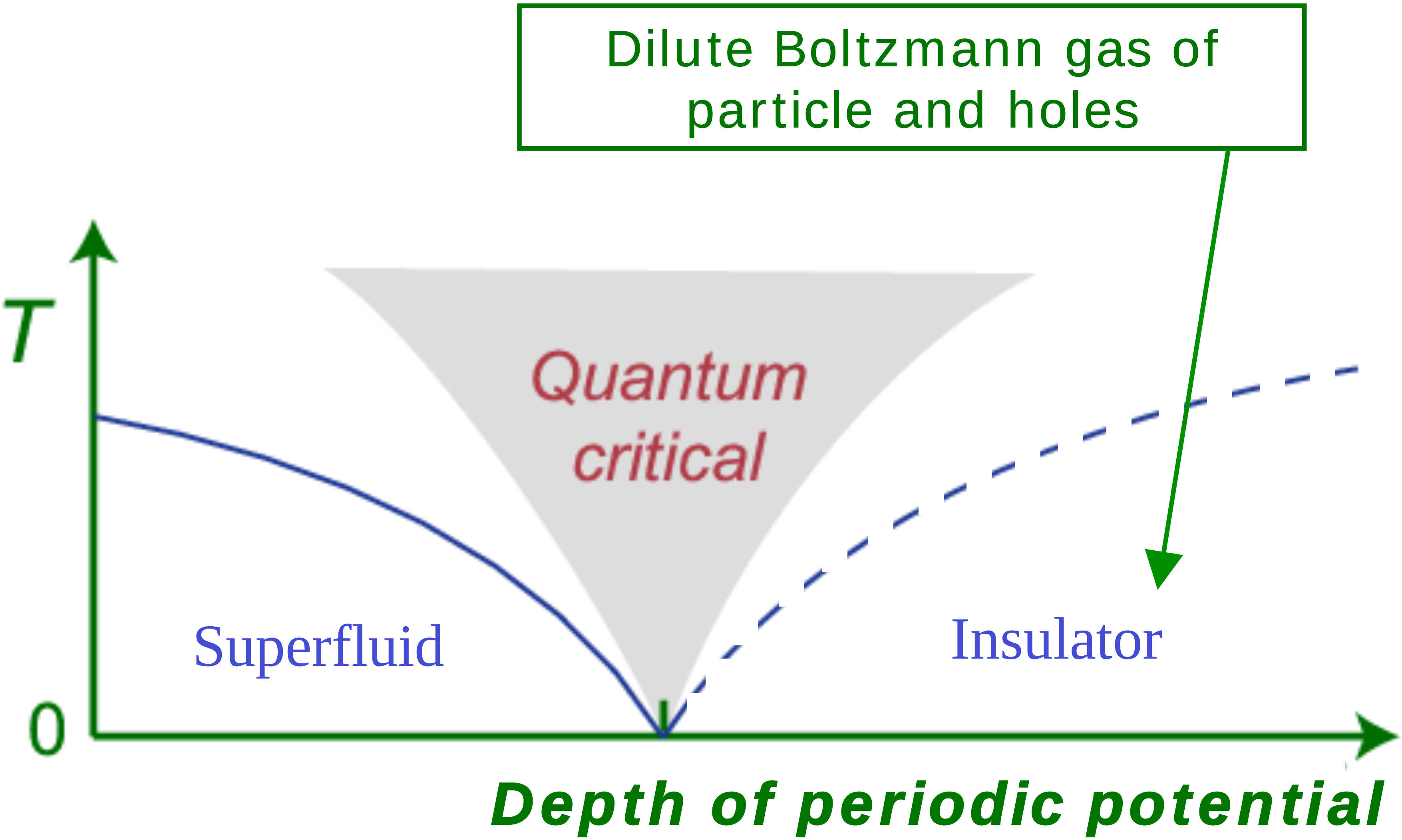

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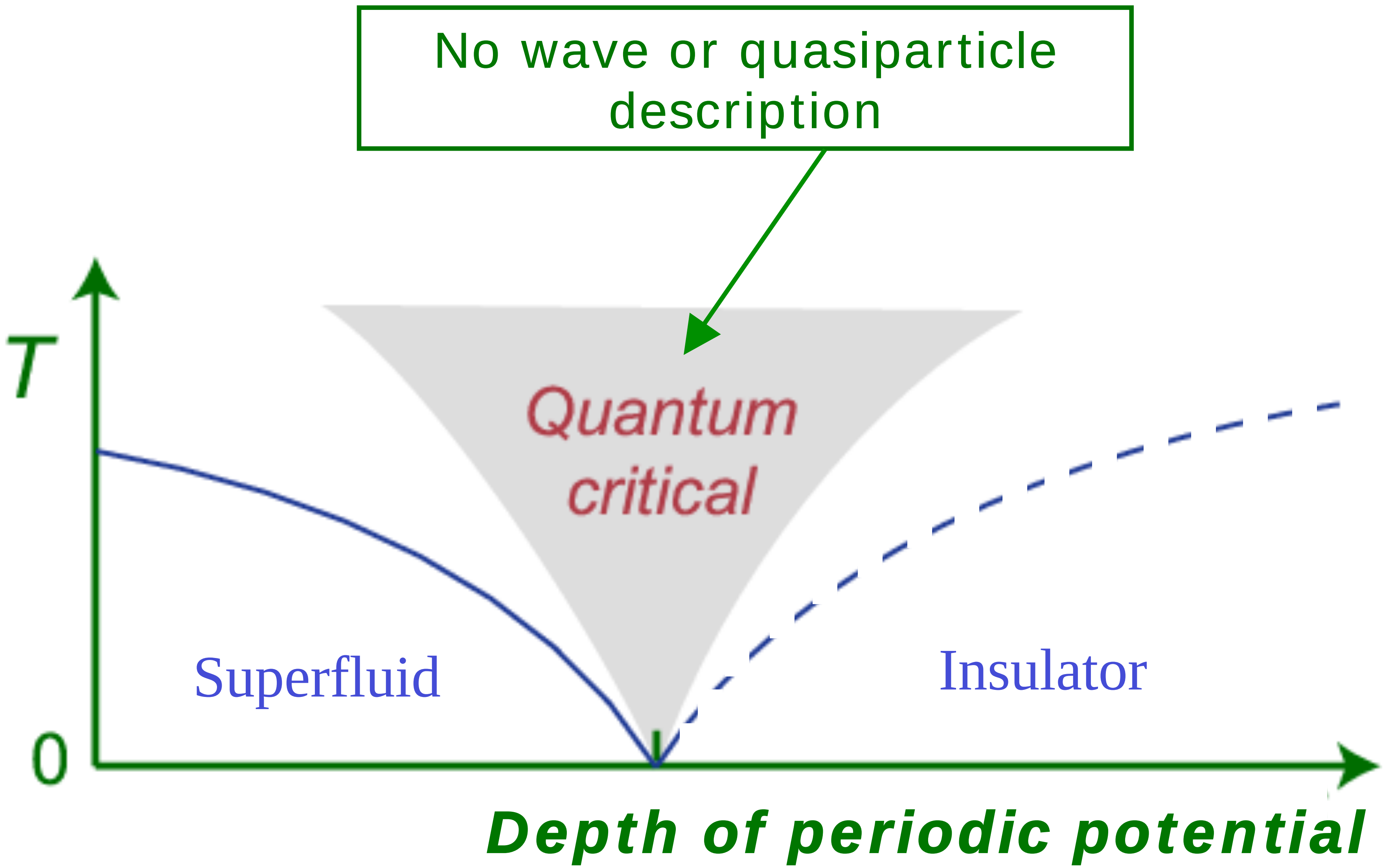


Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}\left(T \to 0\right) = \infty$$

$$\sigma_{\text{Insulator}}\left(T \to 0\right) = 0$$

$$\sigma_{\text{Quantum critical point}}\left(T \to 0\right) \approx \frac{4e^2}{h}$$

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

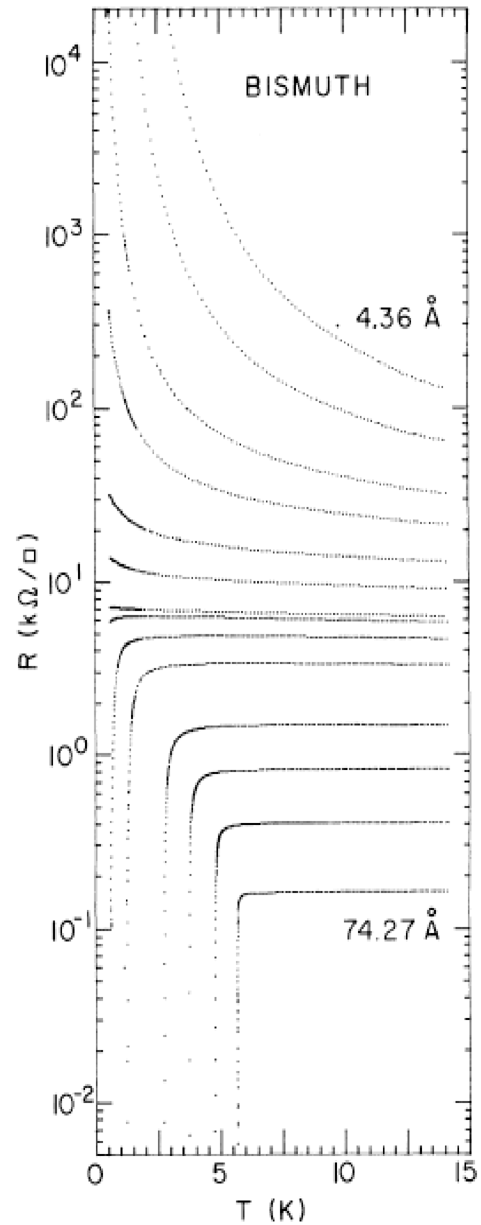
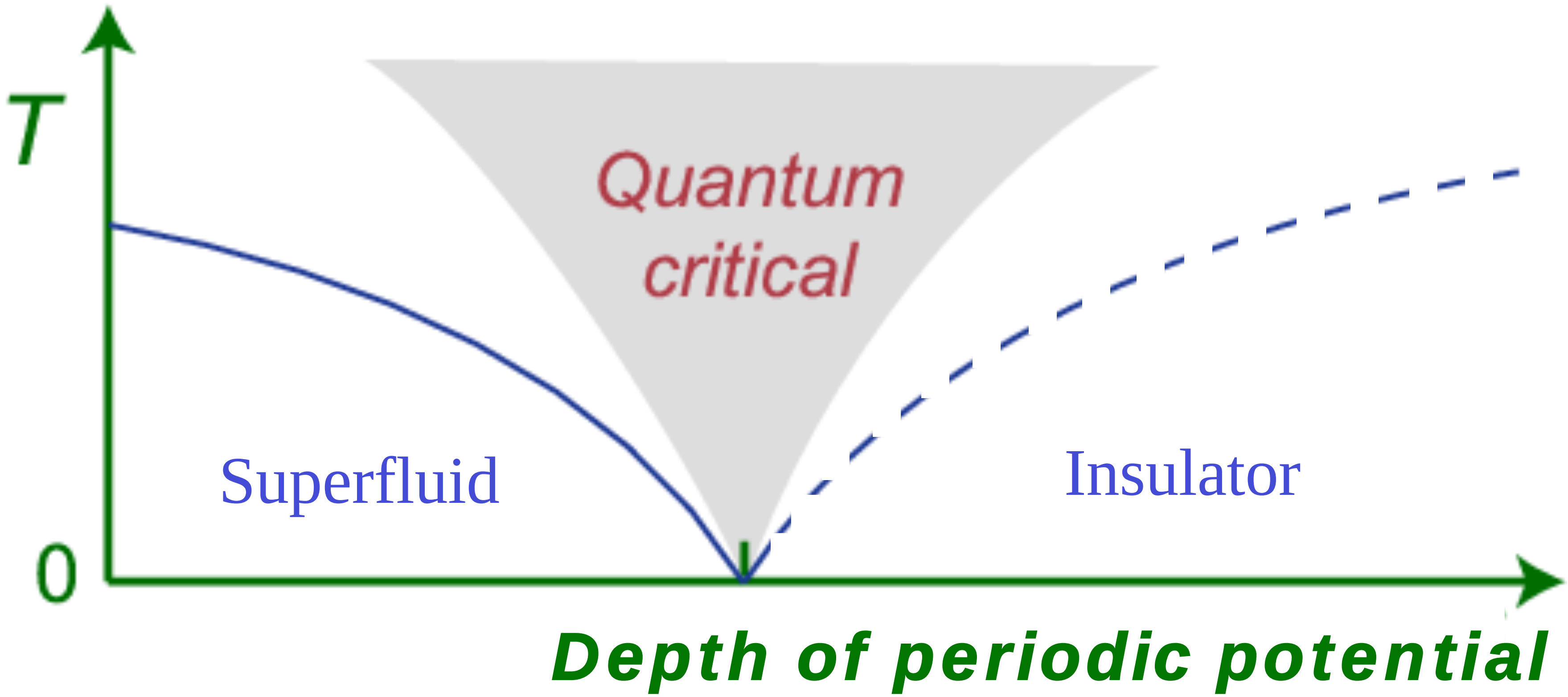


