

Destruction of Neel order
in the cuprates by
electron and hole doping

Ribhu Kaul

Cenke Xu

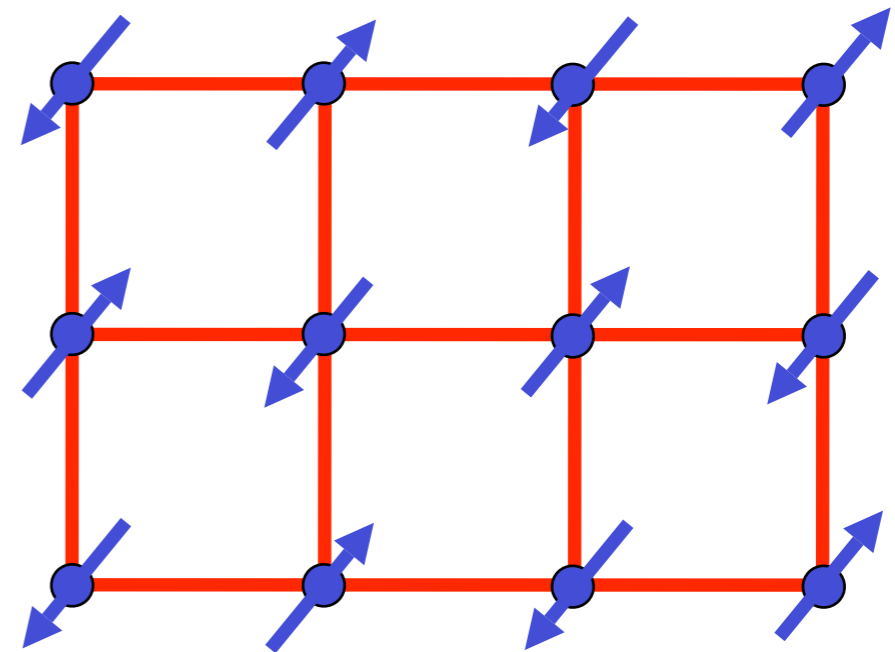
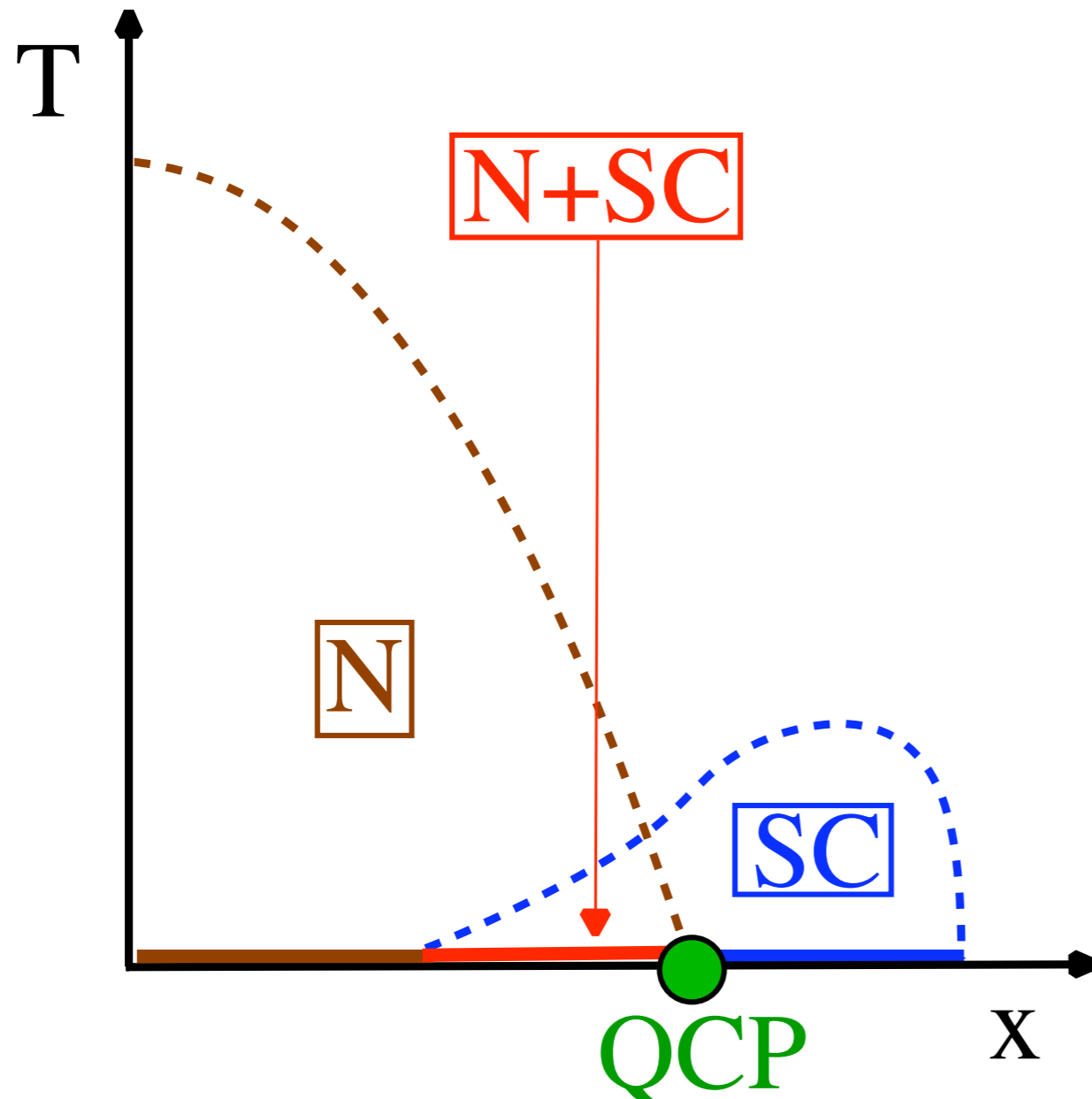
Subir Sachdev

arXiv:0804.1794

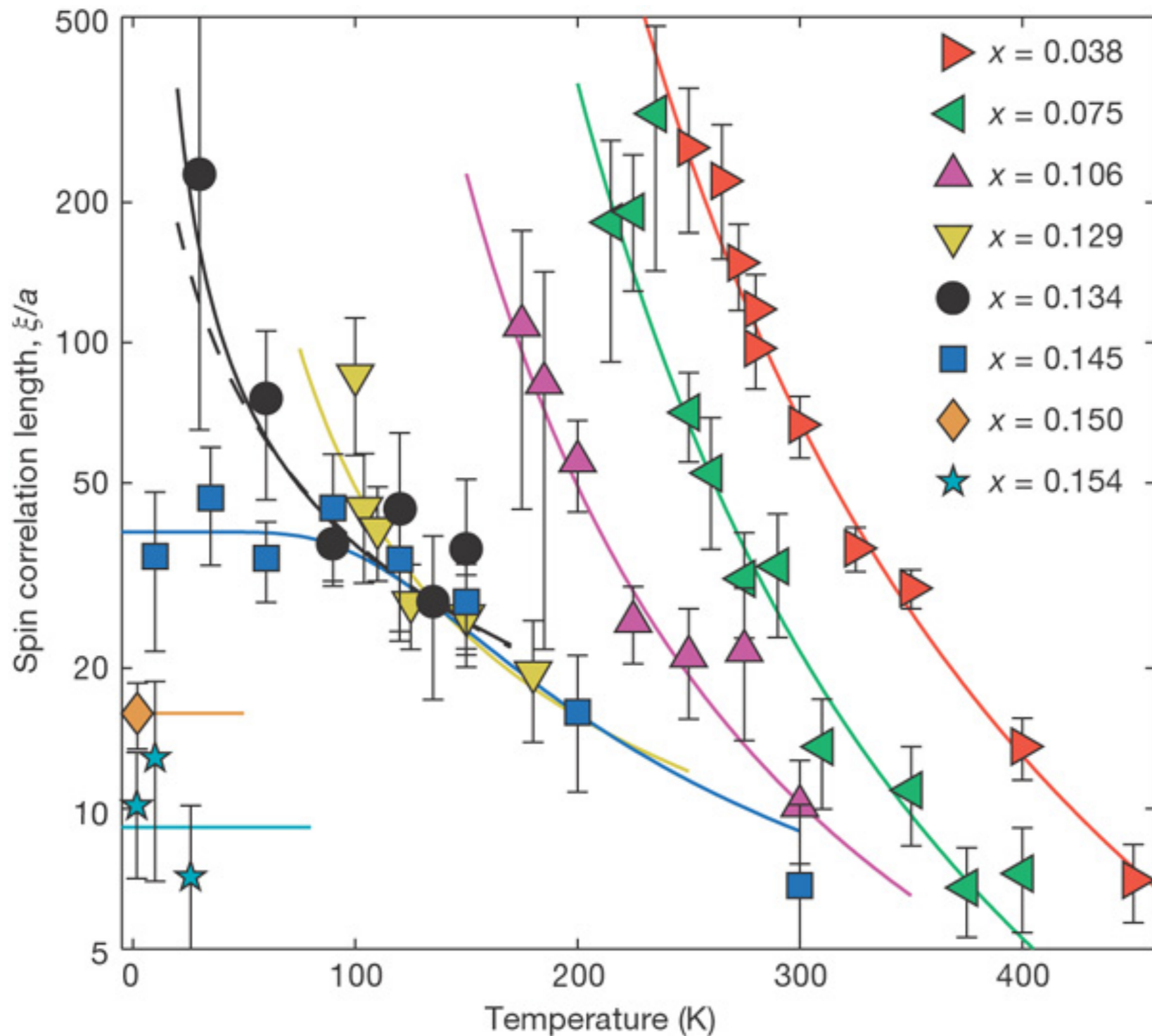
Previous work with A. Kolezhuk, Yong-Baek Kim,
Michael Levin, T. Senthil



Phase diagram of electron-doped superconductors



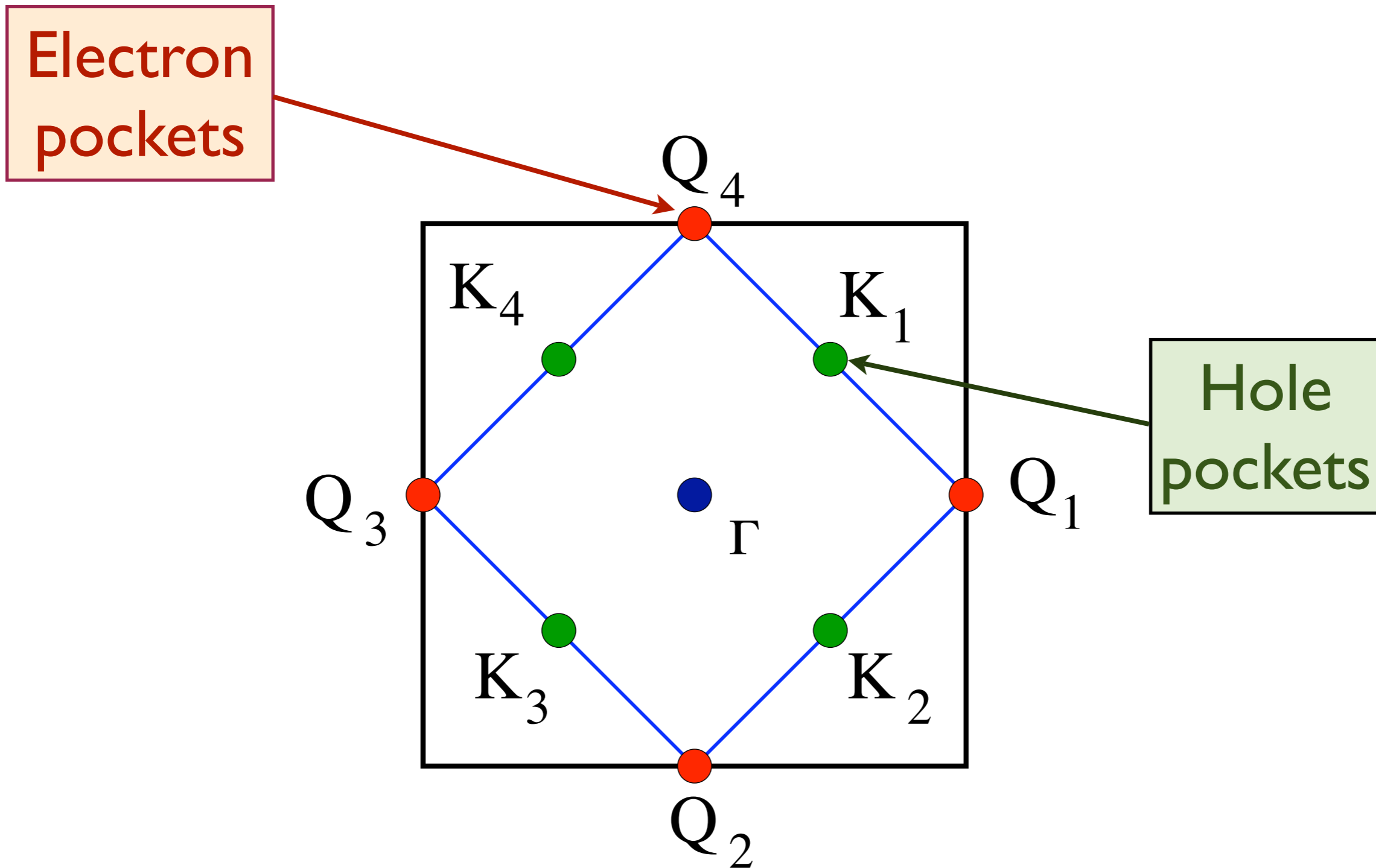
Neel order (N)



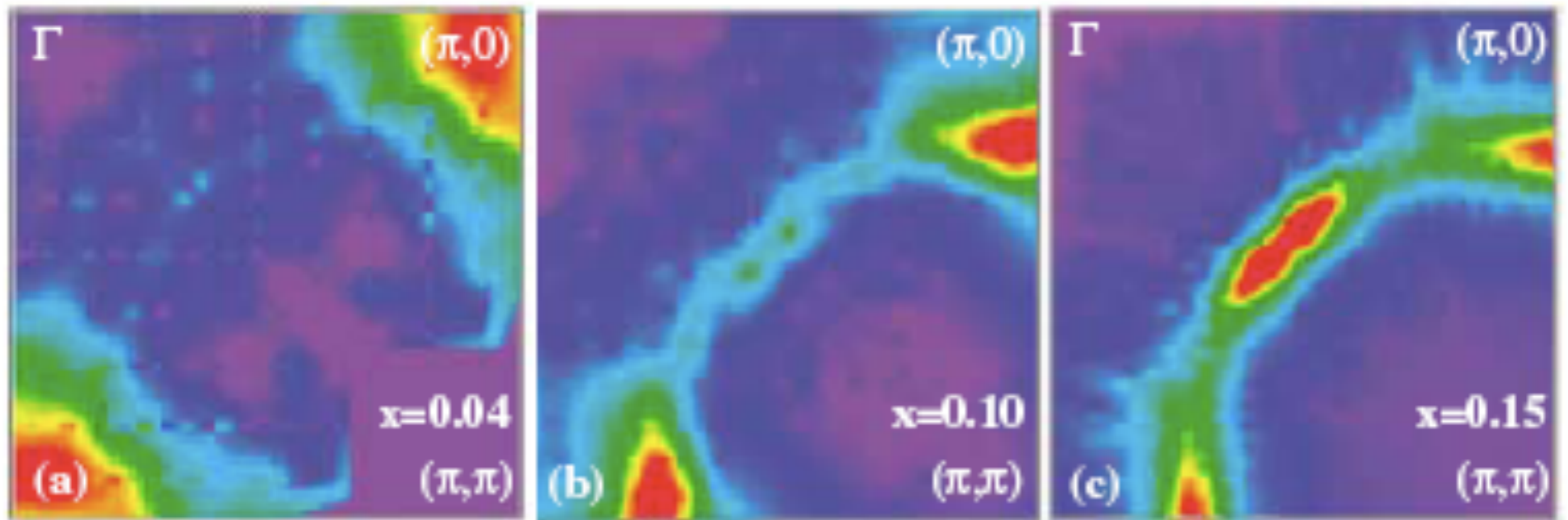
Spin correlations in the electron-doped high-transition-temperature superconductor $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$

E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang and M. Greven
Nature **445**, 186-189(11 January 2007)

Charge carriers in the lightly-doped cuprates with Neel order



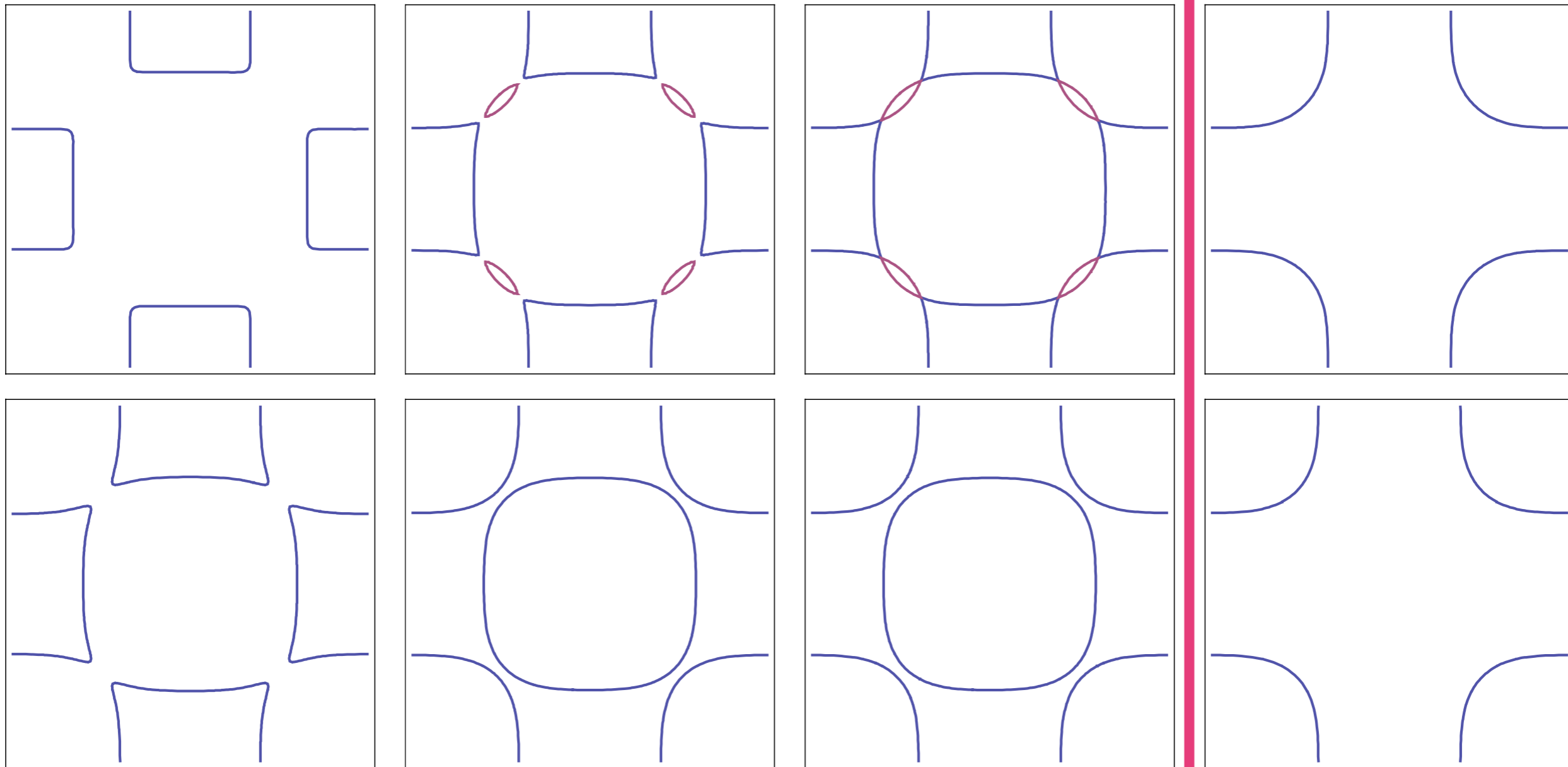
Photoemission in NCCO



N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

Spin density wave theory

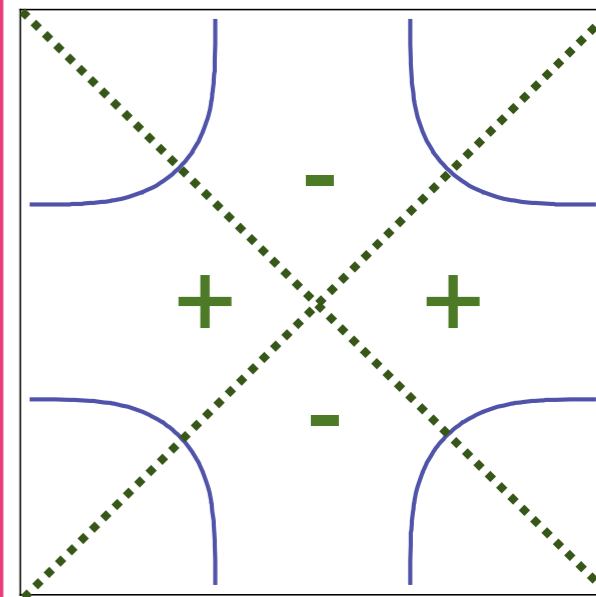
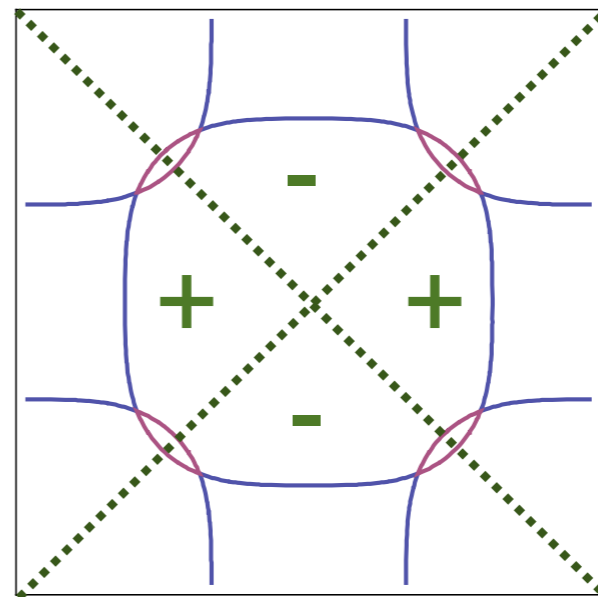
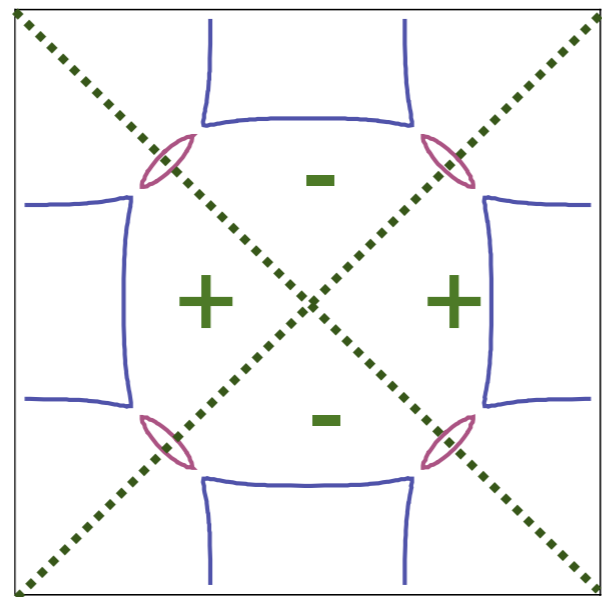
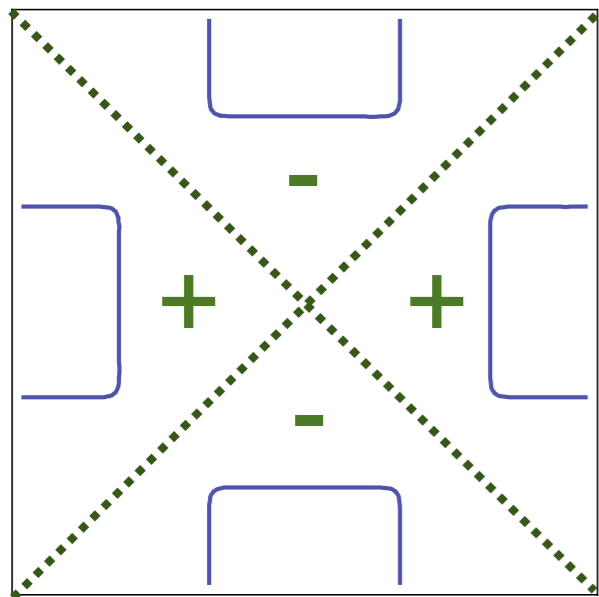
← Néel order →