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Non-zero temperature phase diagram



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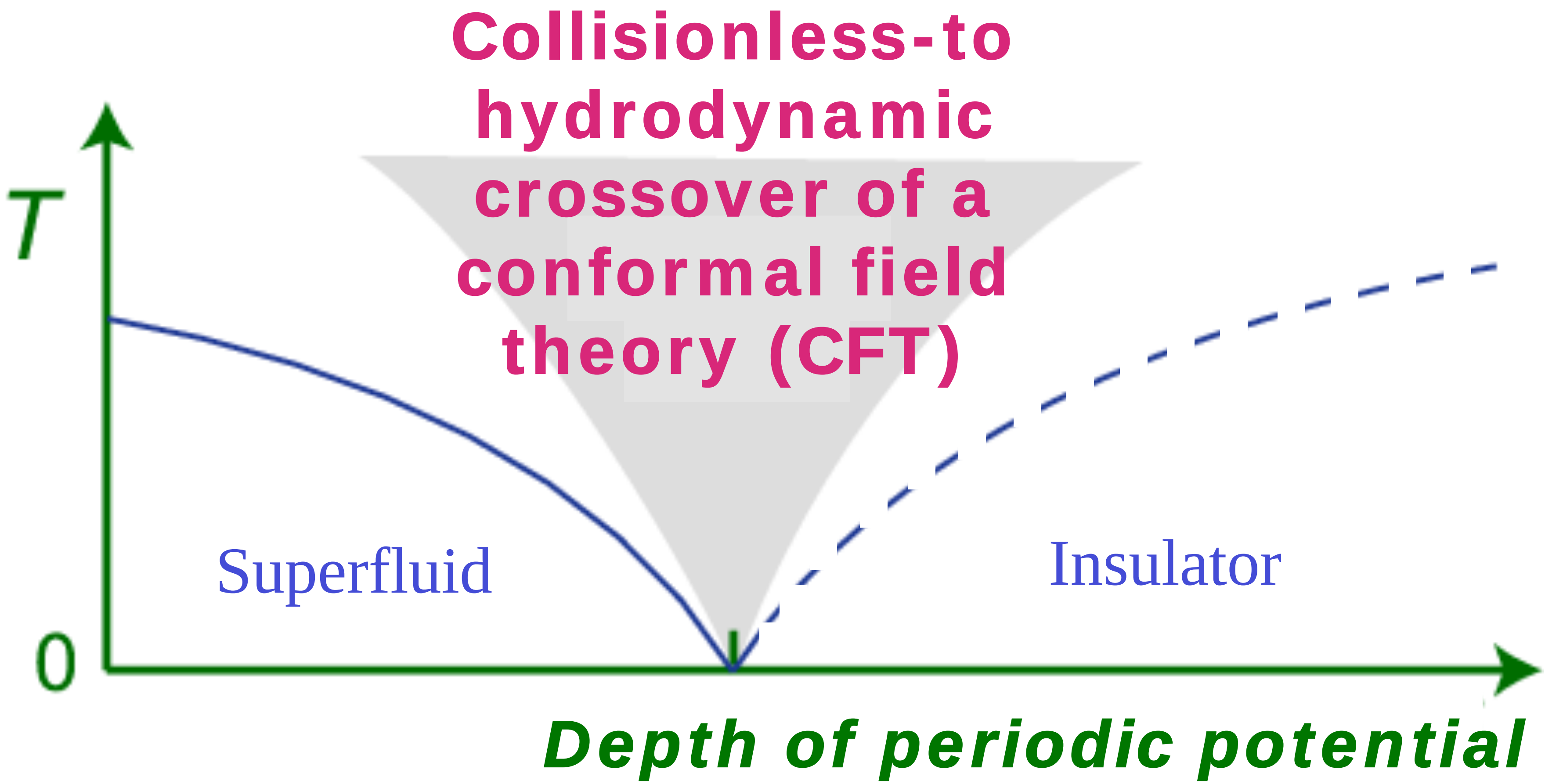


K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

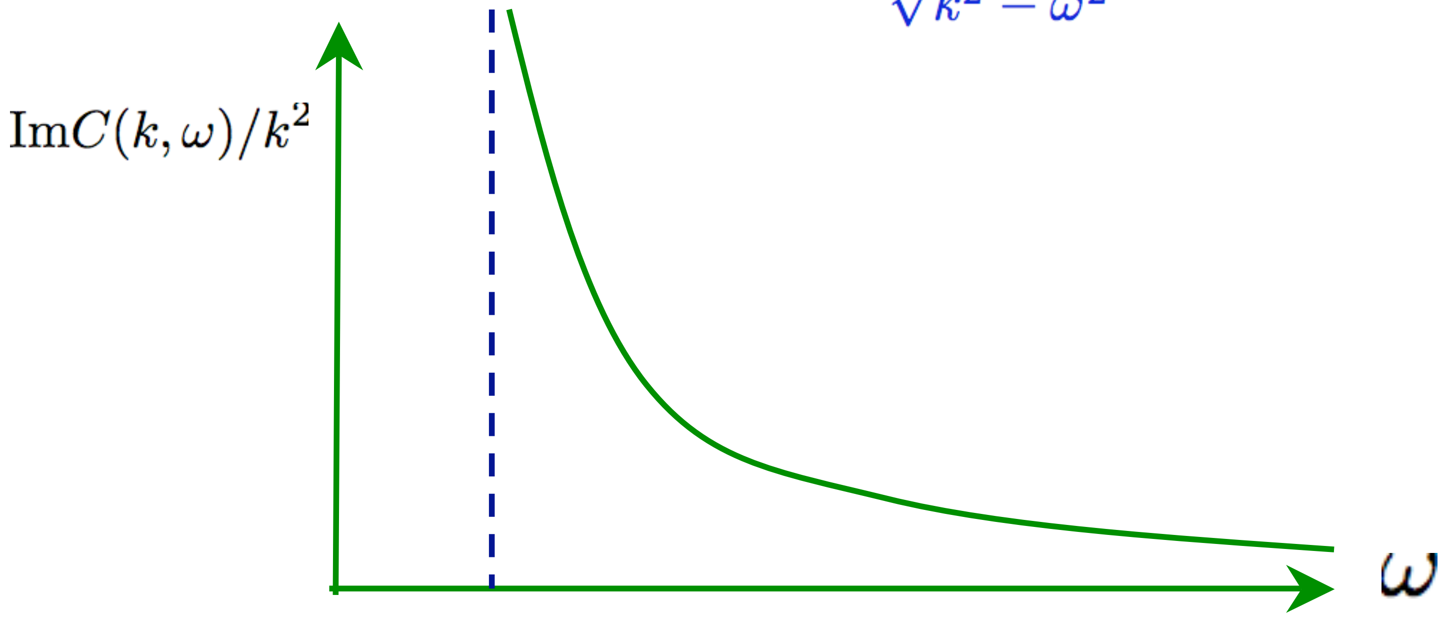
Collisionless-to-hydrodynamic crossover of a CFT in 2+1 dimensions

Consider the retarded density-density correlation function

$$C(k,\omega) = \langle \rho(k,\omega)\rho(-k,-\omega)\rangle_{\text{ret}}$$

The characteristic collision time for excitations of the CFT is $\hbar/k_B T$. So, for $|\omega - k| \gg T$ we have the collisionless conformal behavior

$$C(k,\omega) = K \frac{k^2}{\sqrt{k^2-\omega^2}}$$



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while for $\omega, k \ll T$, we have the hydrodynamic behavior

$$C(k,\omega) = \chi \frac{D_c k^2}{-i\omega + D_c k^2}.$$

So the high frequency conductivity $\sigma(\omega \gg T) = K$, and the hydrodynamic conductivity $\sigma(\omega \ll T) = D_c\chi$, and in general $K \neq D_c\chi$.

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Hydrodynamics of a conformal field theory (CFT)

The scattering cross-section of the thermal excitations is universal and so transport co-efficients are universally determined by $k_B T$

Charge diffusion constant $\quad D_c = \Theta \dfrac{c^2}{k_B T}$

Conductivity $\quad \sigma_Q = D_c \chi = \overline{\Theta} \dfrac{4e^2}{h}$

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Exact solutions of CFTs in 1+1 dimensions

For all 1+1 dimensional CFTs, exact correlators of any conserved current at $T > 0$ can be obtained for all ω, k, and T by the conformal mapping of the plane to the cylinder. After analytic continuation in frequency to the retarded correlator we have, in general

$$C(k, \omega) = K \frac{k^2}{k^2 - \omega^2}$$

where K is a pure number - the central charge of a Kac-Moody algebra.

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There is no collisionless-hydrodynamic crossover for CFTs in 1+1 dimensions. Holomorphic factorization implies absence of collisions between left and right movers.

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The AdS/CFT correspondence (Maldacena, Polyakov) relates the hydrodynamics of CFTs to the quantum gravity theory of the horizon of a black hole in Anti-de Sitter space.

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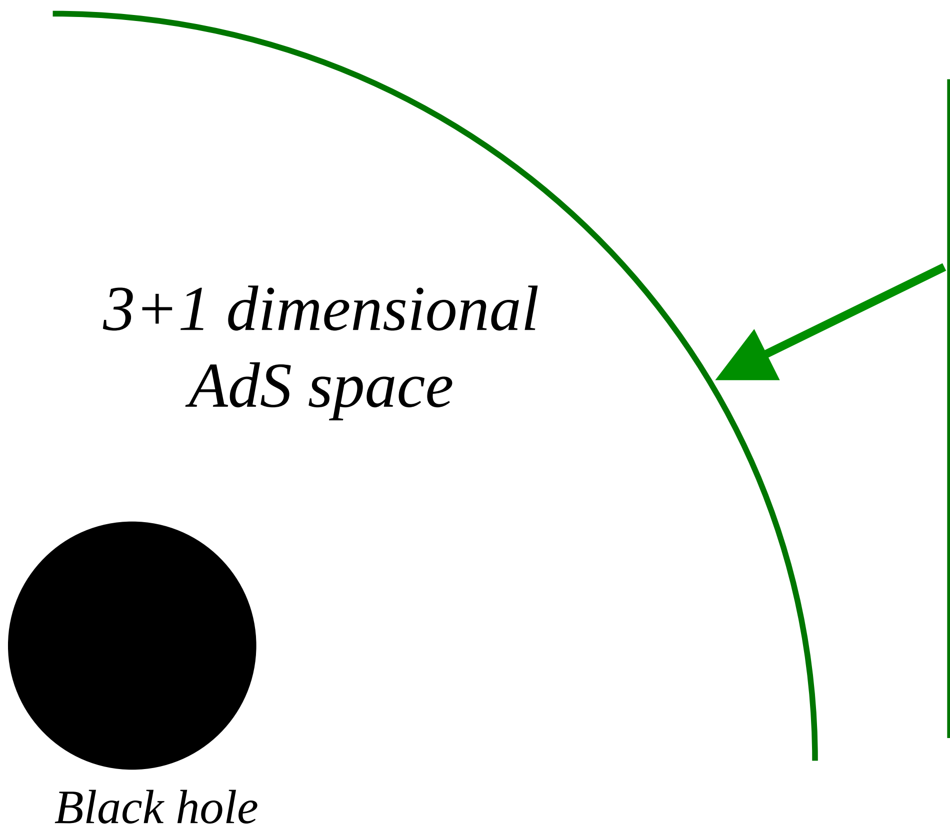
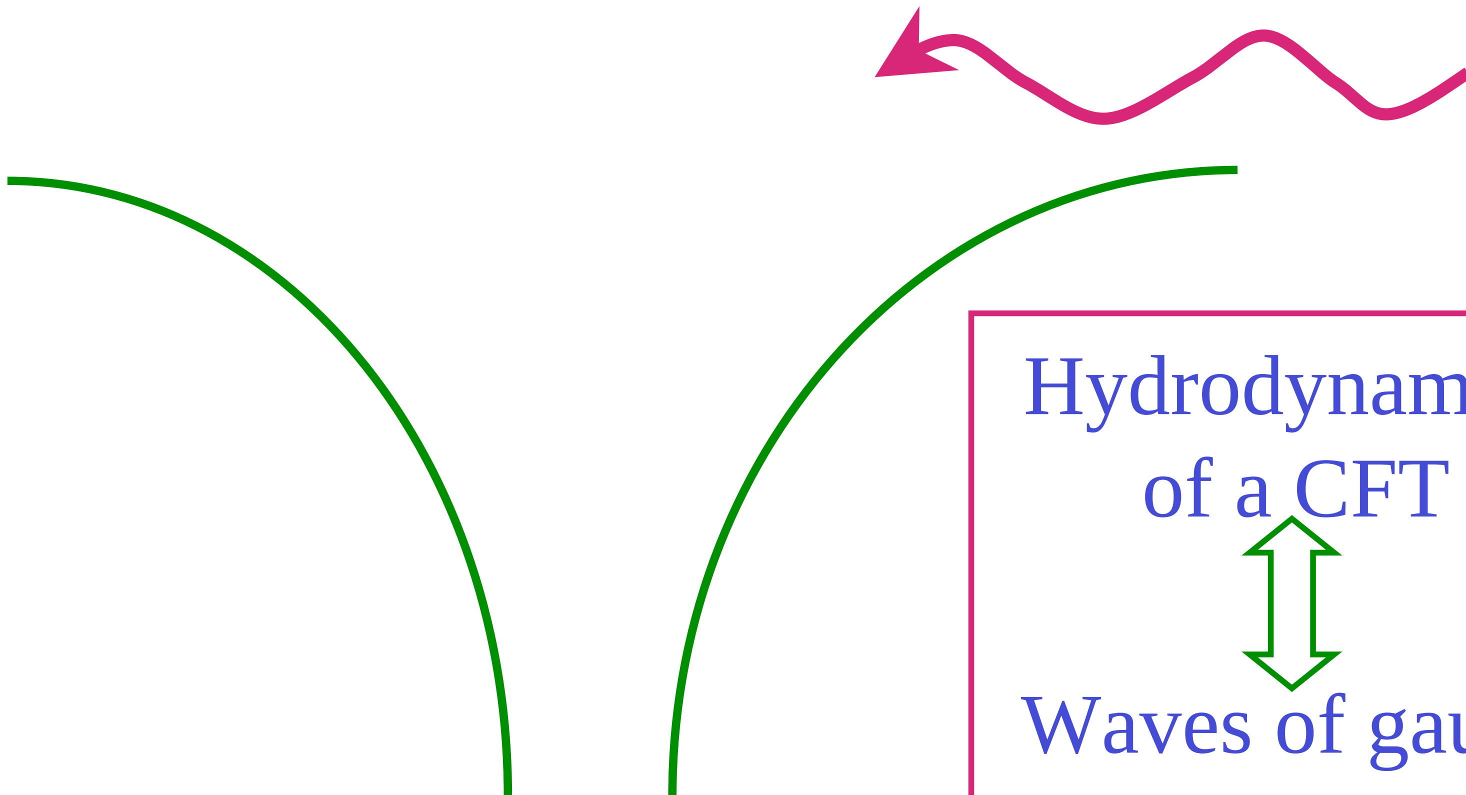