→ x

Spin density wave theory

← Néel order —



dSC - no nodes

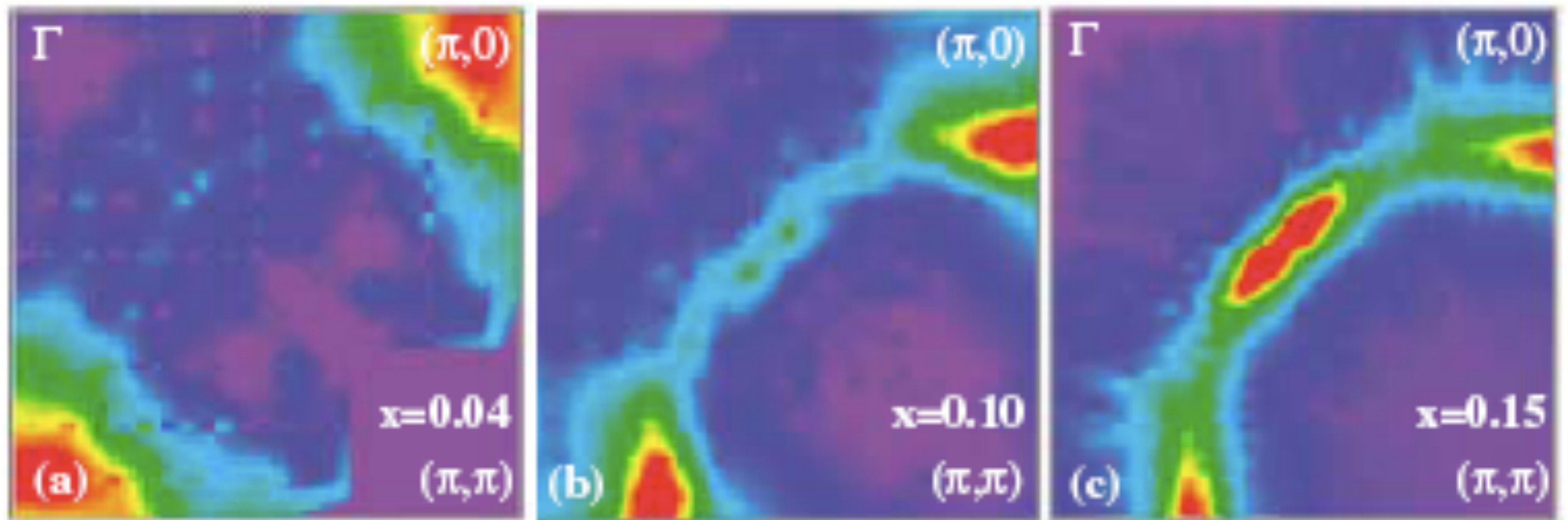
dSC - 8 nodes

dSC - 8 nodes

dSC - 4 nodes

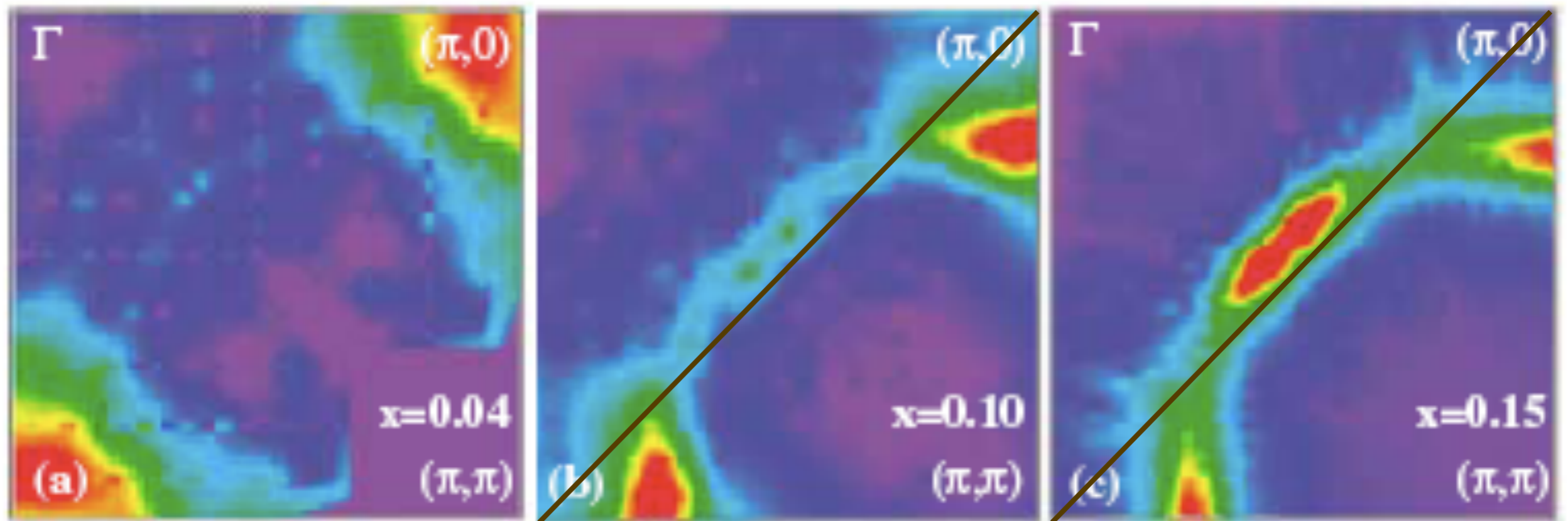
→ x

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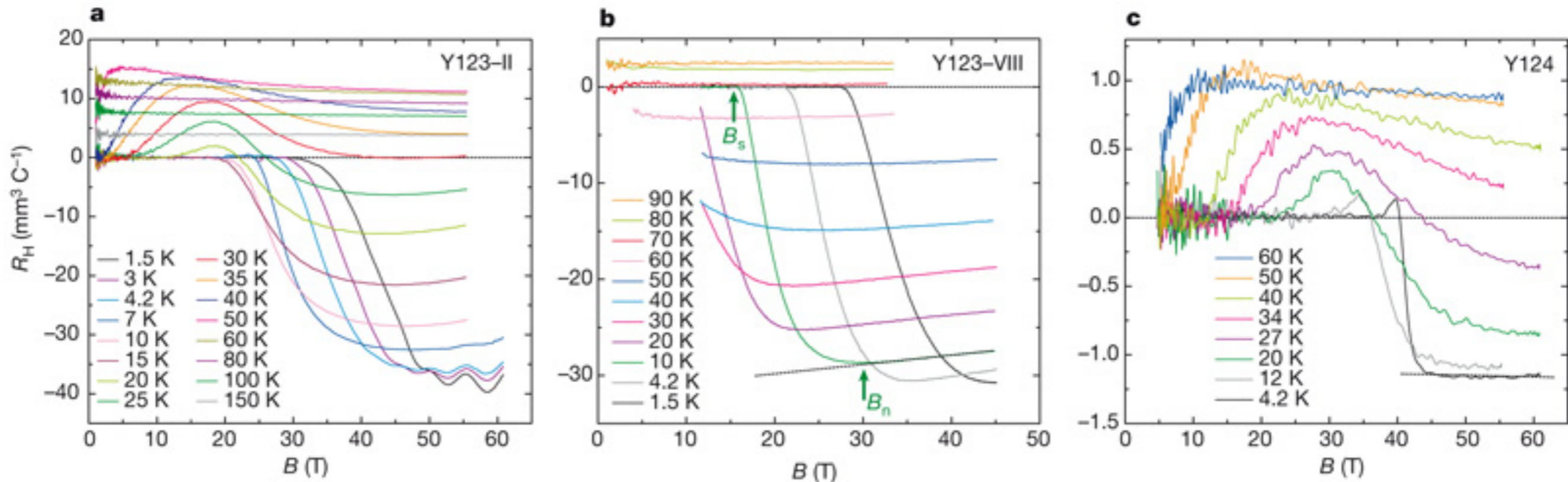
8 zero crossings not observed

N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

Electron pockets in the Fermi surface of hole-doped high- T_c superconductors

David LeBoeuf¹, Nicolas Doiron-Leyraud¹, Julien Levallois², R. Daou¹, J.-B. Bonnemaïson¹, N. E. Hussey³, L. Balicas⁴, B. J. Ramshaw⁵, Ruixing Liang^{5,6}, D. A. Bonn^{5,6}, W. N. Hardy^{5,6}, S. Adachi⁷, Cyril Proust² & Louis Taillefer^{1,6}

Nature **450**, 533 (2007)



Outline

1. Destruction of Neel order in the insulator

Deconfined criticality

2. Conducting Neel states

AF Metals and AF Superconductors

3. Destruction of Neel order in the metal

$O(4)$ transition to doublon/holon metals

4. Destruction of Neel order in the superconductor

(a) The holon superconductor

(b) Confinement transition to BCS superconductors

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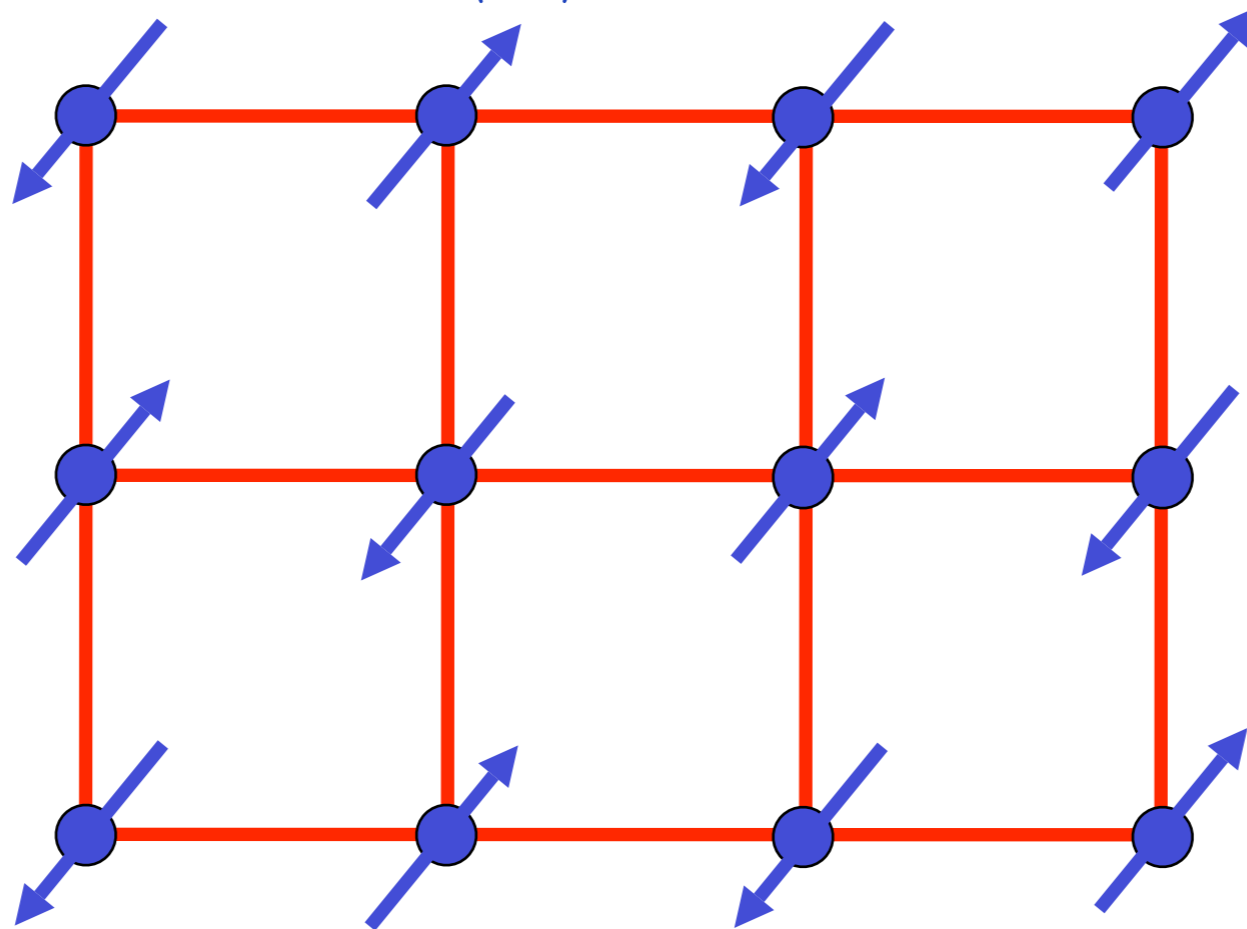
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(a) The holon superconductor

(b) Confinement transition to BCS superconductors

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

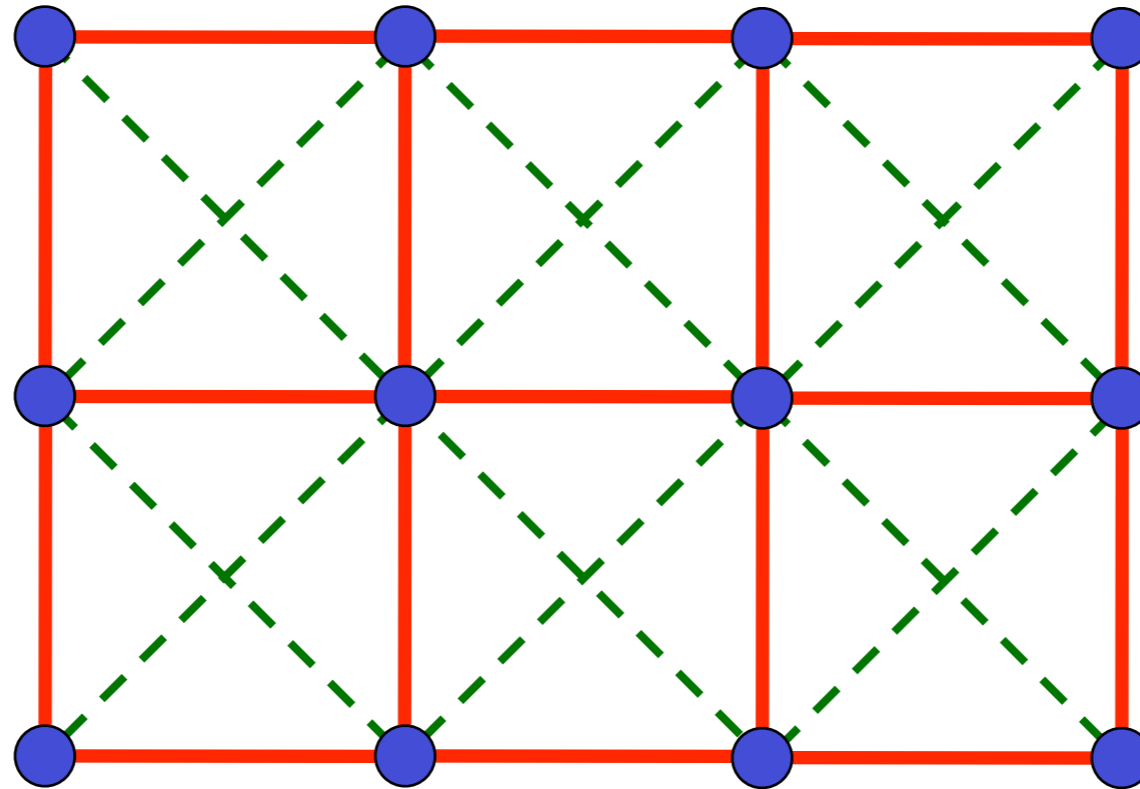
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

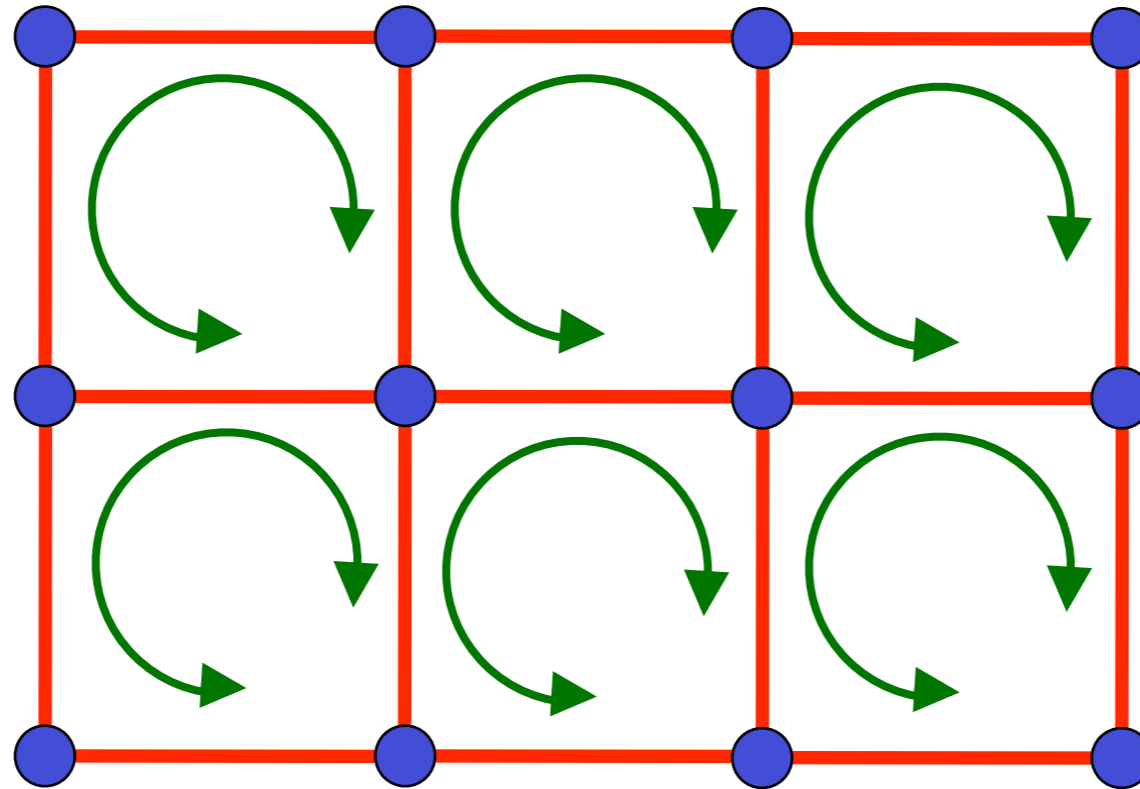


Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

What are possible states with $\langle \vec{\varphi} \rangle = 0$?

Square lattice antiferromagnet

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Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

What are possible states with $\langle \vec{\varphi} \rangle = 0$?

Theory for loss of Néel order

Decompose the Néel order parameter into *spinors*

$$\vec{\varphi} = z_{\alpha}^* \vec{\sigma}_{\alpha\beta} z_{\beta}$$

where $\vec{\sigma}$ are Pauli matrices, and z_{α} are complex spinors which carry spin $S = 1/2$.

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Effective theory for spinons must be invariant under the U(1) gauge transformation

$$z_{\alpha} \rightarrow e^{i\theta} z_{\alpha}$$

Theory for loss of Neel order

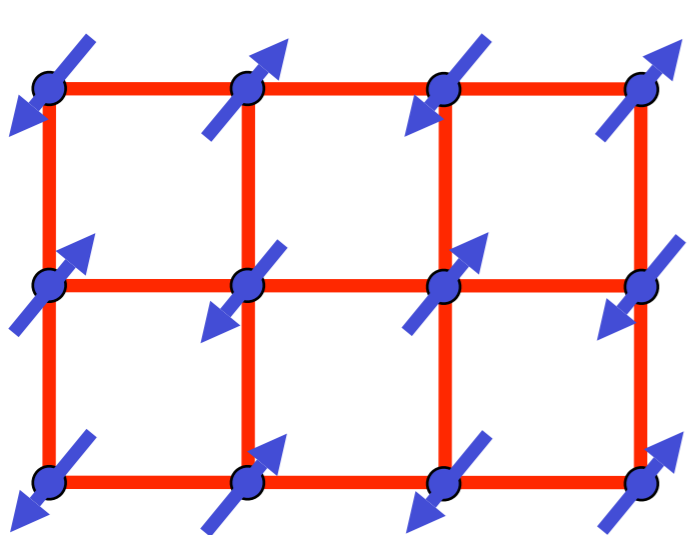
Naive expectation: Low energy spinon theory for “quantum disordering” a Néel state is

$$\mathcal{S}_z = \int d^2x d\tau \left[c^2 |(\nabla_x - iA_x)z_\alpha|^2 + |(\partial_\tau - iA_\tau)z_\alpha|^2 + s |z_\alpha|^2 + u (|z_\alpha|^2)^2 + \frac{1}{4e^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]$$

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$$\langle z_\alpha \rangle \neq 0$$

Néel state

Spin liquid state with stable $S = 1/2$ z_α spinons, and a gapless U(1) photon A_μ representing the topological order.

$$\langle z_\alpha \rangle = 0$$

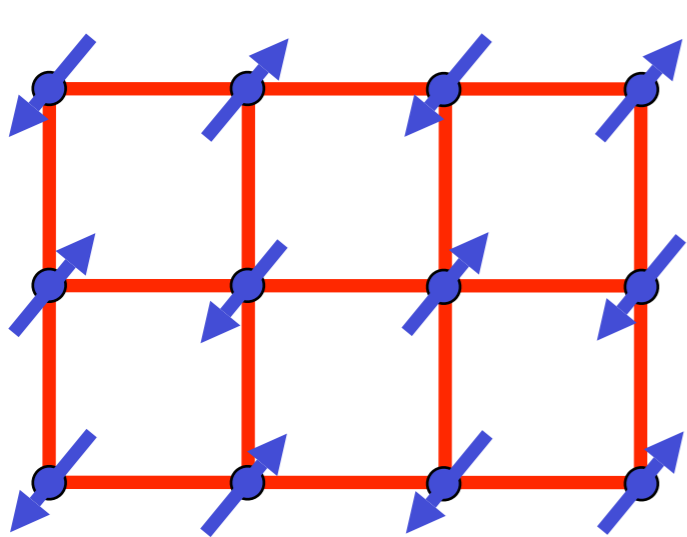
S_c

S

Theory for loss of Neel order

Naive expectation: Low energy spinon theory for “quantum disordering” a Néel state is

$$\mathcal{S}_z = \int d^2x d\tau \left[c^2 |(\nabla_x - iA_x)z_\alpha|^2 + |(\partial_\tau - iA_\tau)z_\alpha|^2 + s |z_\alpha|^2 + u (|z_\alpha|^2)^2 + \frac{1}{4e^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]$$



$$\langle z_\alpha \rangle \neq 0$$

Néel state

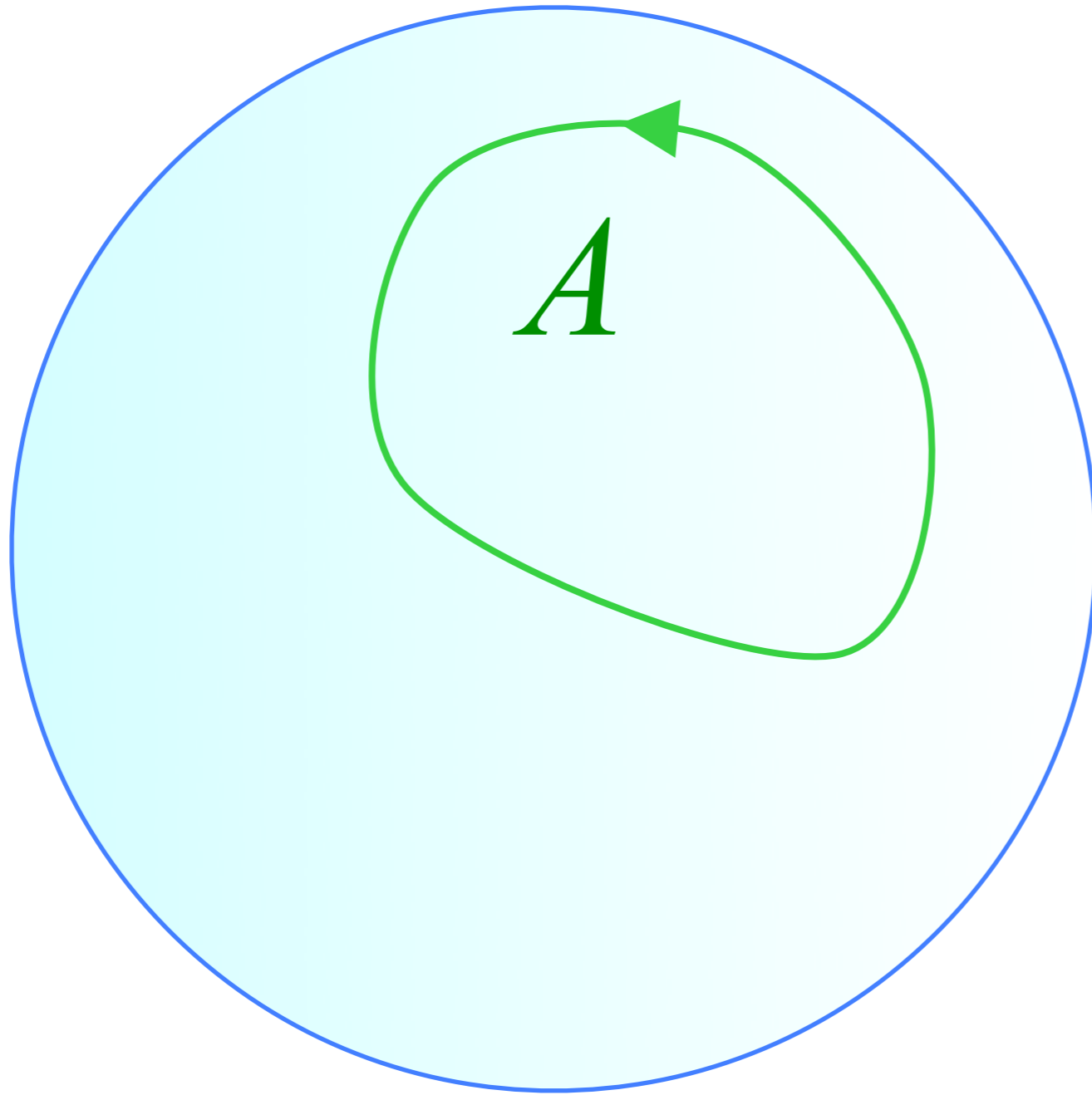
However, **monopoles** in the A_μ field will proliferate because of the gap to z_α excitations, and lead to **confinement** of z_α .

$$\langle z_\alpha \rangle = 0$$

s_c

s

Missing ingredient: Spin Berry Phases



$$e^{iA/2}$$

Quantum theory for destruction of Neel order

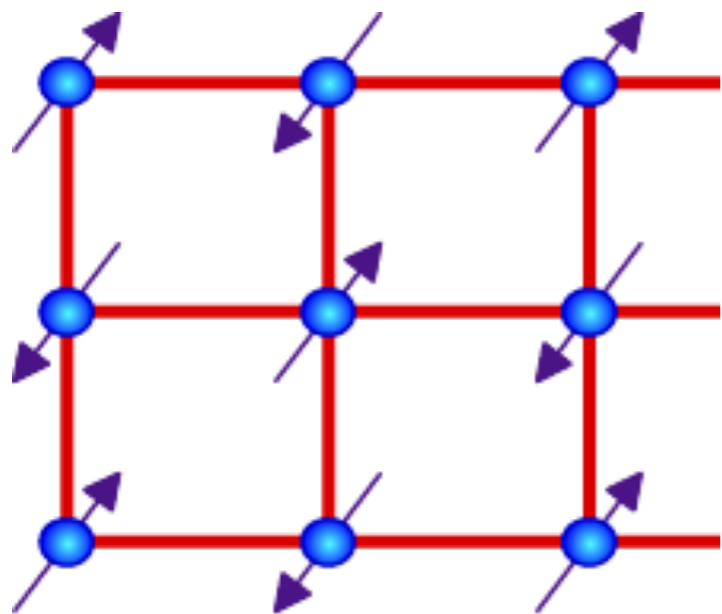
Partition function of spinons and gauge field on
a cubic spacetime lattice

$$\mathcal{Z} = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta \left(\sum_{\alpha} |z_{a\alpha}|^2 - 1 \right) \\ \times \exp \left(\frac{1}{g} \sum_{a,\mu} z_{a\alpha}^* e^{iA_{a\mu}} z_{a+\mu,\alpha} + i \sum_a \eta_a A_{a\tau} \right)$$

$\eta_a = \pm 1$ on the 2 sublattices.

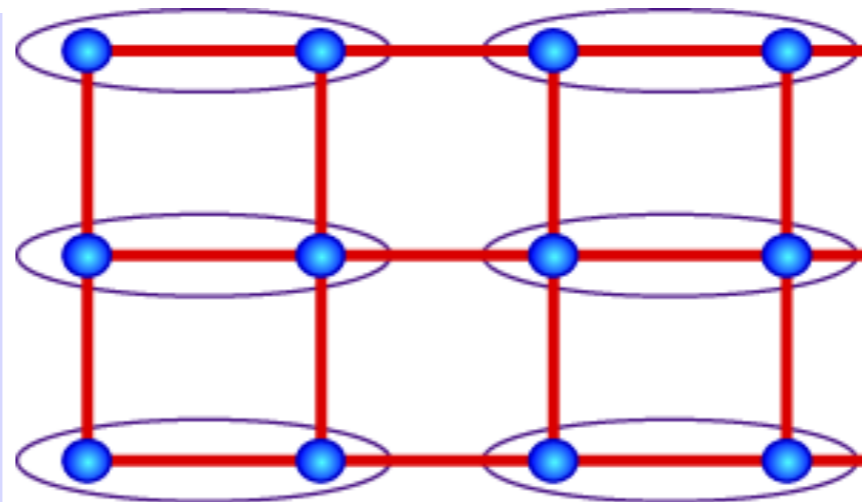
Quantum theory for destruction of Neel order

Berry phases endow monopoles with non-trivial square lattice space group quantum numbers, and so proliferation of monopoles leads to breaking of square lattice symmetry. The resulting state has valence bond solid (VBS) order, with $\langle \Psi_{\text{vbs}} \rangle \neq 0$.