Holographic representation of black hole physics in a 2+1 dimensional CFT at a temperature equal to the Hawking temperature of the black hole.

Hydrodynamics of a conformal field theory (CFT)



Hydrodynamics of a CFT

⇕

Waves of gauge fields in a curved background

Hydrodynamics of a conformal field theory (CFT)

For the (unique) CFT with a SU(N) gauge field and 16 supercharges, we know the exact diffusion constant associated with a global SO(8) symmetry:

Spin diffusion constant $$D_c = \frac{3}{4\pi} \frac{c^2}{k_B T}$$

Spin conductivity $$\sigma_Q = \frac{N^{3/2}}{3\sqrt{2}\pi}$$

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Collisionless-to-hydrodynamic crossover of solvable SYM_3

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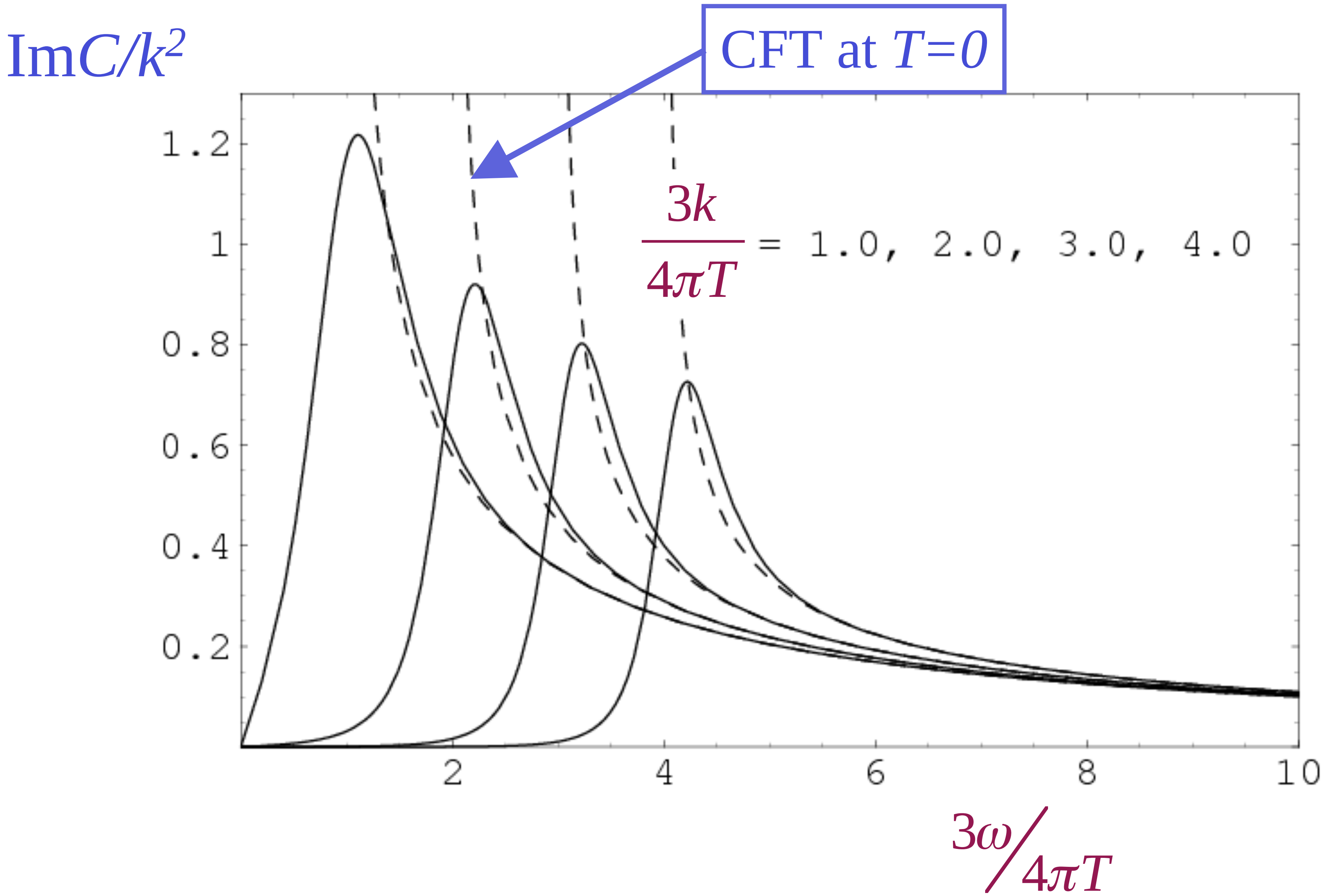

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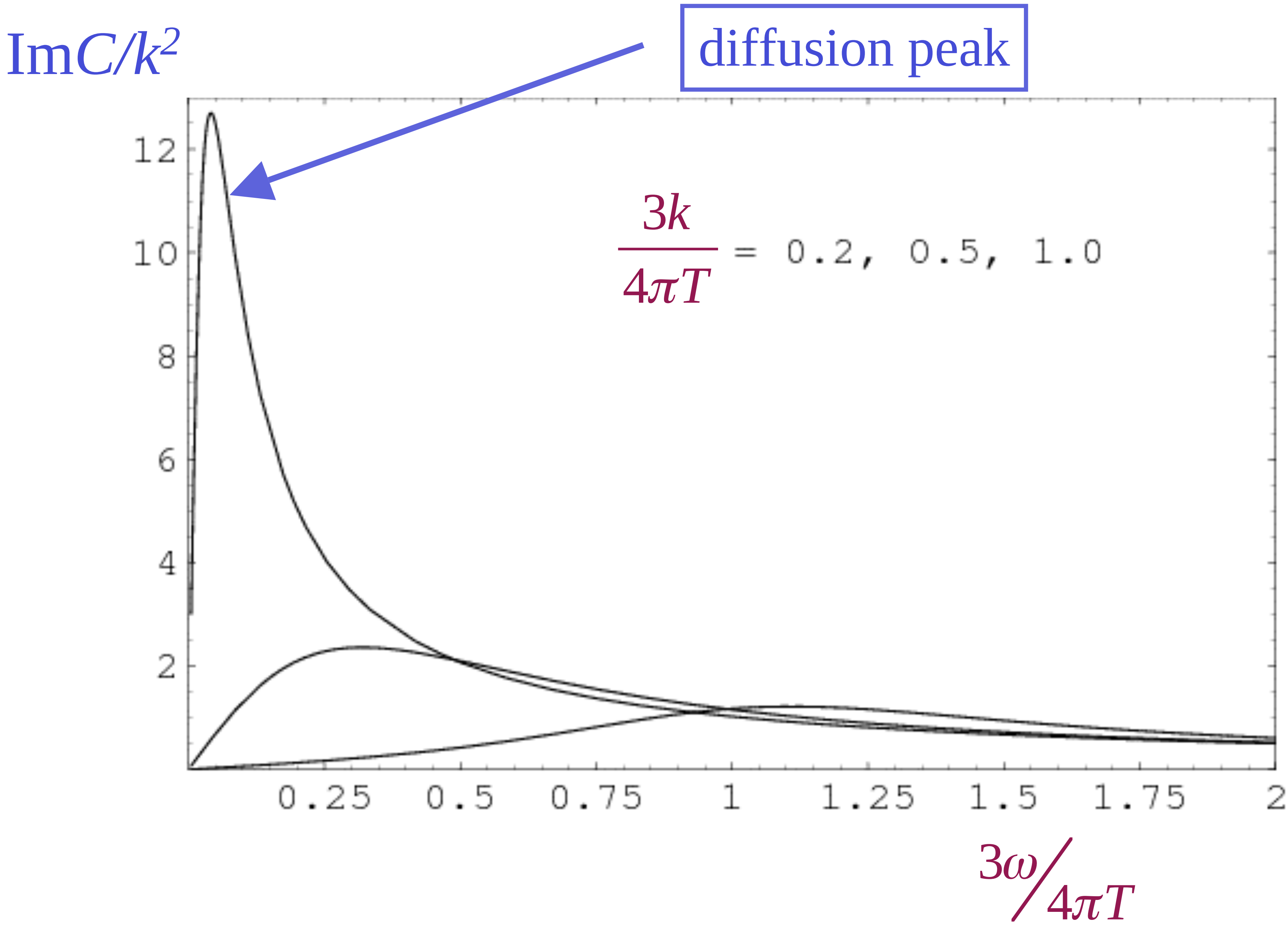

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For experimental applications, we must move away from the ideal CFT

e.g. $$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 \left| \vec{\nabla} \psi \right|^2 - g |\psi|^2 + \frac{u}{2} |\psi|^4 \right]$$

For experimental applications, we must move away from the ideal CFT

- A chemical potential μ

e.g. $$\mathcal{S} = \int d^2r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 \left|\vec{\nabla}\psi\right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

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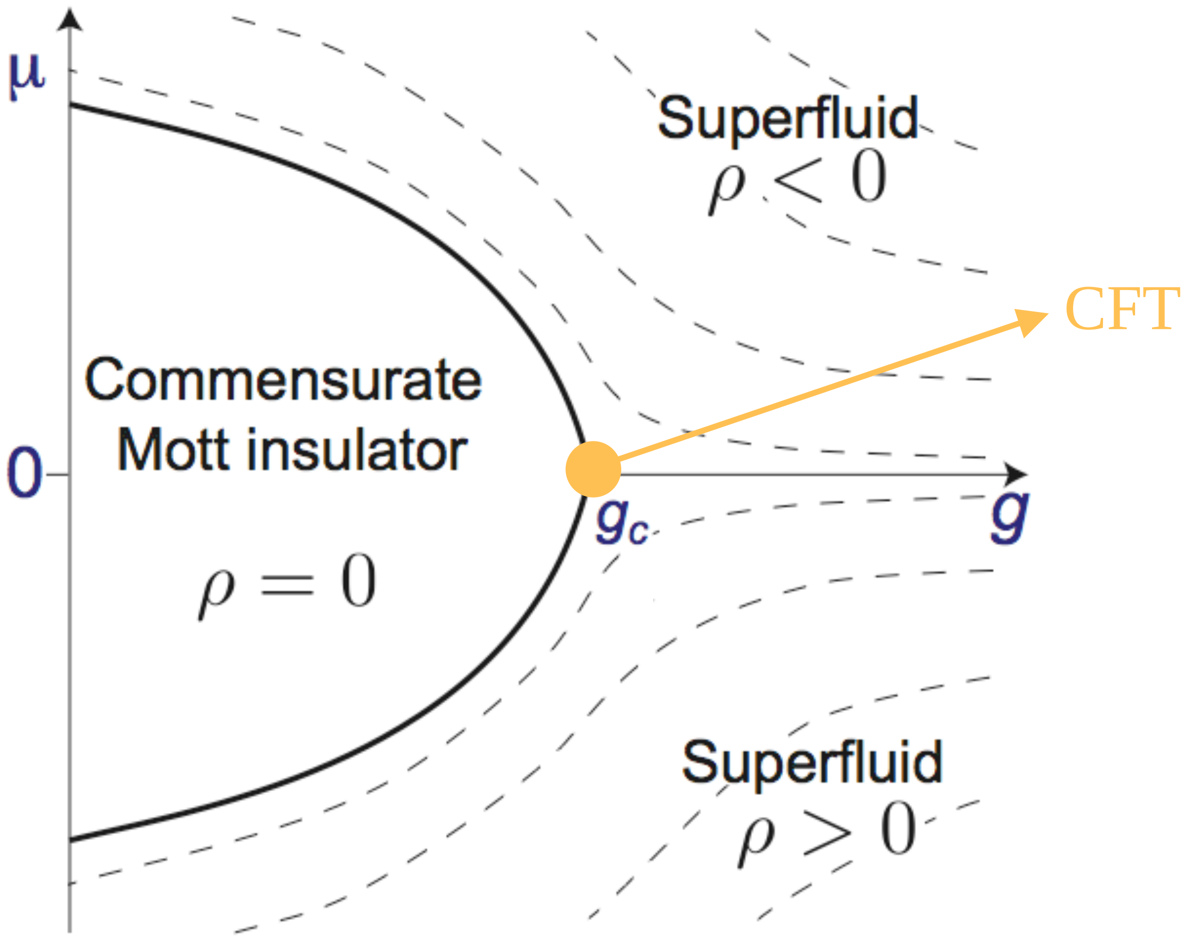