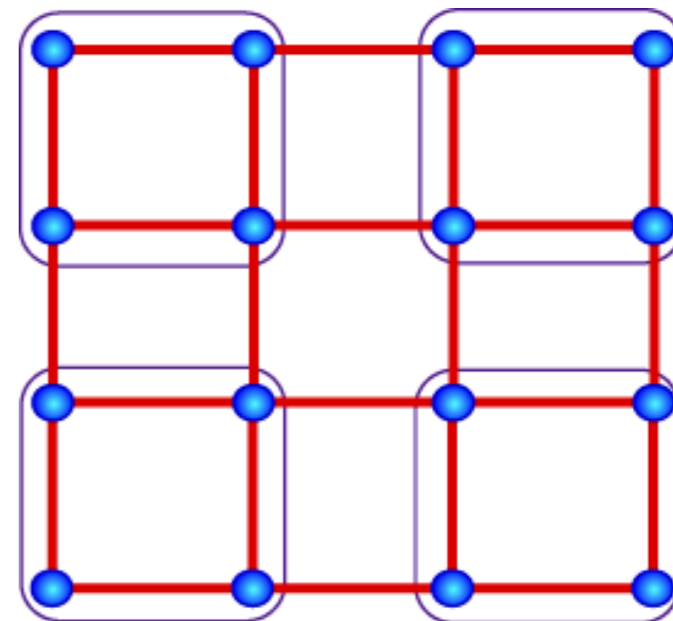


Néel order

$$\langle \vec{\varphi} \rangle \neq 0$$



or



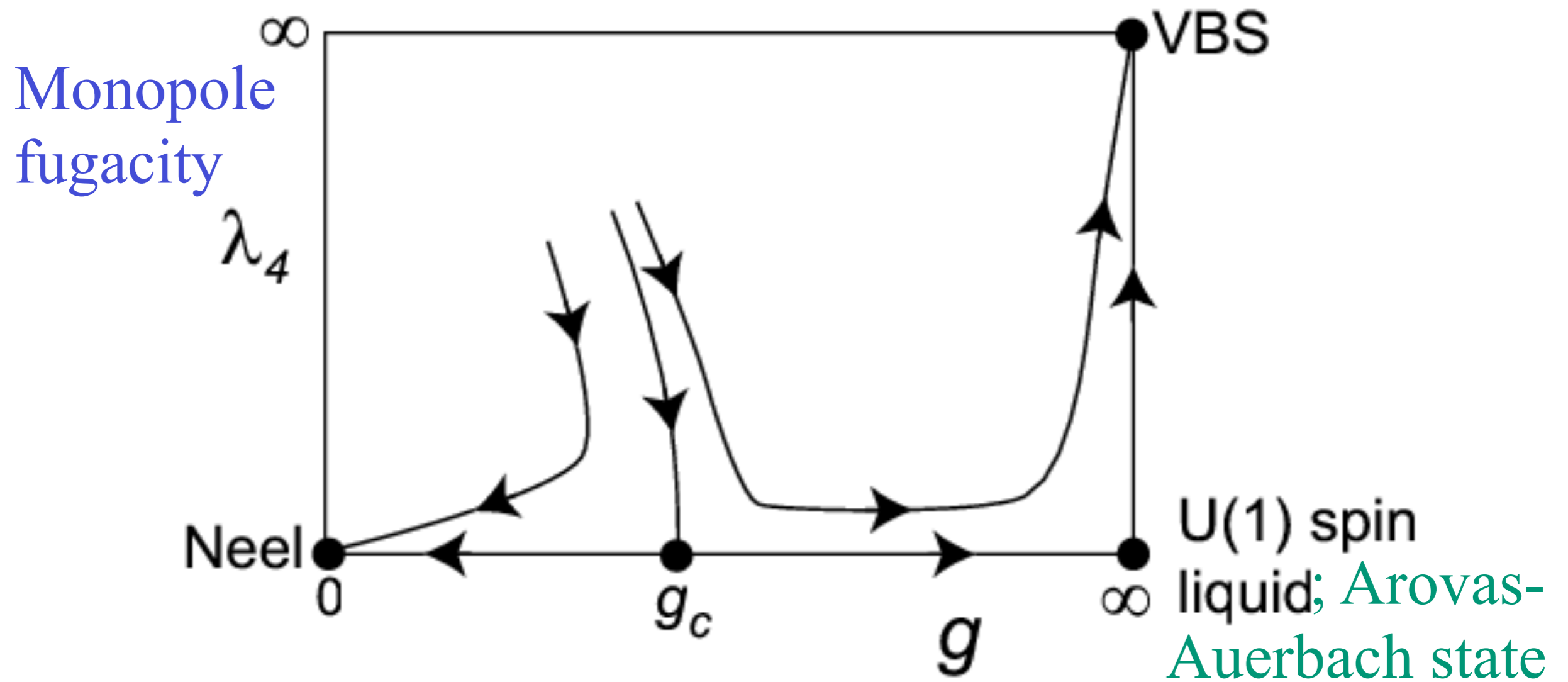
VBS order

$$\langle \Psi_{\text{vbs}} \rangle \neq 0$$

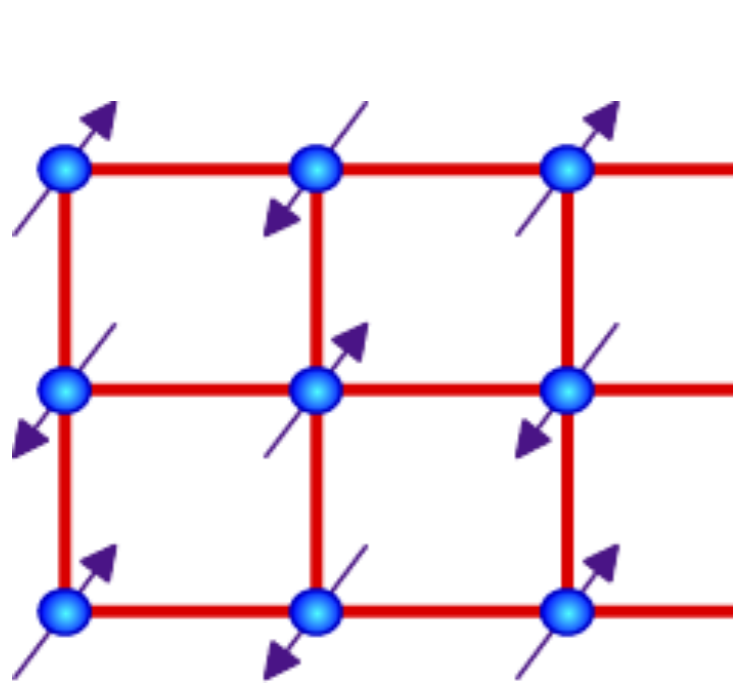
0

g



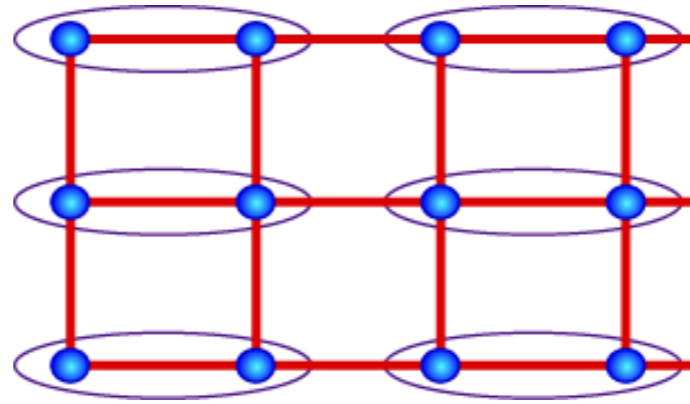


Phase diagram of S=1/2 square lattice antiferromagnet

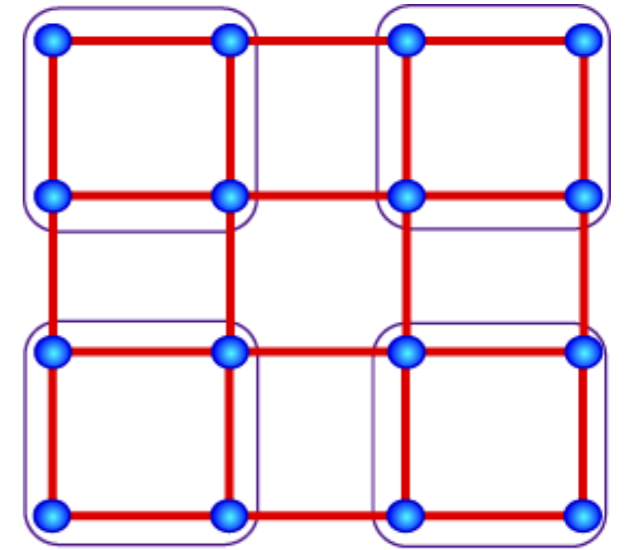


Neel order

$$\langle \vec{\varphi} \rangle \sim \langle z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \rangle \neq 0$$



or



$$\text{VBS order } \langle \Psi_{\text{vbs}} \rangle \neq 0$$

(associated with condensation of monopoles in A_μ),

$S = 1/2$ spinons z_α confined,

$S = 1$ triplon excitations



Second-order critical point described by CP^1 model

$$\mathcal{S}_z = \int d^2x d\tau \left[|(\partial_\mu - iA_\mu)z_\alpha|^2 + r|z_\alpha|^2 + \frac{u}{2} (|z_\alpha|^2)^2 + \frac{1}{4e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right]$$

at its critical point $r = r_c$ where A_μ is *non-compact*.

Large scale Quantum Monte Carlo studies

Easy-plane model:

$$\mathcal{H}_{XY} = 2J \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y) - K \sum_{\langle ijkl \rangle} (S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+)$$

A.W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett.* **89**, 247201 (2002)

A.W. Sandvik and R.G. Melko, *Phys. Rev. E* **72**, 026702 (2005).

SU(2)-invariant model:

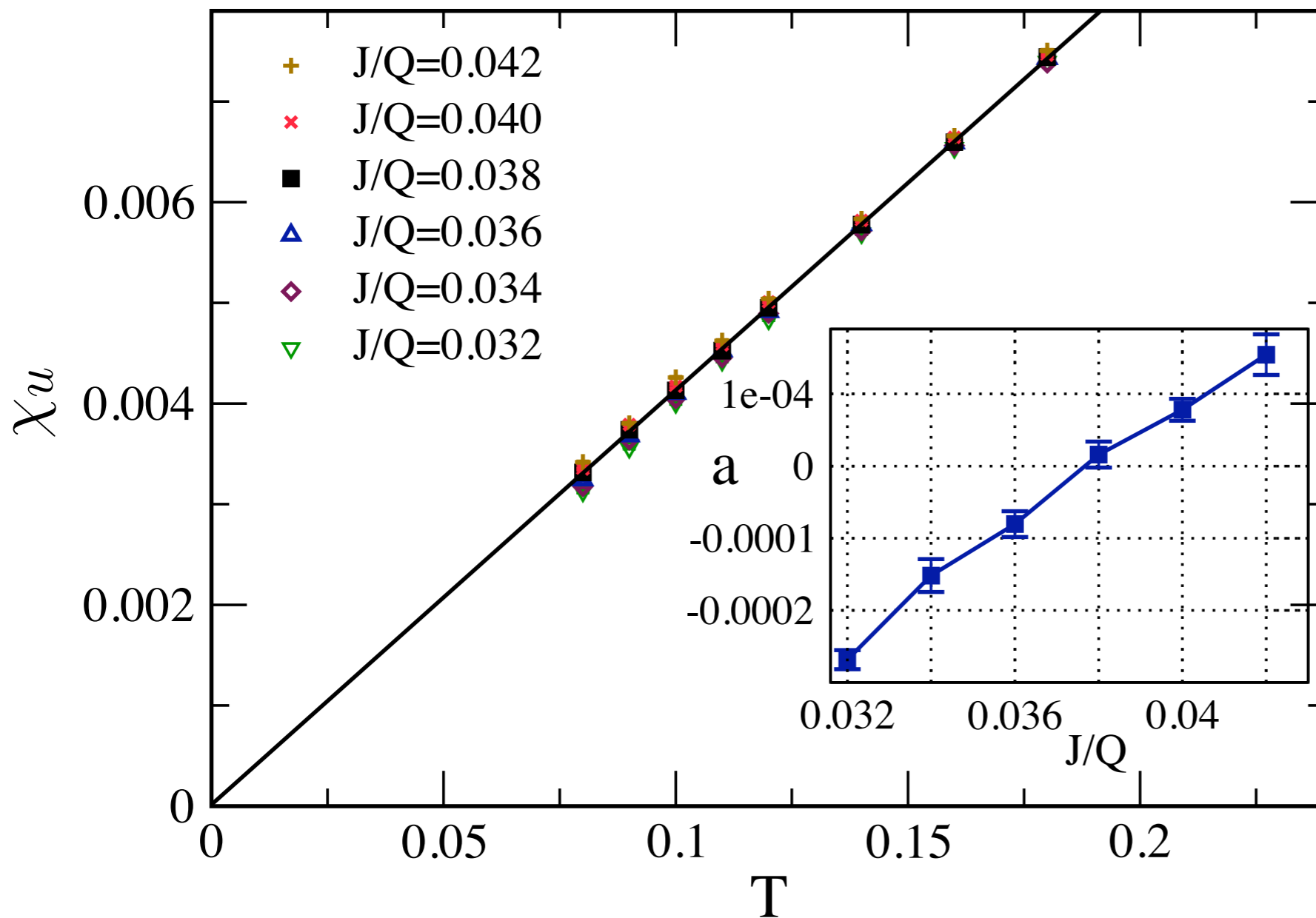
$$\mathcal{H}_{\text{SU}(2)} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} \right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \frac{1}{4} \right)$$

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 227202 (2007).

R.G. Melko and R.K. Kaul, *Phys. Rev. Lett.* **100**, 017203 (2008).

SU(2) invariant model

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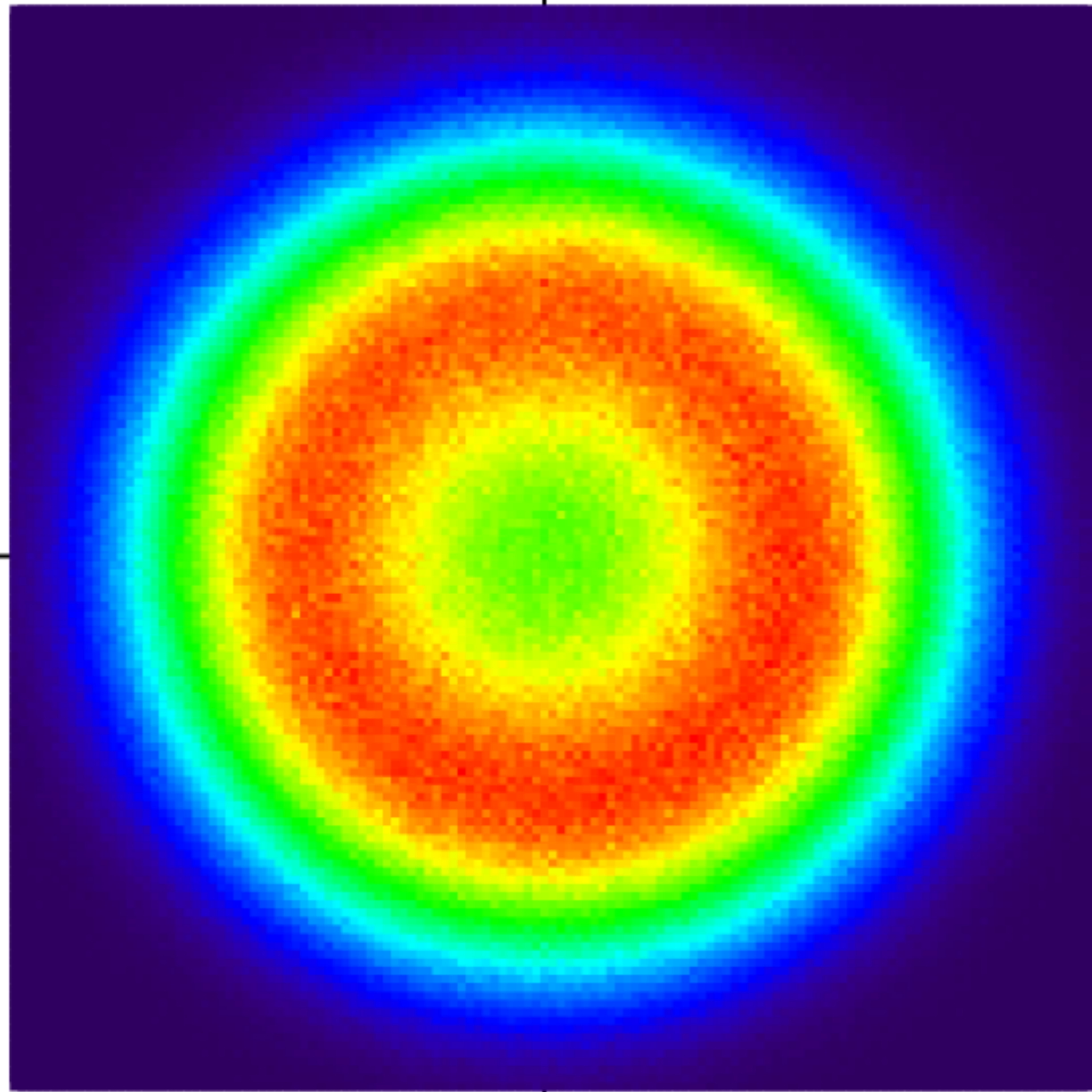


Strong evidence for a continuous “deconfined” quantum critical point

SU(2) invariant model

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|Dy



Dx

Histogram
of VBS order Ψ at
quantum critical point

*Emergent circular
symmetry is
evidence for U(1)
photon and
topological order*

Outline

1. Destruction of Neel order in the insulator

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AF Metals and AF Superconductors

3. Destruction of Neel order in the metal

$O(4)$ transition to doublon/holon metals

4. Destruction of Neel order in the superconductor

(a) The holon superconductor

(b) Confinement transition to BCS superconductors

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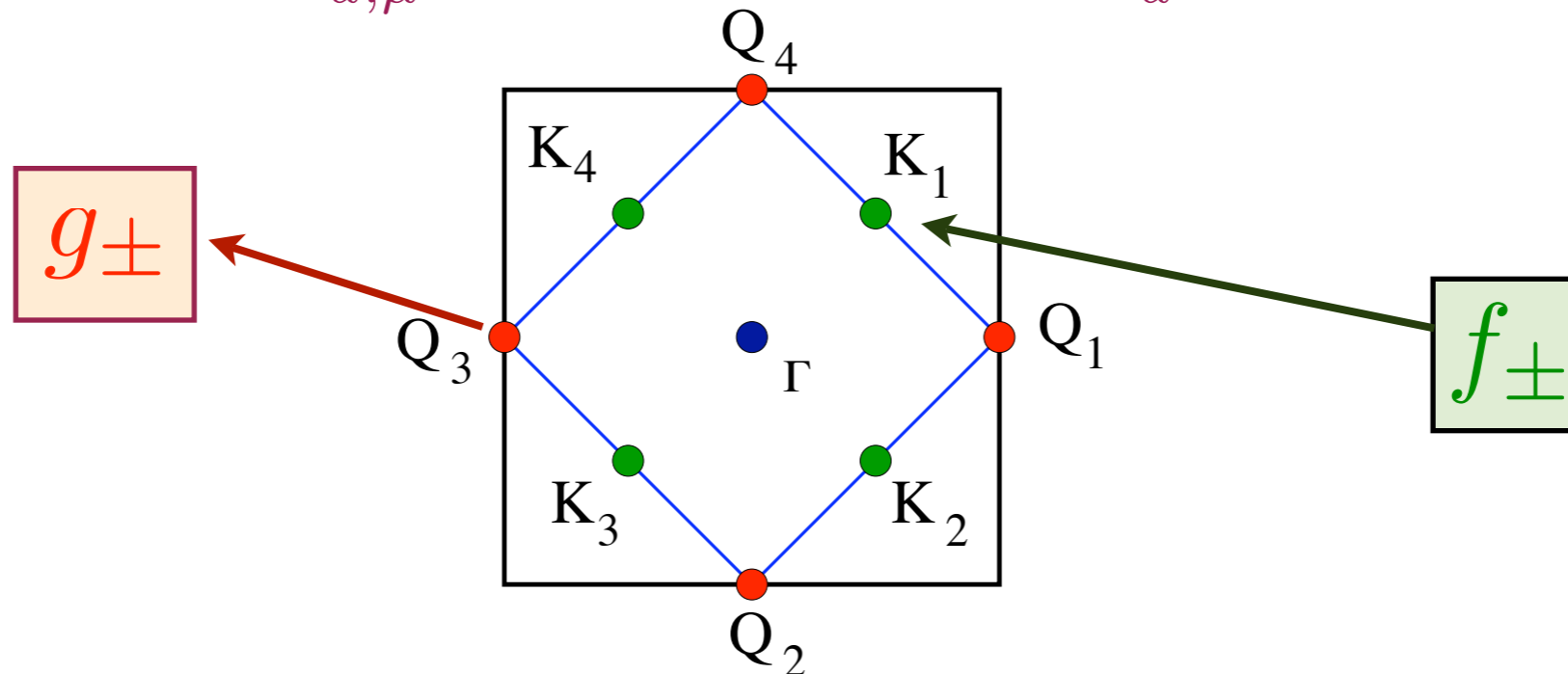
(b) Confinement transition to BCS superconductors

- Begin with the representation of the quantum antiferromagnet as the lattice $\mathbb{C}\mathbb{P}^1$ model:

$$\mathcal{S}_z = -\frac{1}{g} \sum_{a,\mu} z_{a\alpha}^* e^{iA_{a\mu}} z_{a+\mu,\alpha} + i \sum_a \eta_a A_{a\tau}$$

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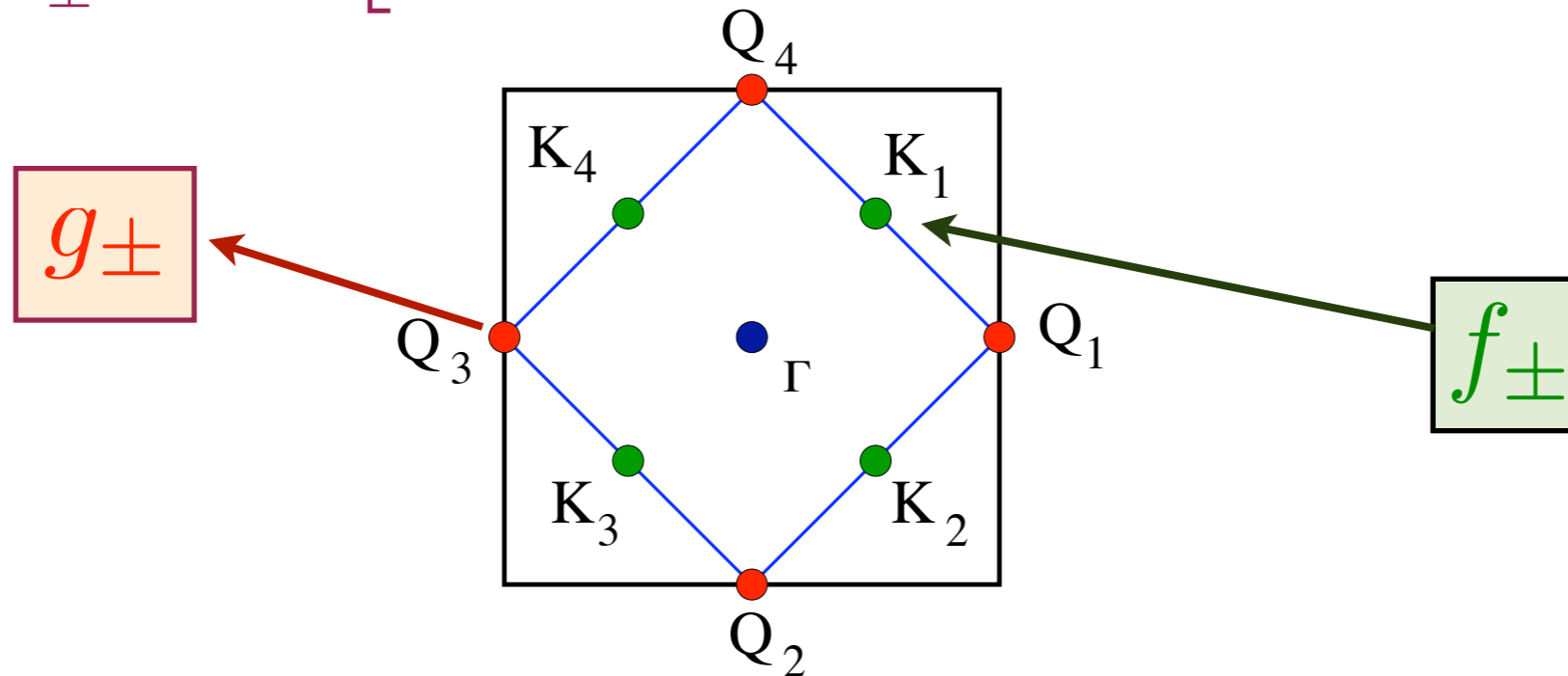
- Write the electron operator at site r , $c_{\alpha}(r)$ in terms of **doublon** operators g_{\pm} and **holon** operators f_{\pm}

$$c_{\alpha}(r) = \begin{cases} (g_{+}(r) + f_{+}^{\dagger}(r))z_{r\alpha} & \text{for } r \text{ on sublattice A} \\ \varepsilon_{\alpha\beta}(g_{-}(r) + f_{-}^{\dagger}(r))z_{r\beta}^* & \text{for } r \text{ on sublattice B} \end{cases}$$

Note that the fermions g_s, f_s have charge s under the $U(1)$ gauge field A_{μ} .

- Choose the fermion dispersions to match the positions on electron/hole pockets

$$\mathcal{S}_f = \int d\tau \sum_{s=\pm} \int_{\diamond} \frac{d^2k}{4\pi^2} \left[g_s^\dagger(\vec{k}) \left(\partial_\tau - isA_\tau + \epsilon(\vec{k} - s\vec{A}) \right) g_s(\vec{k}) + (g \rightarrow f) \right]$$



- Include the hopping between opposite sublattices (Shraiman-Siggia term):

$$\mathcal{S}_t = -t \sum_{\langle rr' \rangle} c_\alpha^\dagger(r) c_\alpha(r') + \text{h.c.}$$

- Complete theory for doped antiferromagnet:

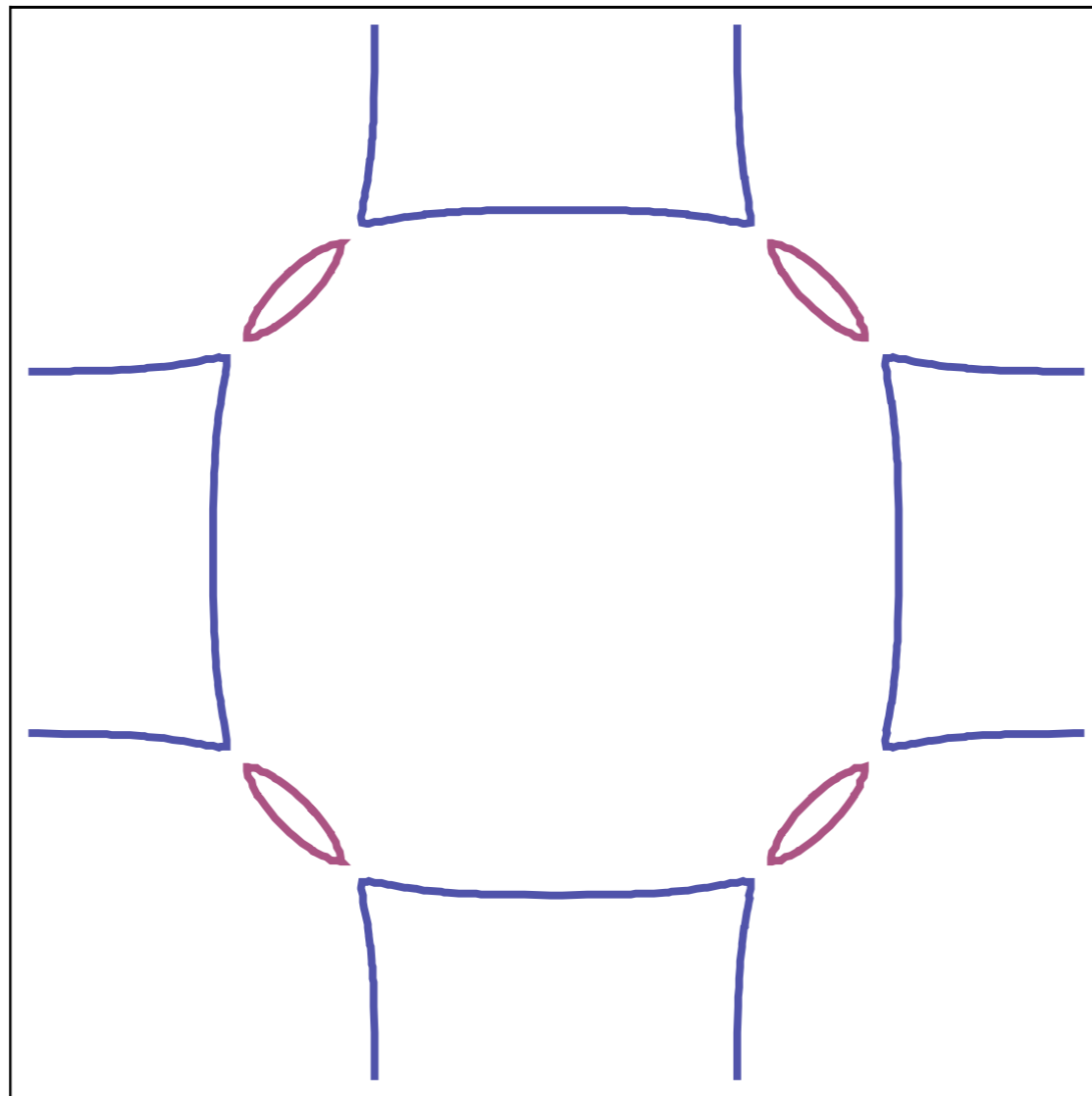
$$\mathcal{S} = \mathcal{S}_z + \mathcal{S}_f + \mathcal{S}_t$$

Conventional phases

AF Metal

$\langle z_\alpha \rangle \neq 0$, Fermi surfaces of f_\pm and/or g_\pm

“Meissner” effect ties U(1) gauge charge to conserved spin along the direction of Néel order