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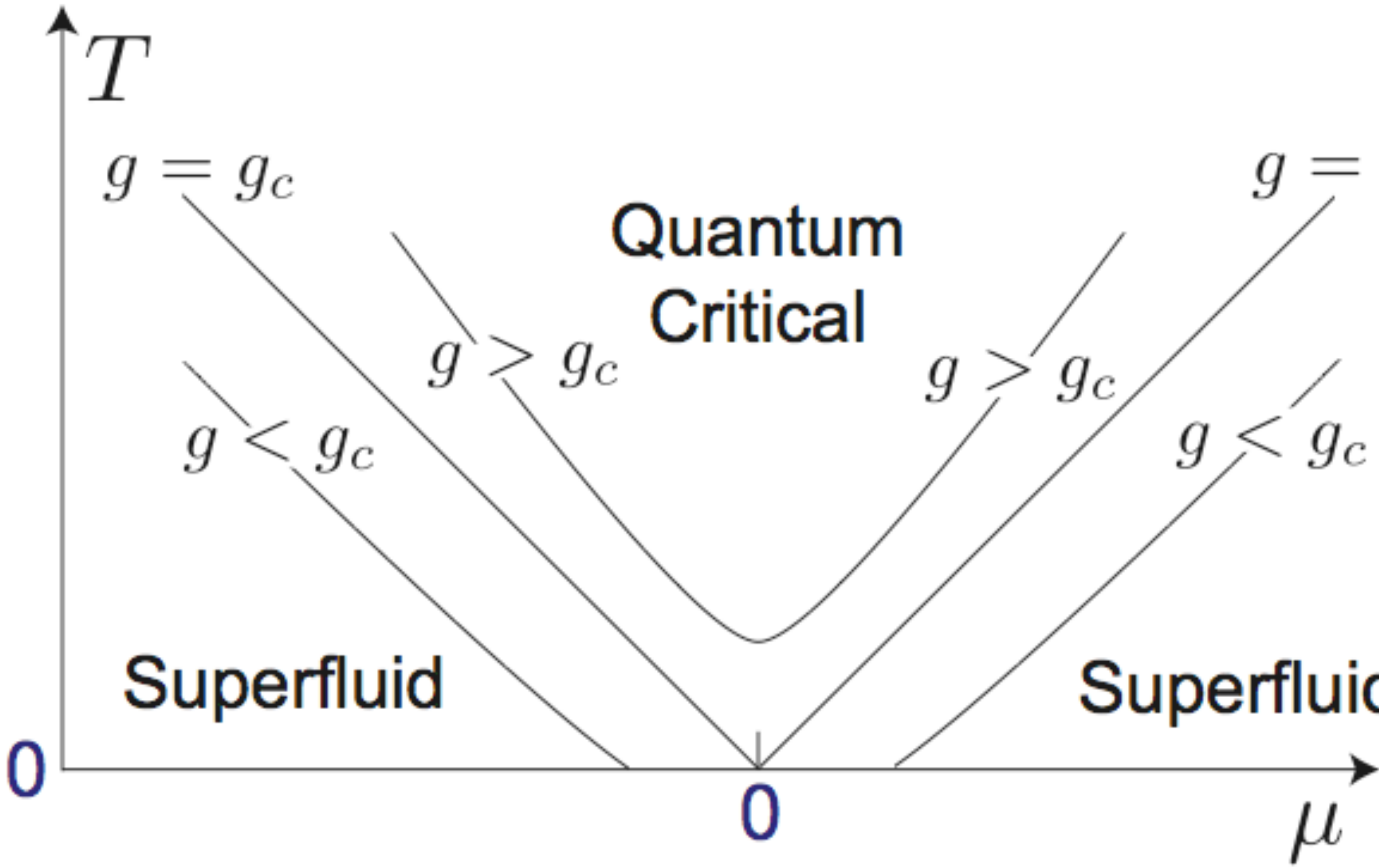

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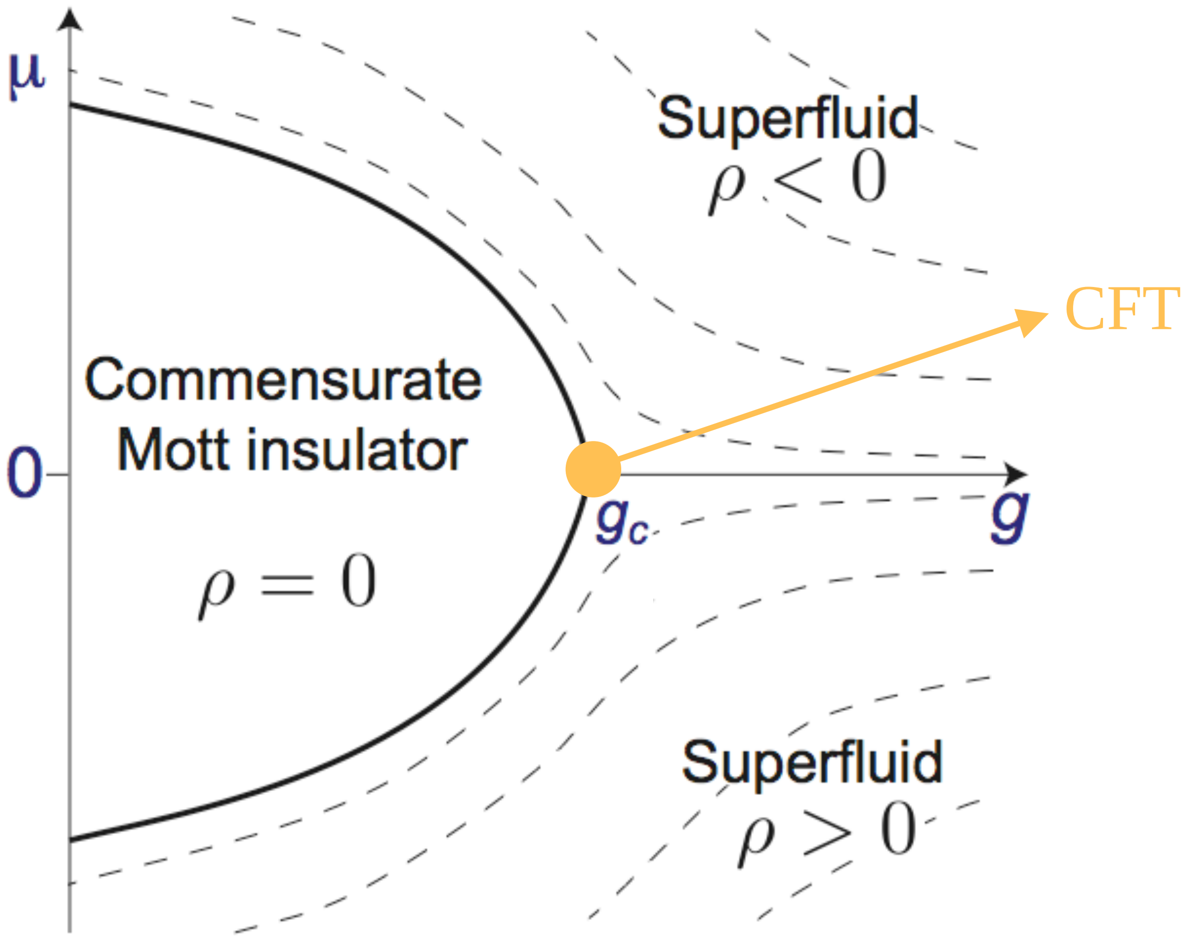

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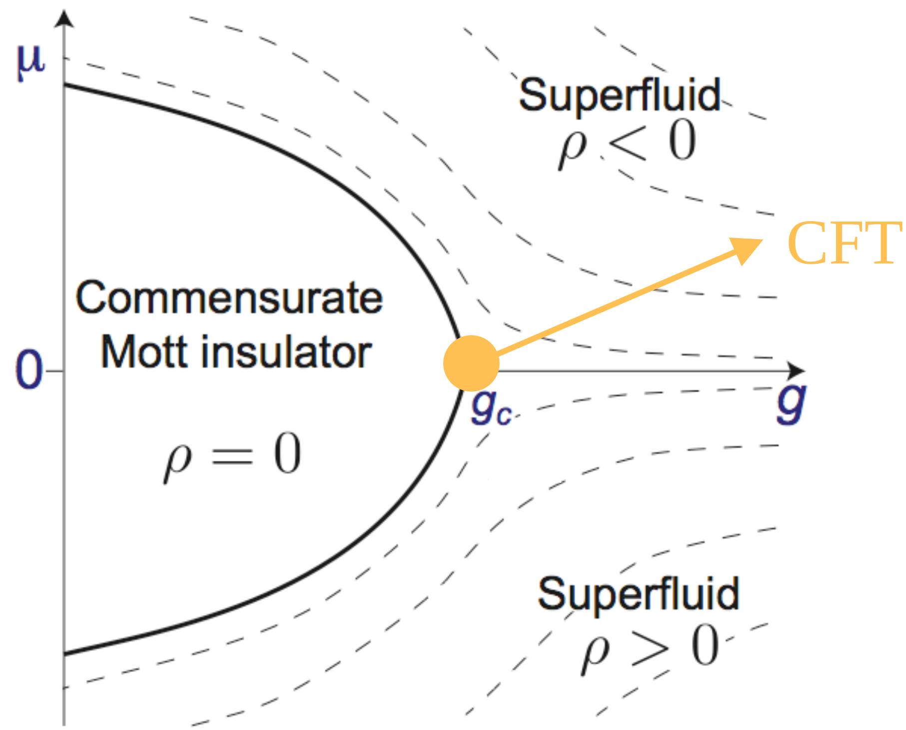
- A chemical potential μ
- A magnetic field B



e.g.

$$\mathcal{S} = \int d^2r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 \left|(\vec{\nabla} - i\vec{A})\psi\right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B$$

In the hydrodynamic regime, $\hbar\omega \ll k_B T$, we can use classical principles involving relaxation to local equilibrium to understand these perturbations.
The variables entering the hydrodynamic theory are

- the external magnetic field $F^{\mu\nu}$,

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix},$$

- $T^{\mu\nu}$, the stress energy tensor,
- J^{μ}, the current,
- ρ, the local number density,
- ε, the local energy density,
- P, the local pressure,
- u^{μ}, the local velocity, and
- σ_Q, a universal conductivity, which is the **single transport co-efficient**.