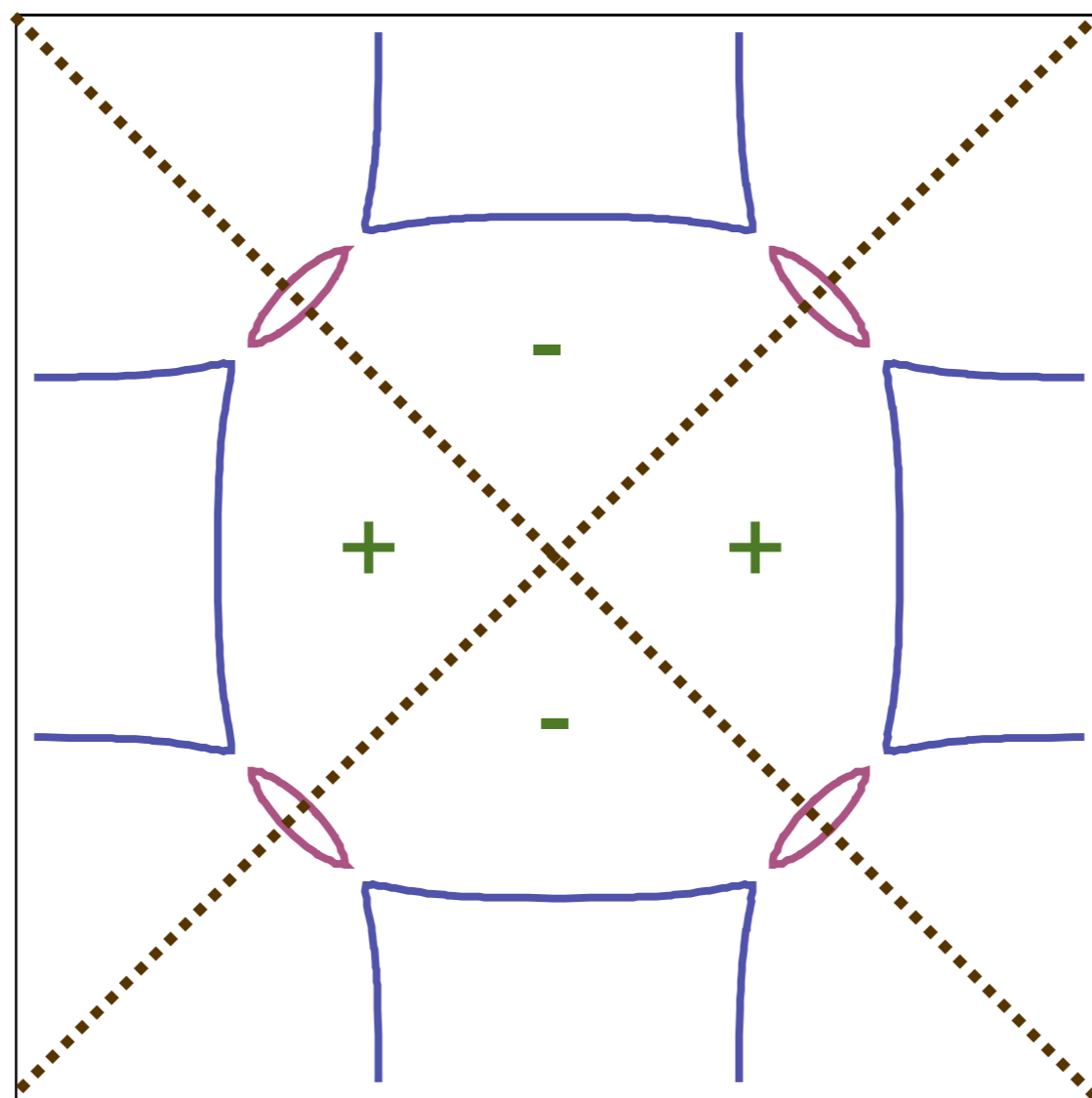
Conventional phases

AF d-wave superconductor

$$\langle z_\alpha \rangle \neq 0$$

$$\langle g_+ g_- \rangle \neq 0, \text{ } s\text{-wave pairing}$$

$$\langle f_+ f_- \rangle \neq 0, \text{ } p\text{-wave pairing}$$



4 Dirac points
(+ 4 shadows)

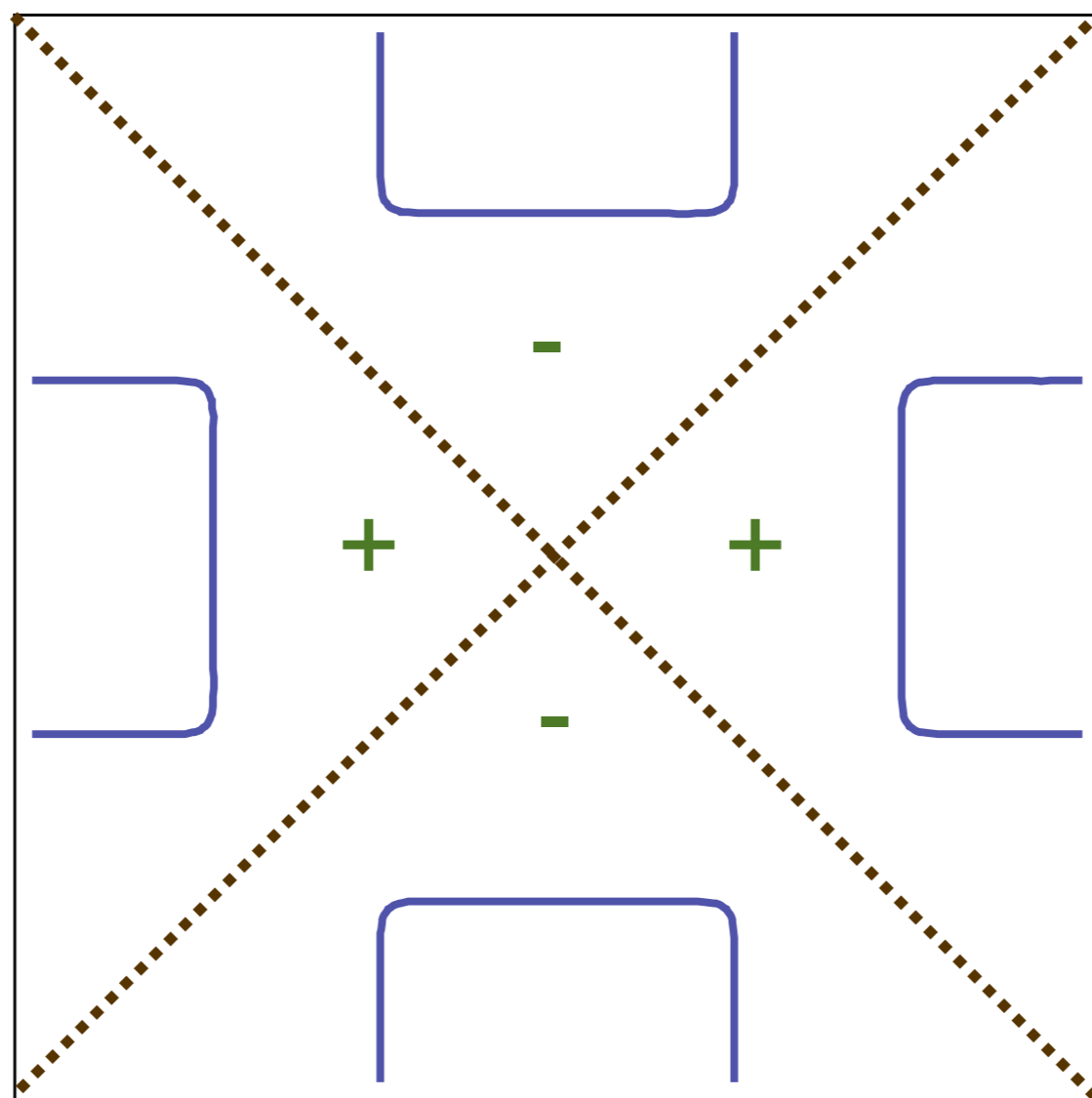
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Fermions fully gapped

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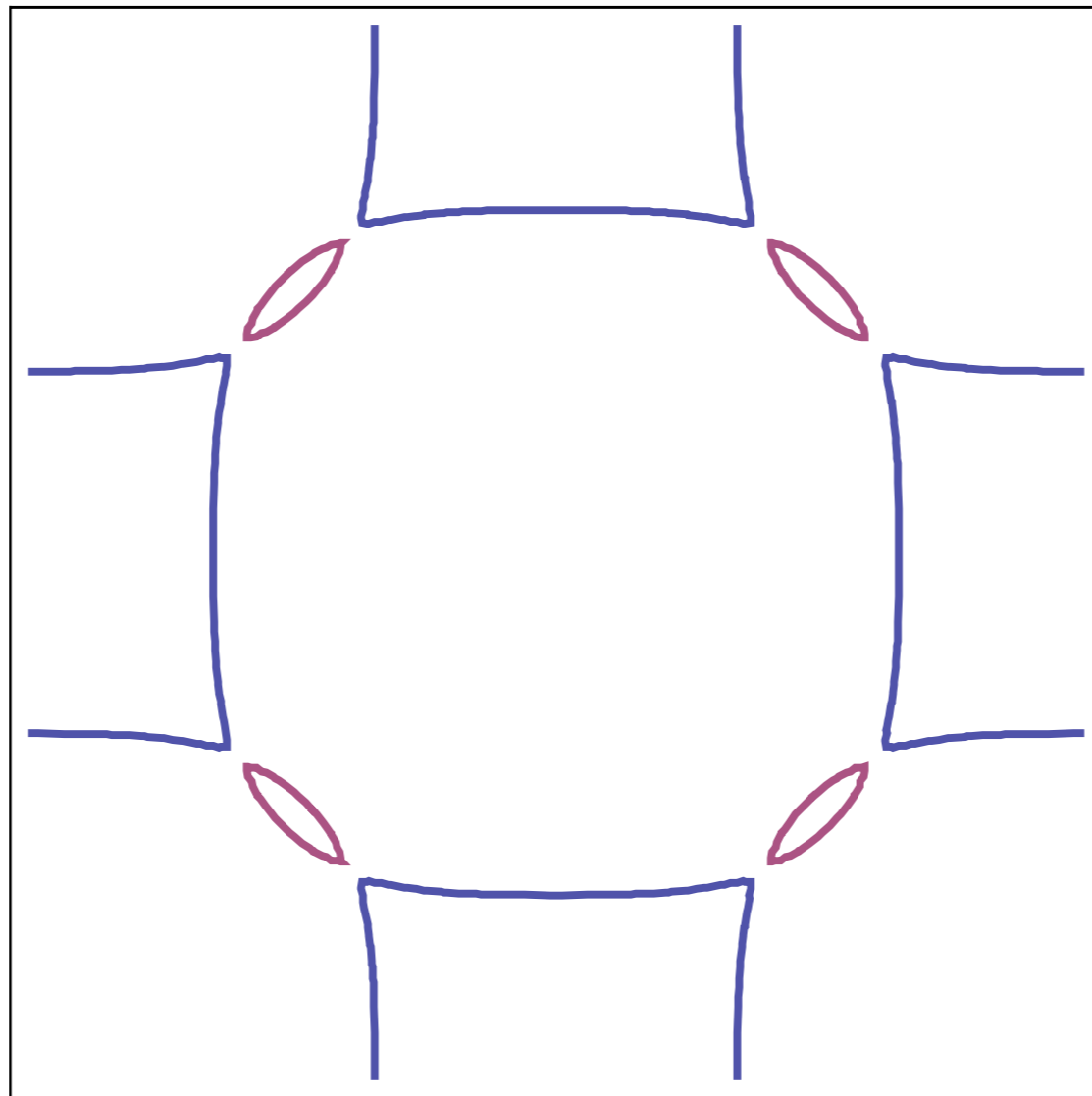
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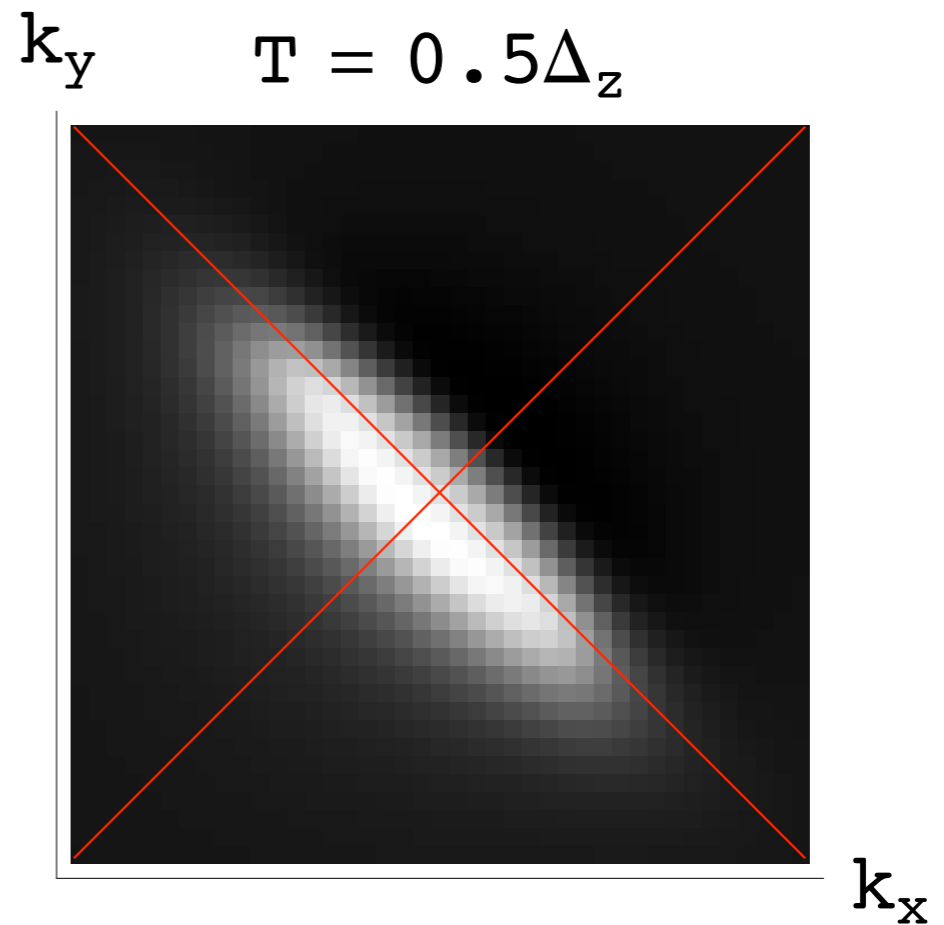
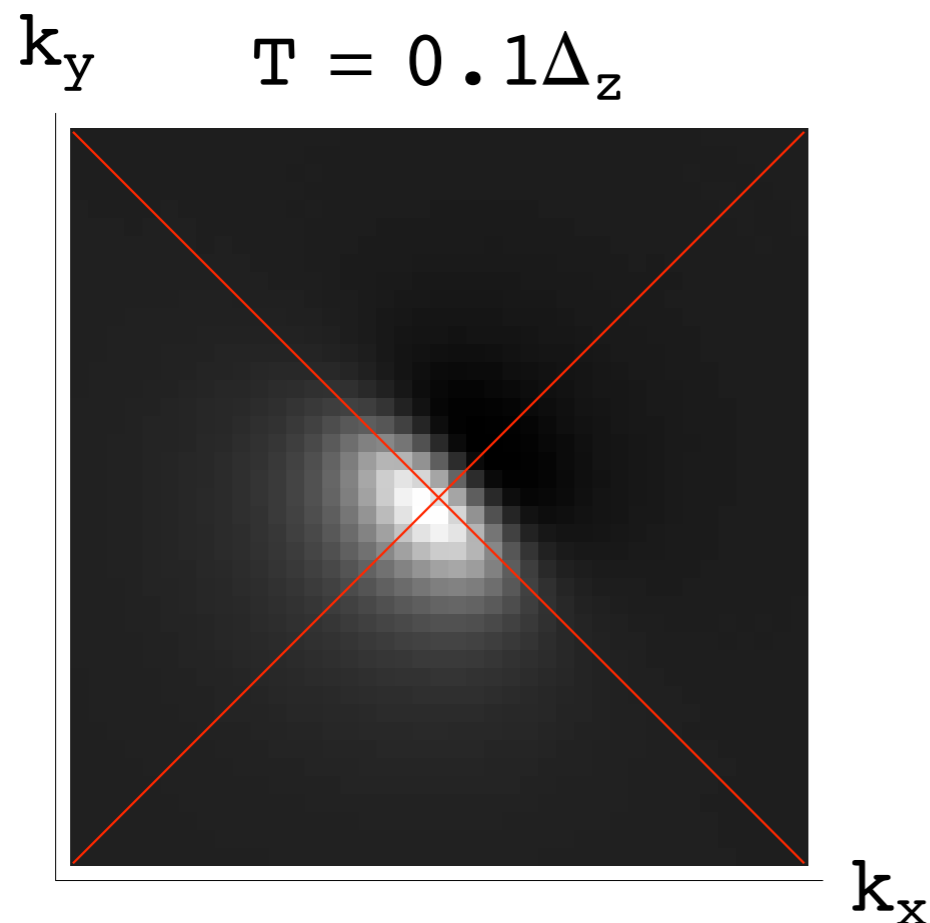