The dependence of ε, P, σ_Q on T and v follows from simple scaling arguments

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$
\begin{aligned}
\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu
\end{aligned}
$$

Conservation laws/equations of motion

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\
T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu
\end{aligned}
$$

Constitutive relations which follow from Lorentz transformation to moving frame

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[\left(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda \right) + \mu \frac{\partial_\mu T}{T} \right]
\end{aligned}
$$

Single dissipative term allowed by requirement of positive entropy production. There is only one independent transport co-efficient

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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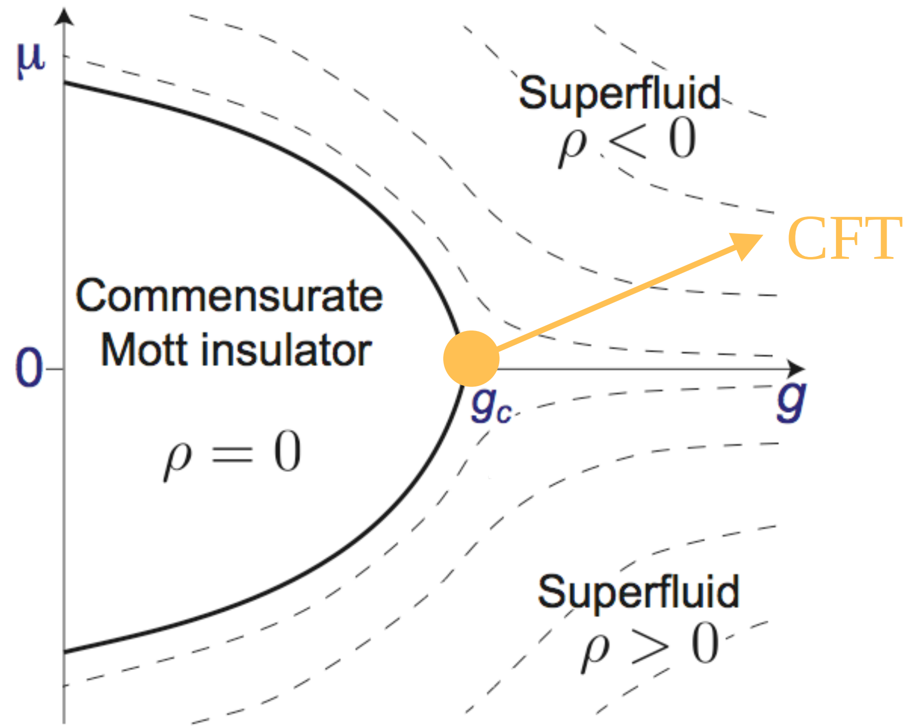
- A magnetic field B



e.g.

$$\mathcal{S} = \int d^2r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 \left| (\vec{\nabla} - i\vec{A})\psi \right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

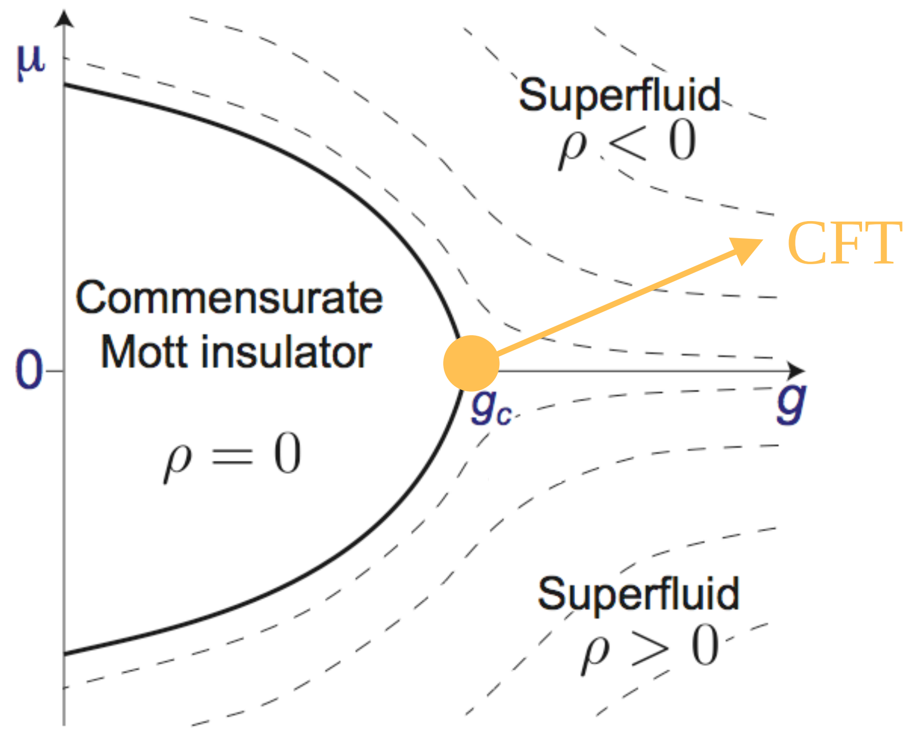
$$\nabla \times \vec{A} = B$$

For experimental applications, we must move away from the ideal CFT

- A chemical potential μ

- A magnetic field B

- An impurity scattering rate $1/\tau_{\text{imp}}$ (its T dependence follows from scaling arguments)



e.g.

$$\mathcal{S} = \int d^2r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 \left| (\vec{\nabla} - i\vec{A})\psi \right|^2 - g|\psi|^2 + V(r)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B \quad , \quad \overline{V(r)} = 0 \quad , \quad \overline{V(r)V(r')} = V_{\text{imp}}^2 \delta^2(r - r')$$

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$
\begin{aligned}
\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu + \frac{1}{\tau_{\rm imp}} \left(\delta^\mu_\nu + u^\mu u_\nu\right) T^{\nu\gamma} u_\gamma \\
T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[\left(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda \right) + \mu \frac{\partial_\mu T}{T} \right]
\end{aligned}
$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

From these relations, we obtained results for the transport co-efficients, expressed in terms of a "cyclotron" frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Longitudinal conductivity

$$\sigma_{xx} = \sigma_Q \left[\frac{(\omega + i/\tau_{\rm imp})(\omega + i\gamma + i\omega_c^2/\gamma + i/\tau_{\rm imp})}{(\omega + i\gamma + i/\tau_{\rm imp})^2 - \omega_c^2} \right] .$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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$$= \sigma_Q + \frac{4e^2\rho^2 v^2}{(\varepsilon + P)} \frac{1}{(-i\omega + 1/\tau_{\text{imp}})} \quad \text{as } B \to 0$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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Hall conductivity

$$\begin{aligned} \sigma_{xy} &= -\frac{2e\rho c}{B}\left[\frac{\gamma^2 + \omega_c^2 - 2i\gamma\omega + 2\gamma/\tau_{\text{imp}}}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2}\right] \\ &= B\left[\sigma_Q \frac{4e\rho v^2}{(\varepsilon + P)(1/\tau_{\text{imp}} - i\omega)} + \frac{8e^3\rho^3 v^4}{(\varepsilon + P)^2(1/\tau_{\text{imp}} - i\omega)^2}\right] \\ &\qquad \text{as } B \to 0 \end{aligned}$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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Thermal conductivity

$$\begin{aligned}
\kappa_{xx} &= \sigma_Q \left(\frac{k_B^2 T}{4e^2}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)^2 \left[\frac{(\omega_c^2/\gamma)(\omega_c^2/\gamma + 1/\tau_{\rm imp})}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right] \\
&= \frac{1}{\sigma_Q} k_B^2 T \left(\frac{c(\varepsilon + P)}{k_B T B}\right)^2 \left[\frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\rm imp})}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right]
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S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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$$= \frac{1}{\sigma_Q} k_B^2 T \left(\frac{c(\varepsilon + P)}{k_B T B}\right)^2 \left[\frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2}\right]$$

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S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Nernst signal

$$e_N = \left(\frac{k_B}{2e}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)\left[\frac{\omega_c/\tau_{\rm imp}}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right]$$

$$\frac{k_B}{2e} = 43.086\mu\text{V/K}$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

Outline

Transport near strongly interacting quantum critical points

1. The superfluid-insulator transition in the boson Hubbard model:
   Hydrodynamic-collisionless crossover of a CFT

2. Exact solutions of CFTs in 1+1 dimensions
   No hydrodynamics

3. Exact solution of a CFT in 2+1 dimensions - Yang-Mills theory
   with N=8 supersymmetry:
   Black holes in AdS_4

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   chemical potential and impurities:
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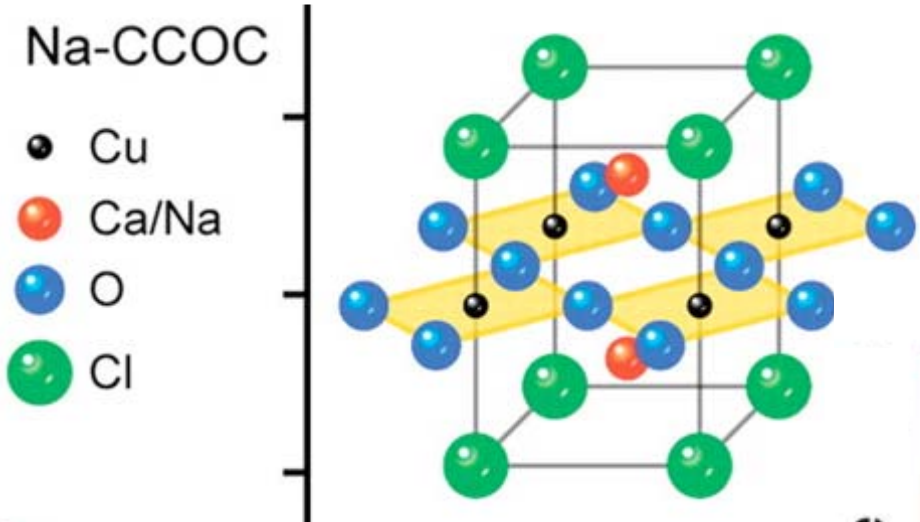
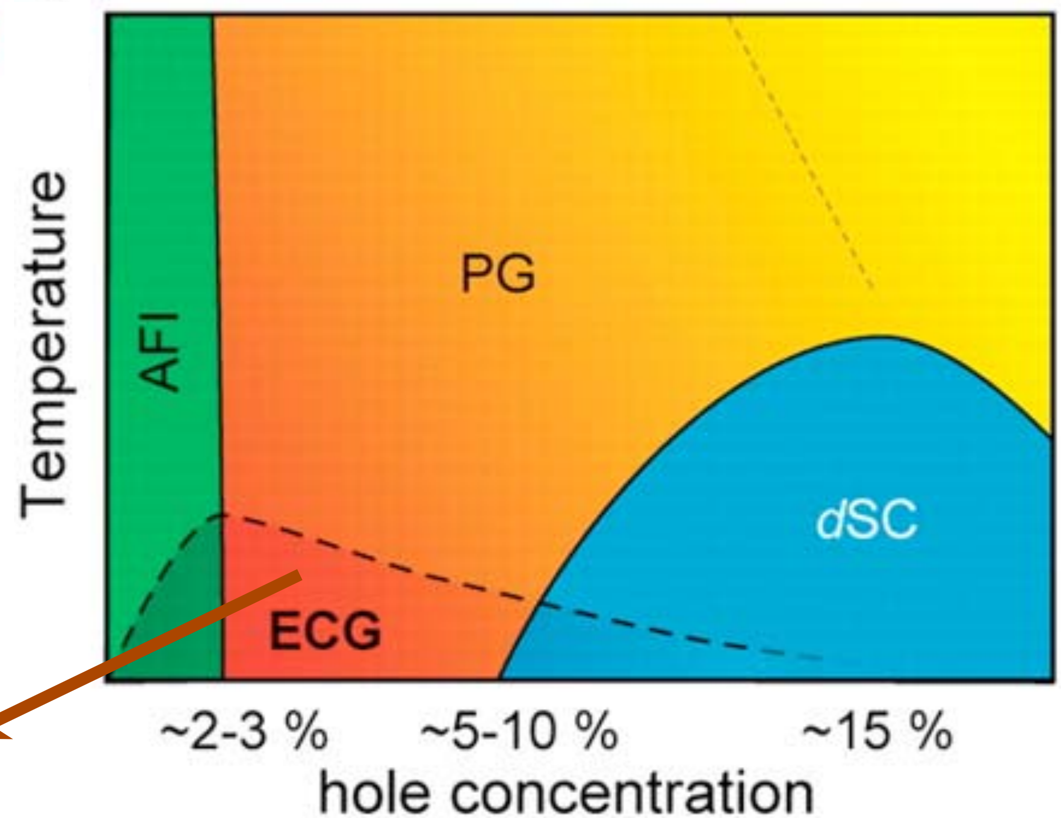
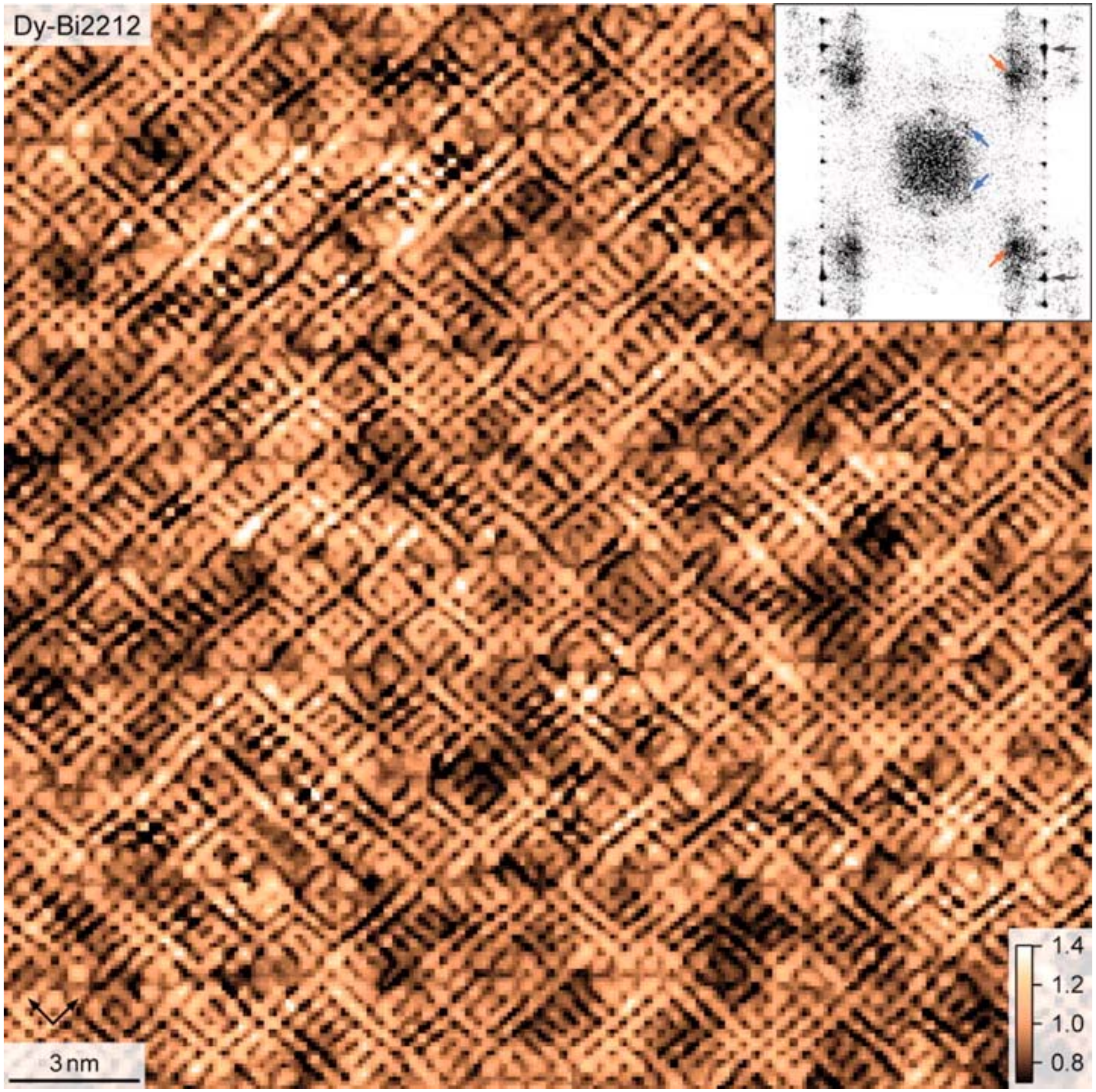
Temperature-doping phase diagram of the cuprate superconductors

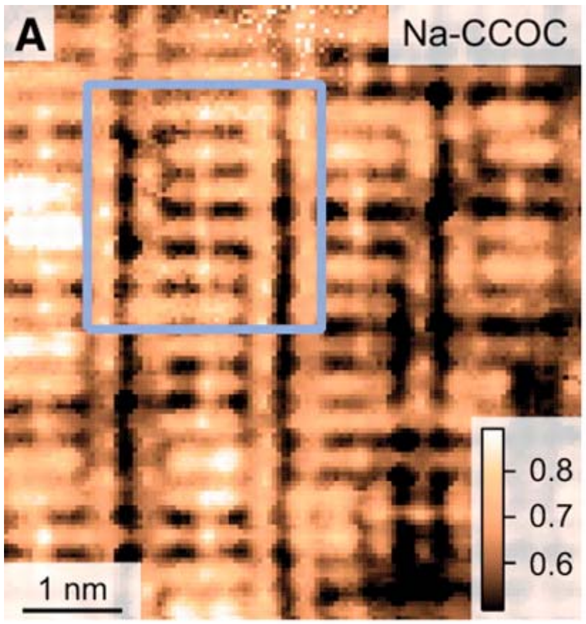




STM in zero field



Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007)



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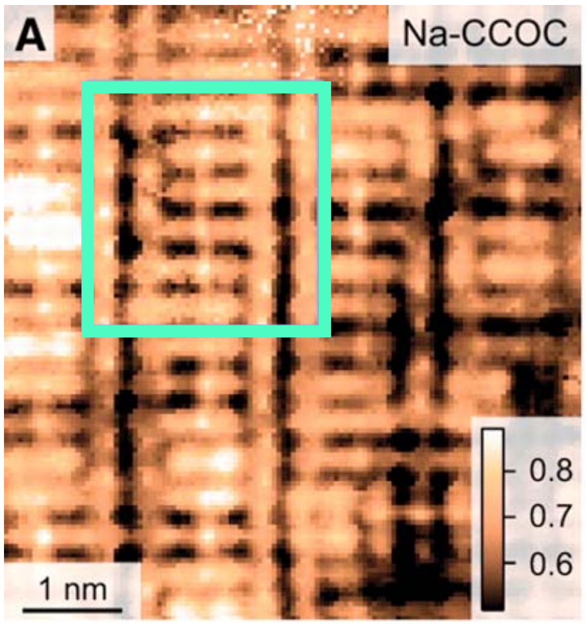

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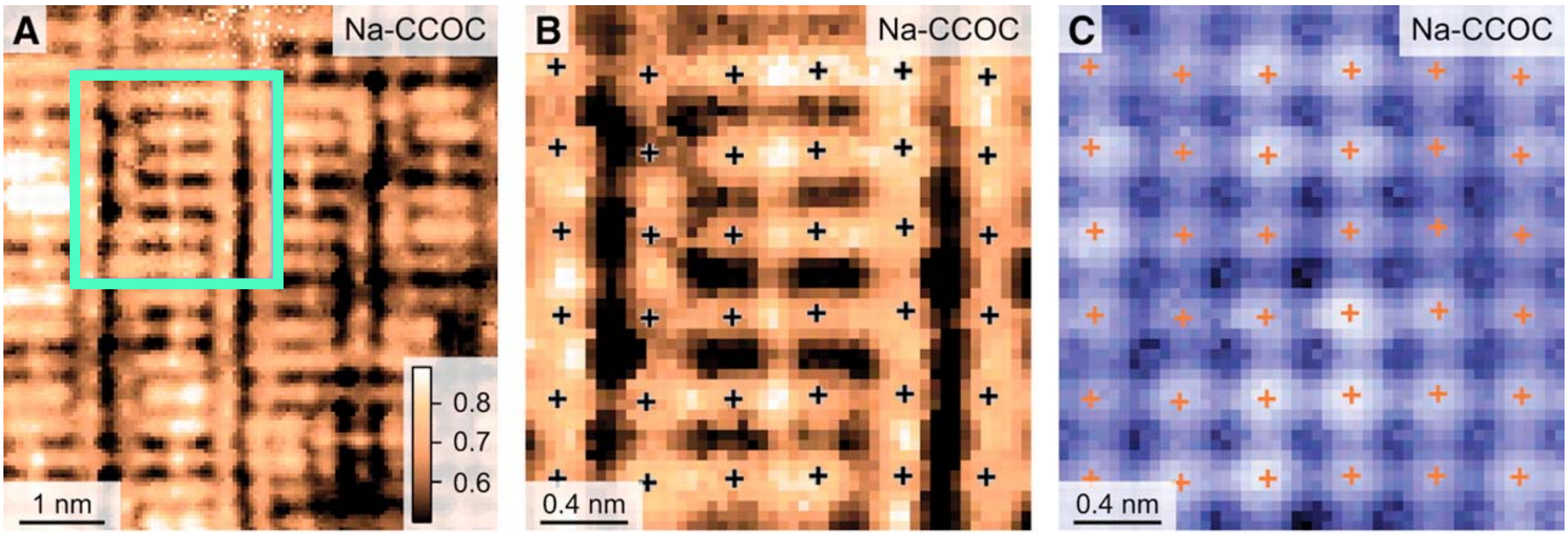

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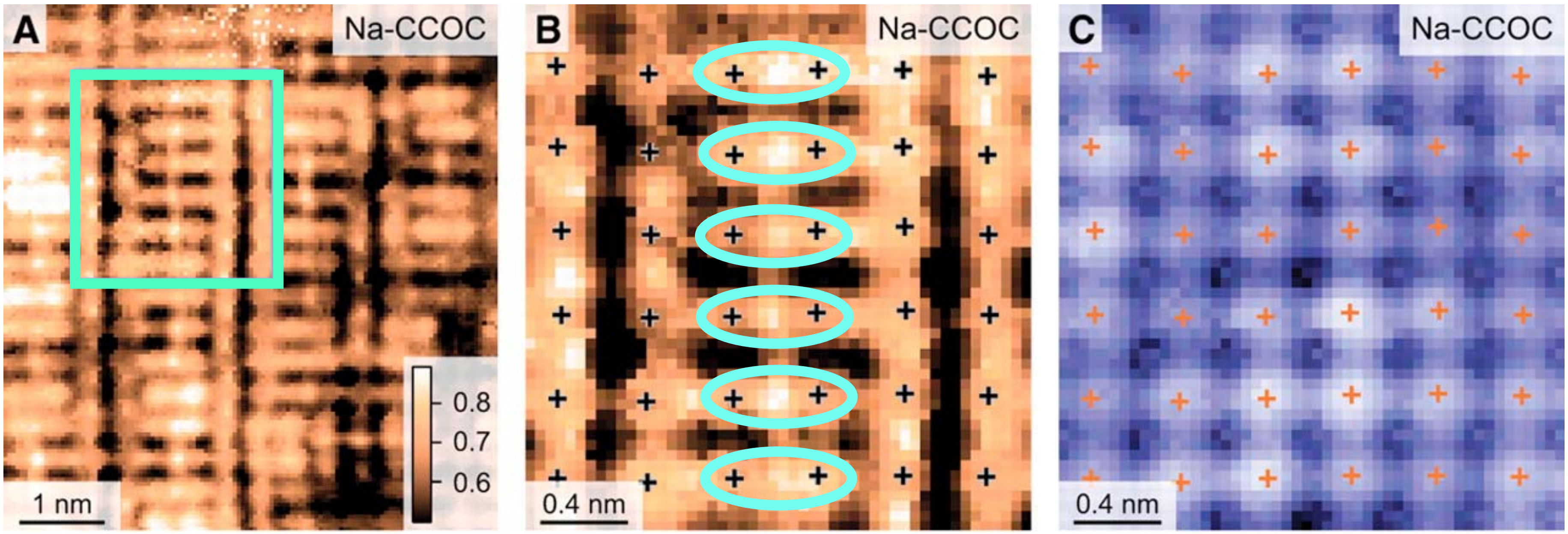

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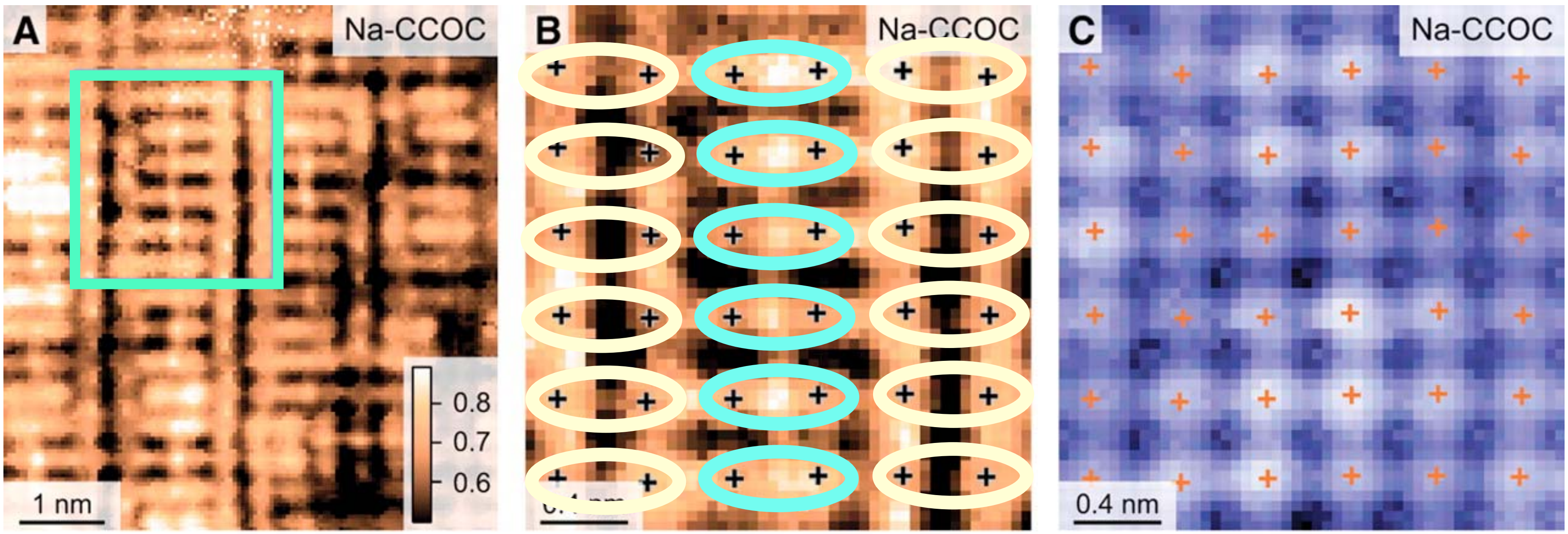

“Glassy” Valence Bond Solid (VBS) ?

Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007)

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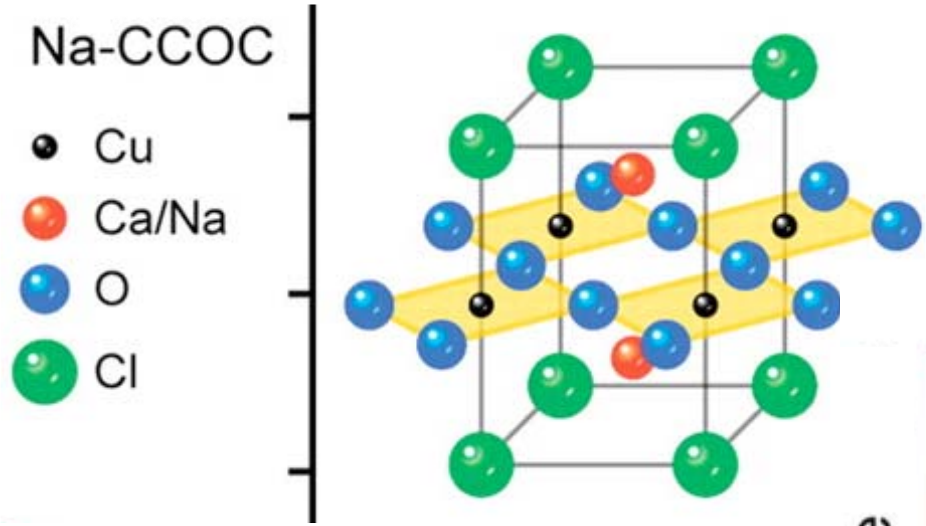

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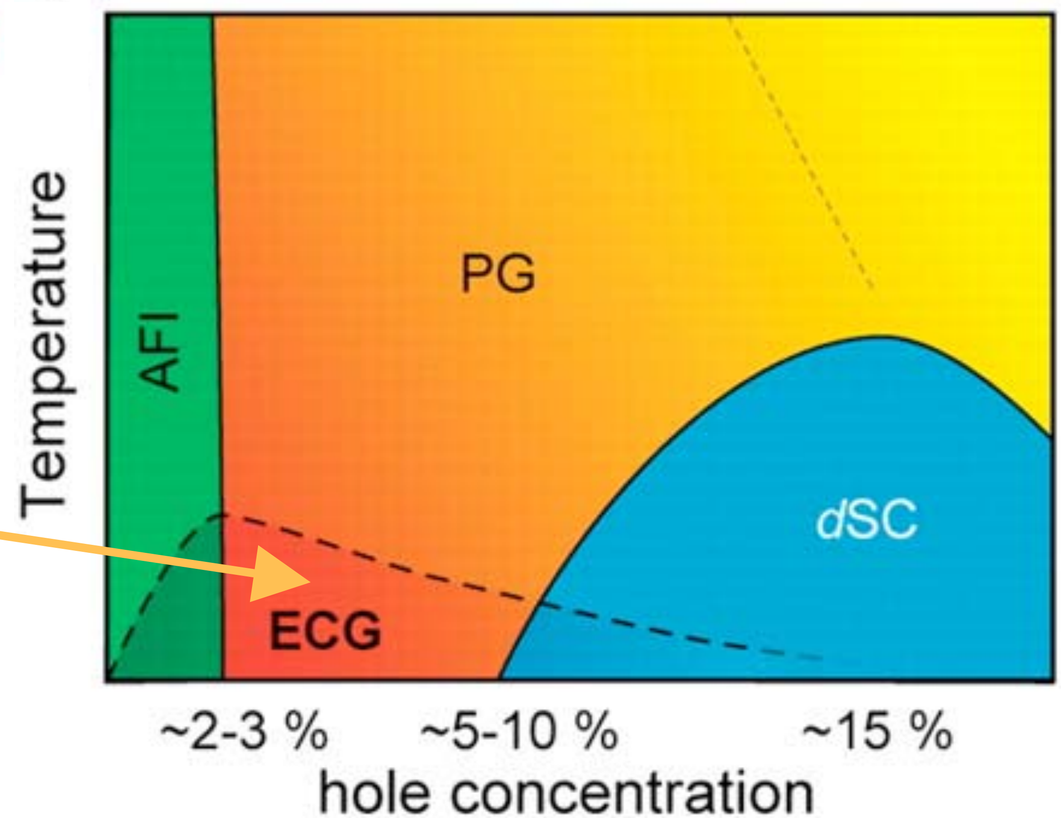
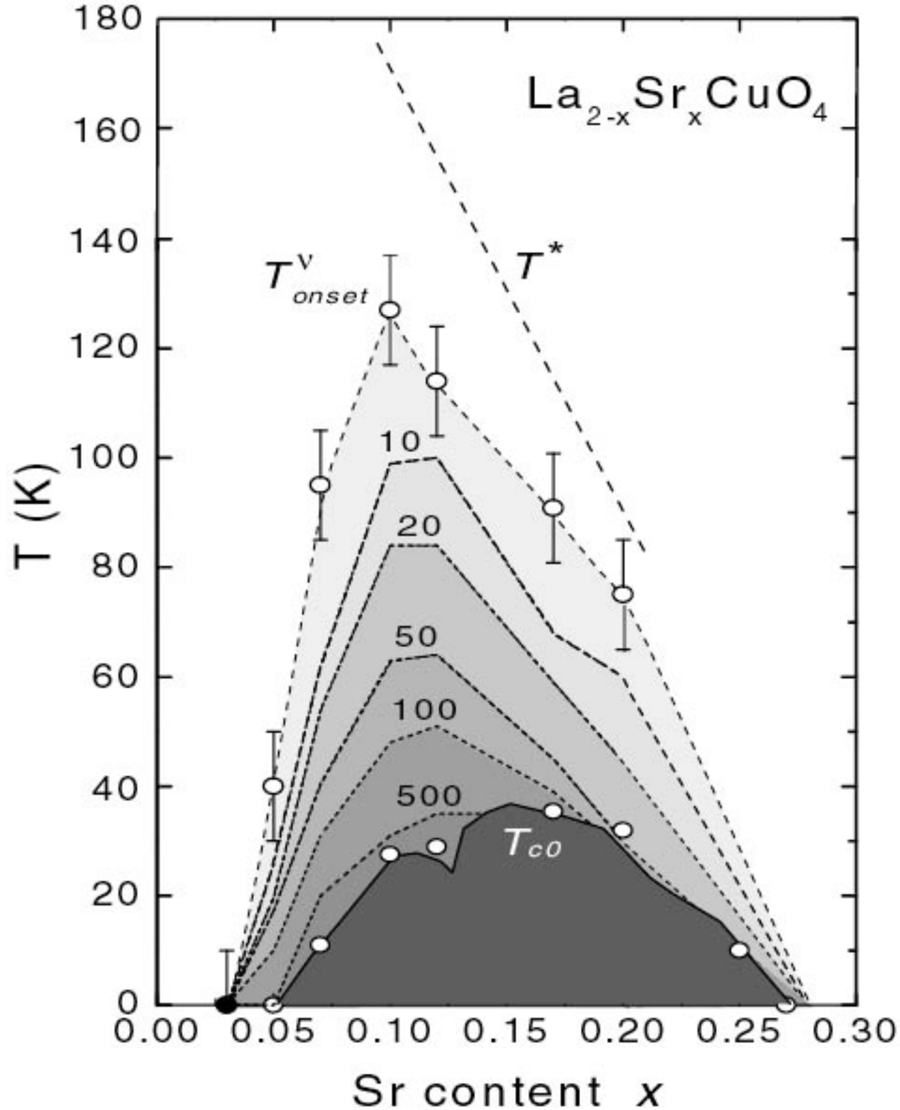


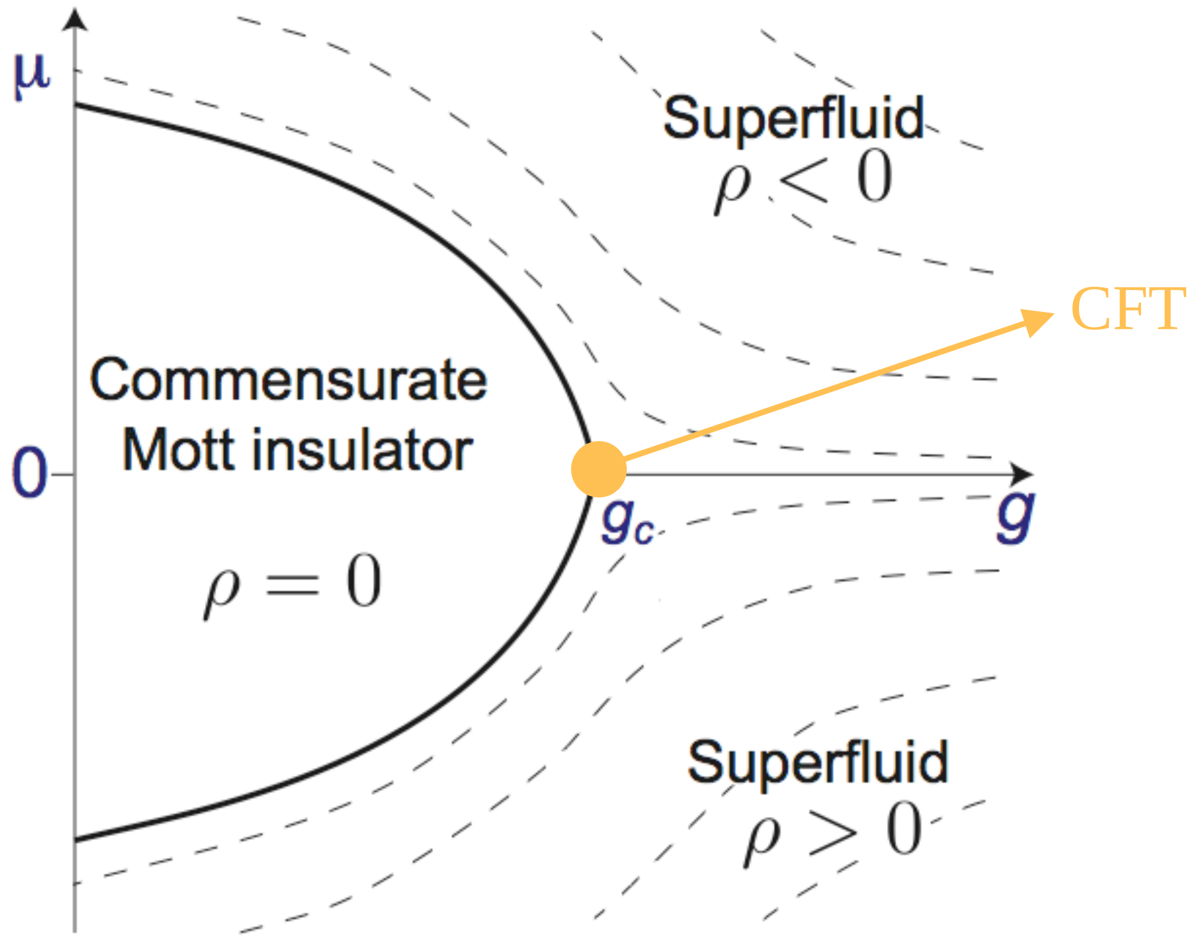
LSCO Phase diagram



Dip in T_c near x=1/8 indicates proximity of insulator

For experimental applications, we must move away from the ideal CFT

- A chemical potential μ



e.g. $$\mathcal{S} = \int d^2r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 \left|\vec{\nabla}\psi\right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

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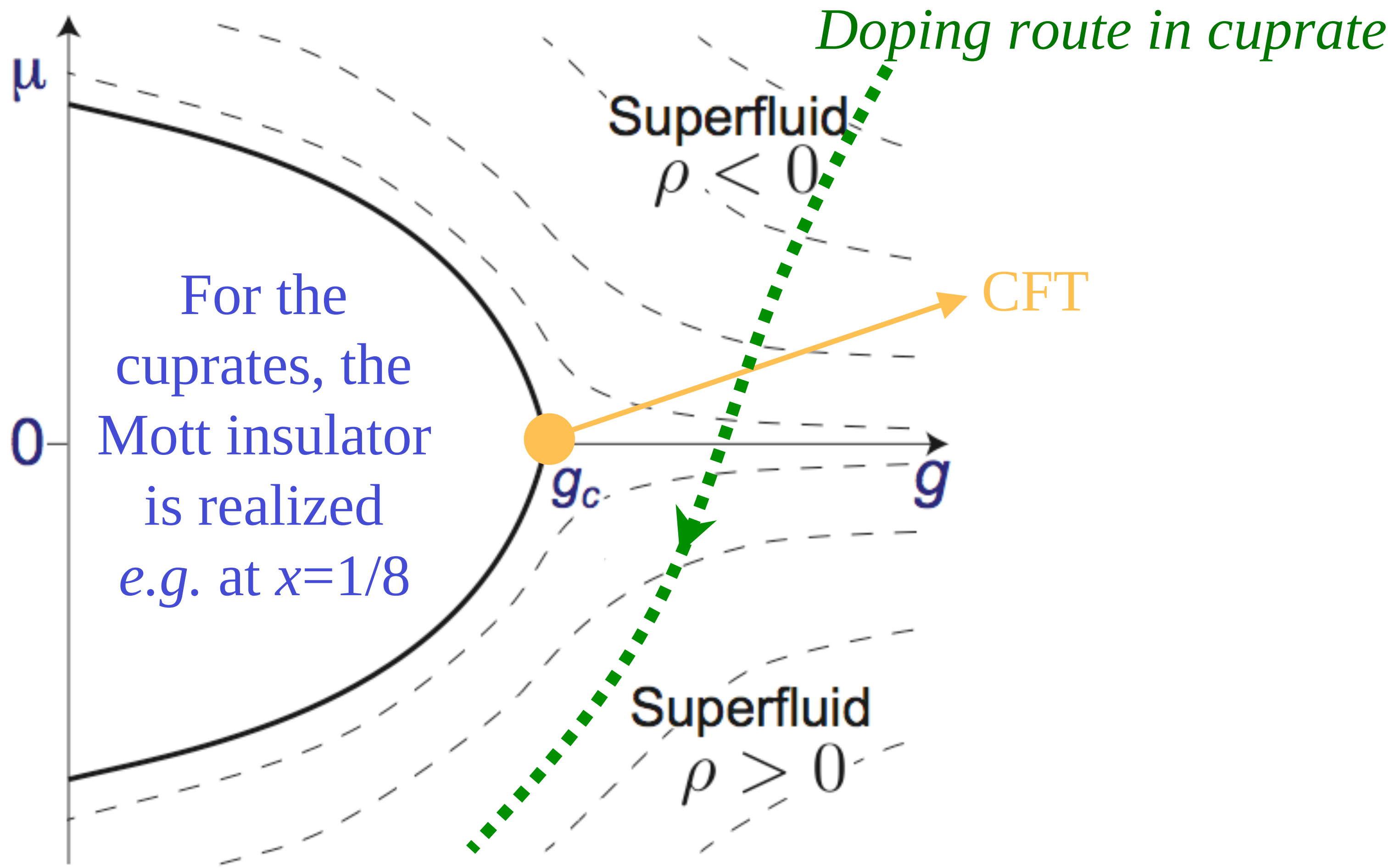

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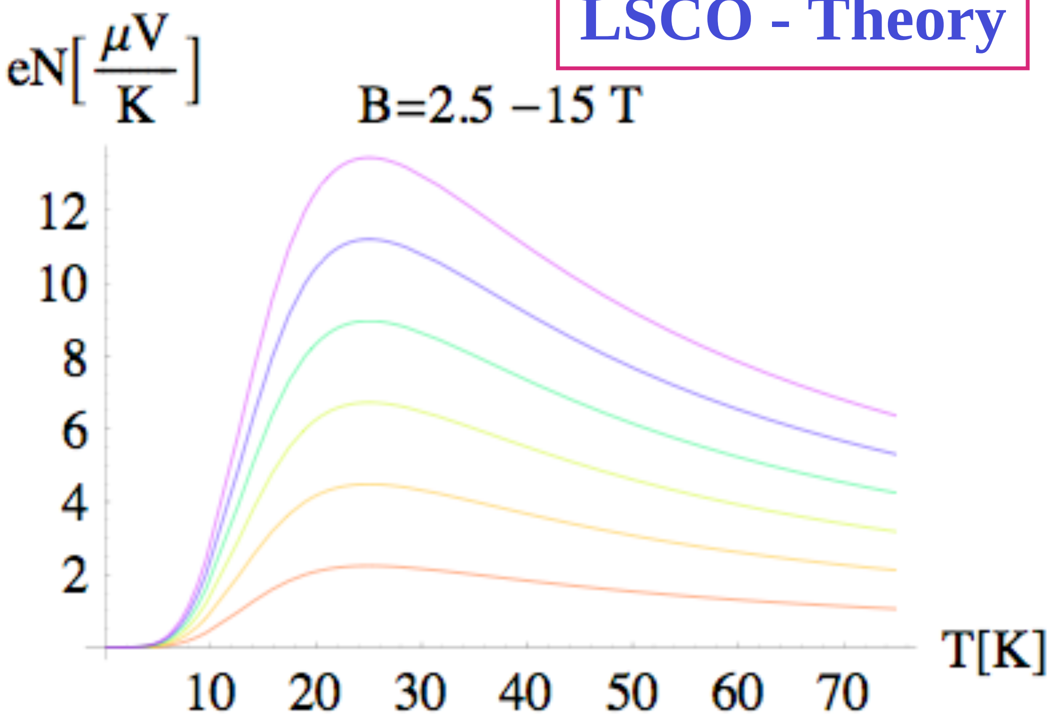
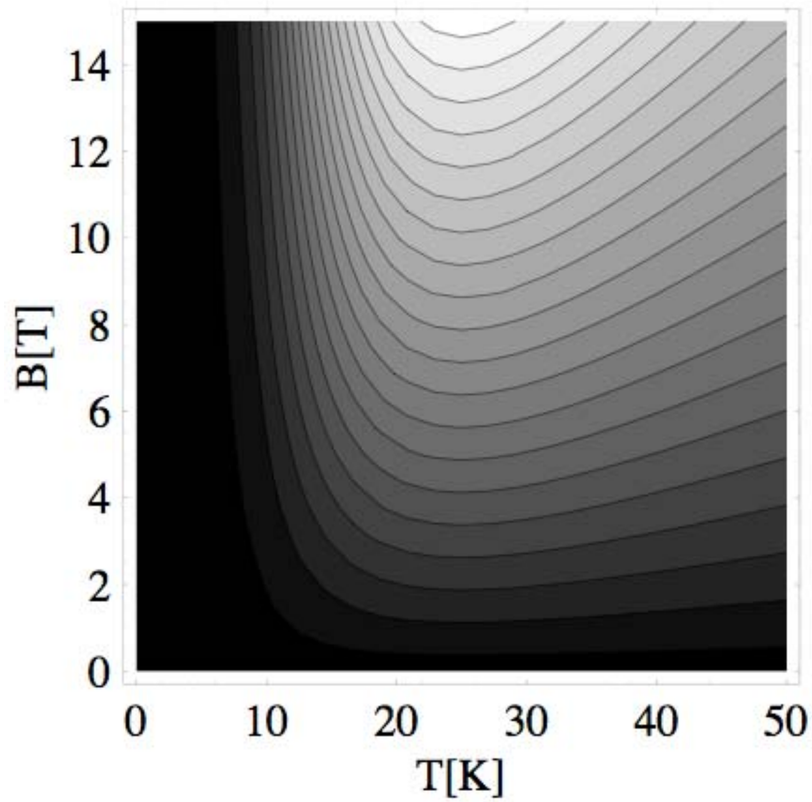


LSCO - Theory



Only input parameters

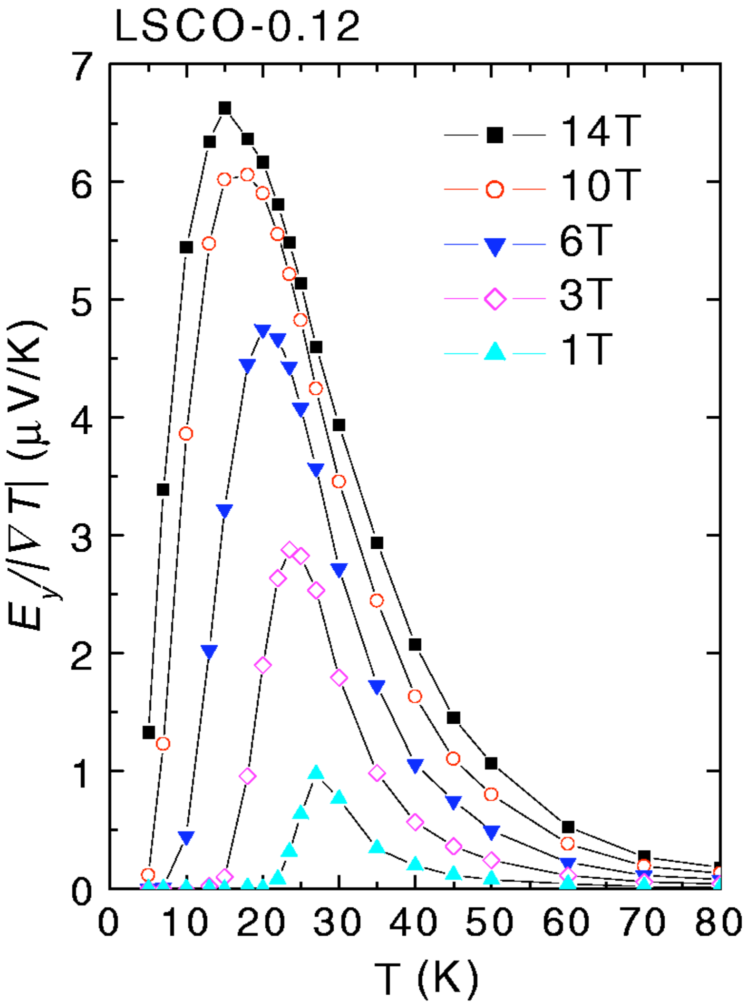
$$\hbar v = 15\text{meV Å}$$
$$\tau_{\text{imp}} = 6.25 \cdot 10^{-12}\text{s}$$

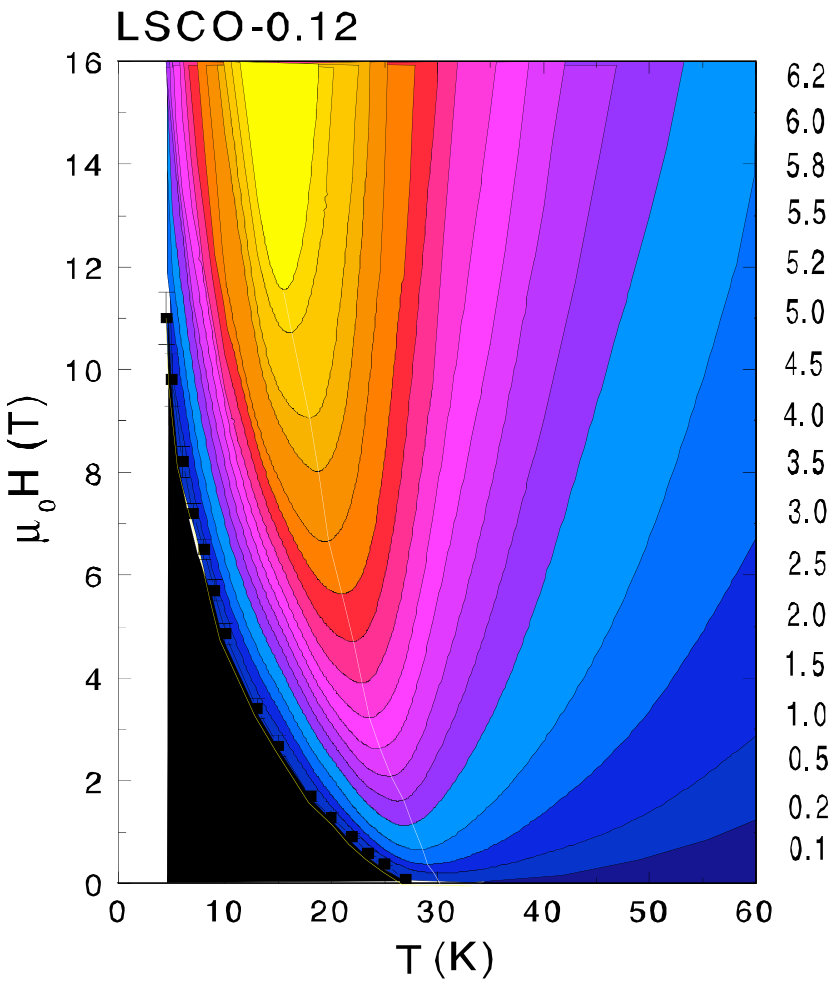
Output

$$\omega_c = 180 \text{ MHz} \cdot \frac{B}{1\text{ T}} \left(\frac{25\text{K}}{T}\right)^3$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

LSCO - Experiments





Y. Wang, L. Li, and N. P. Ong, Phys. Rev. B **73**, 024510 (2006).

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To the solvable supersymmetric, Yang-Mills theory CFT, we add

- A chemical potential μ
- A magnetic field B

After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

- An electric charge
- A magnetic charge

The exact results are found to be in *precise* accord with *all* hydrodynamic results presented earlier

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

Conclusions

- General theory of transport in a weakly disordered ``vortex liquid’’ state.
- “Relativistic” magnetohydrodynamics offers an efficient approach to disentangling momentum and charge transport
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Simplest model reproduces many trends of the Nernst measurements in cuprates.