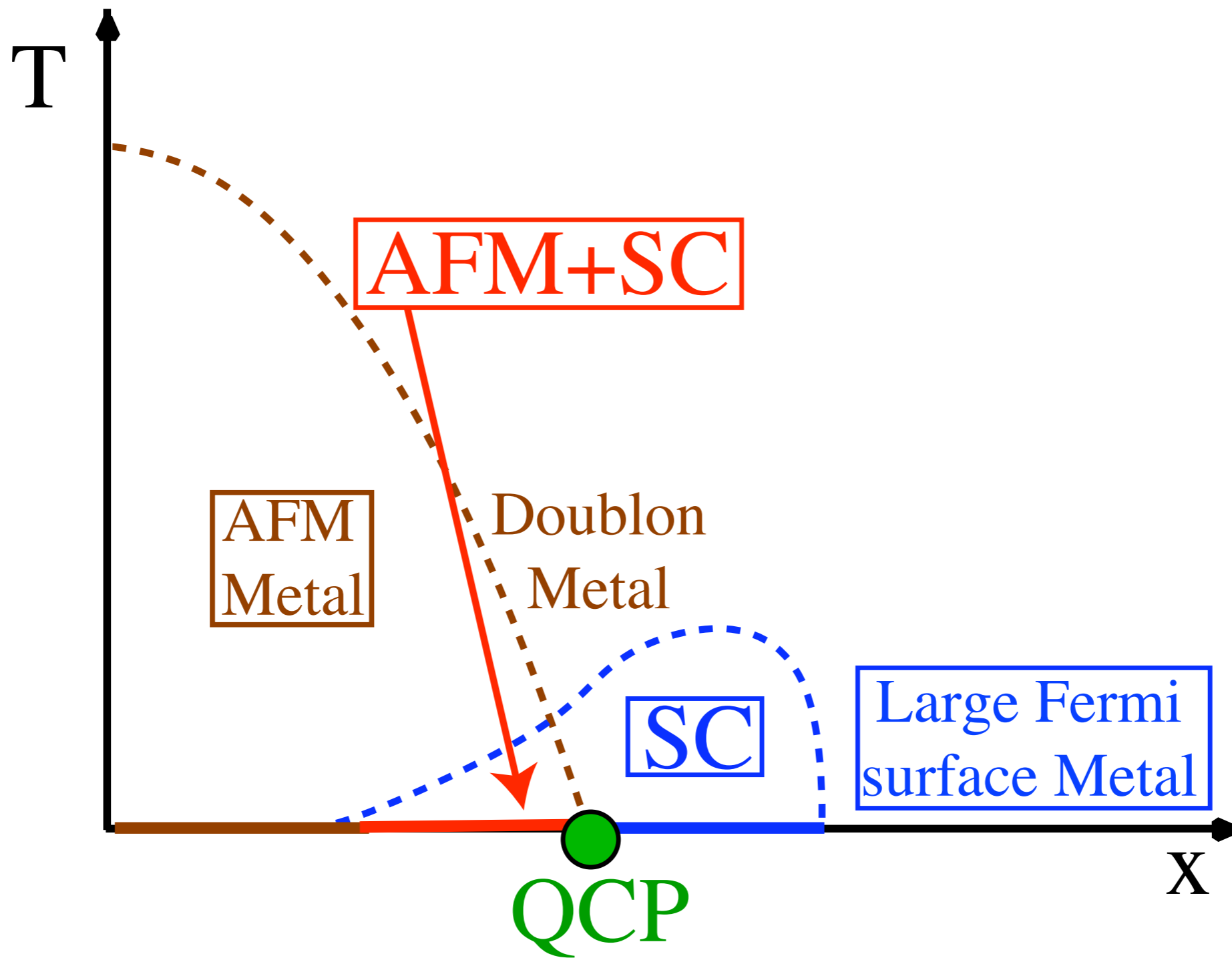
Topological phases

Doublon/holon Metal

$\langle z_\alpha \rangle = 0$, spinons gapped , Fermi surfaces of f_\pm, g_\pm

Monopoles suppressed by Fermi surfaces. Captures features of finite T pseudogap regime of the cuprates





Quantum phase transition between AFM metal and holon/doublon metal:

Fermi surfaces quench A_μ fluctuations in a “fermionic Higgs mechanism”:

$$\mathcal{S}_A = \int d\omega \int d^2k \left[A_i^2 \left(\frac{|\omega|}{k} + \chi_d k^2 \right) + A_\tau^2 \left(\chi + \frac{|\omega|}{k} \right) \right]$$

So the critical theory is just the O(4) model:

$$\mathcal{S}_z = \int d^2r d\tau \left[|\partial_\mu z_\alpha|^2 + s |z_\alpha|^2 + u (|z_\alpha|^2)^2 \right]$$

Important consequence: The Néel order parameter has anomalous dimension $\eta_N \approx 1.37$.

t - J model of electrons

AFM metal

g_{\pm} Fermi pockets, $\langle z_{\alpha} \rangle \neq 0$
Monopoles suppressed

O(4) criticality

Doublon metal

g_{\pm} Fermi pockets, $\langle z_{\alpha} \rangle = 0$
Monopoles suppressed

t - J model of bosons

AFM boson superfluid

$\langle g_{\pm} \rangle \neq 0$, $\langle z_{\alpha} \rangle \neq 0$
Monopoles suppressed

O(4) criticality

Paired boson superfluid

$\langle g_{\pm} \rangle \neq 0$, $\langle z_{\alpha} \rangle = 0$
Monopoles suppressed

Comparison with a toy t - J model in which the g_{\pm} and f_{\pm} are bosons

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4. Destruction of Neel order in the superconductor

(a) The holon superconductor

(b) Confinement transition to BCS superconductors

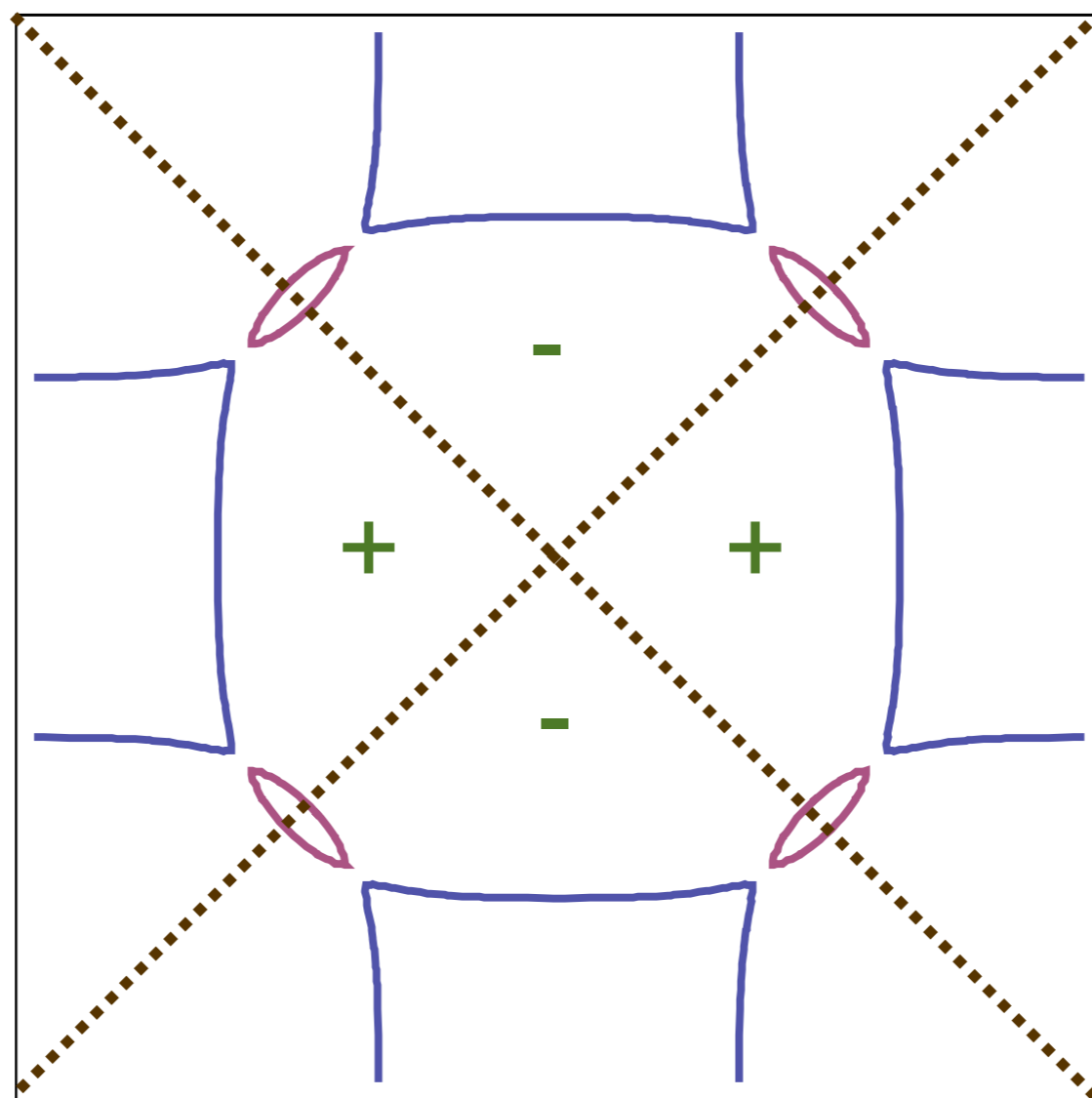
Conventional phases

AF d-wave superconductor

$$\langle z_\alpha \rangle \neq 0$$

$$\langle g_+ g_- \rangle \neq 0, \text{ } s\text{-wave pairing}$$

$$\langle f_+ f_- \rangle \neq 0, \text{ } p\text{-wave pairing}$$



4 Dirac points
(+ 4 shadows)

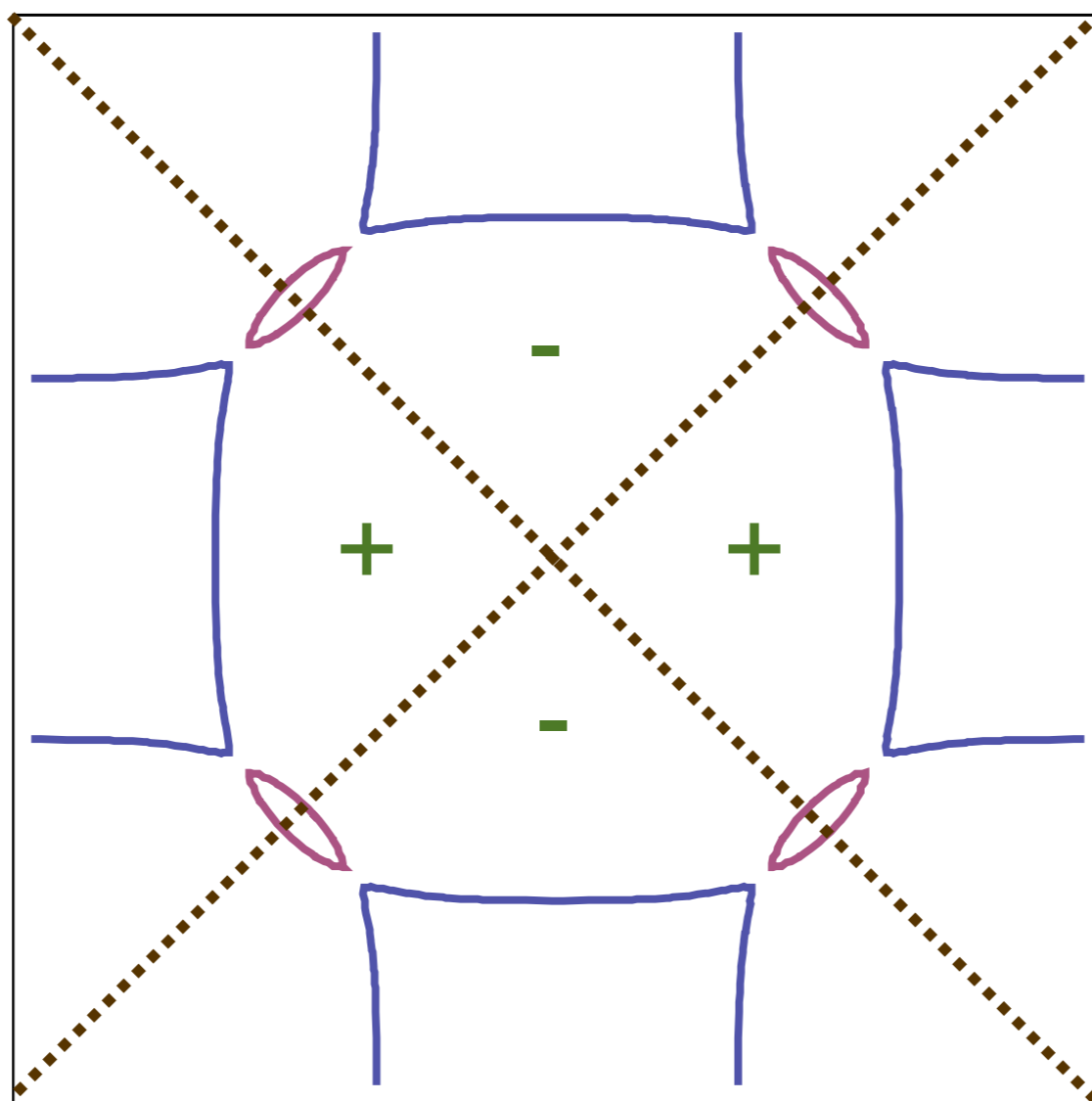
(a)

Topological phases

Holon superconductor

$$\langle z_\alpha \rangle = 0, \quad \langle f_+ f_- \rangle \neq 0, \quad \langle g_+ g_- \rangle \neq 0, \quad \text{Dirac fermions of } f_\pm$$

Possibly stable if monopoles are suppressed by 4 Dirac fermions.
Exotic superconductor with a spin gap, but with heat/charge transport controlled by a CFT.



4 Dirac points
(+ 4 shadows)

(a) Topological phases

Holon superconductor

$\langle z_\alpha \rangle = 0$, $\langle f_+ f_- \rangle \neq 0$, $\langle g_+ g_- \rangle \neq 0$, Dirac fermions of f_\pm

Possibly stable if monopoles are suppressed by 4 Dirac fermions. Exotic superconductor with a spin gap, but with heat/charge transport controlled by a CFT.

Fundamental property: The superfluid density, ρ_s , has the following x and T dependence:

$$\rho_s(x, T) = cx - \mathcal{R}k_B T$$

where c is a non-universal constant and \mathcal{R} is a universal constant obtained in a $1/N$ expansion ($N = 4$ is the number of Dirac fermions):

$$\mathcal{R} = 0.4412 + \frac{0.307}{N} + \dots$$

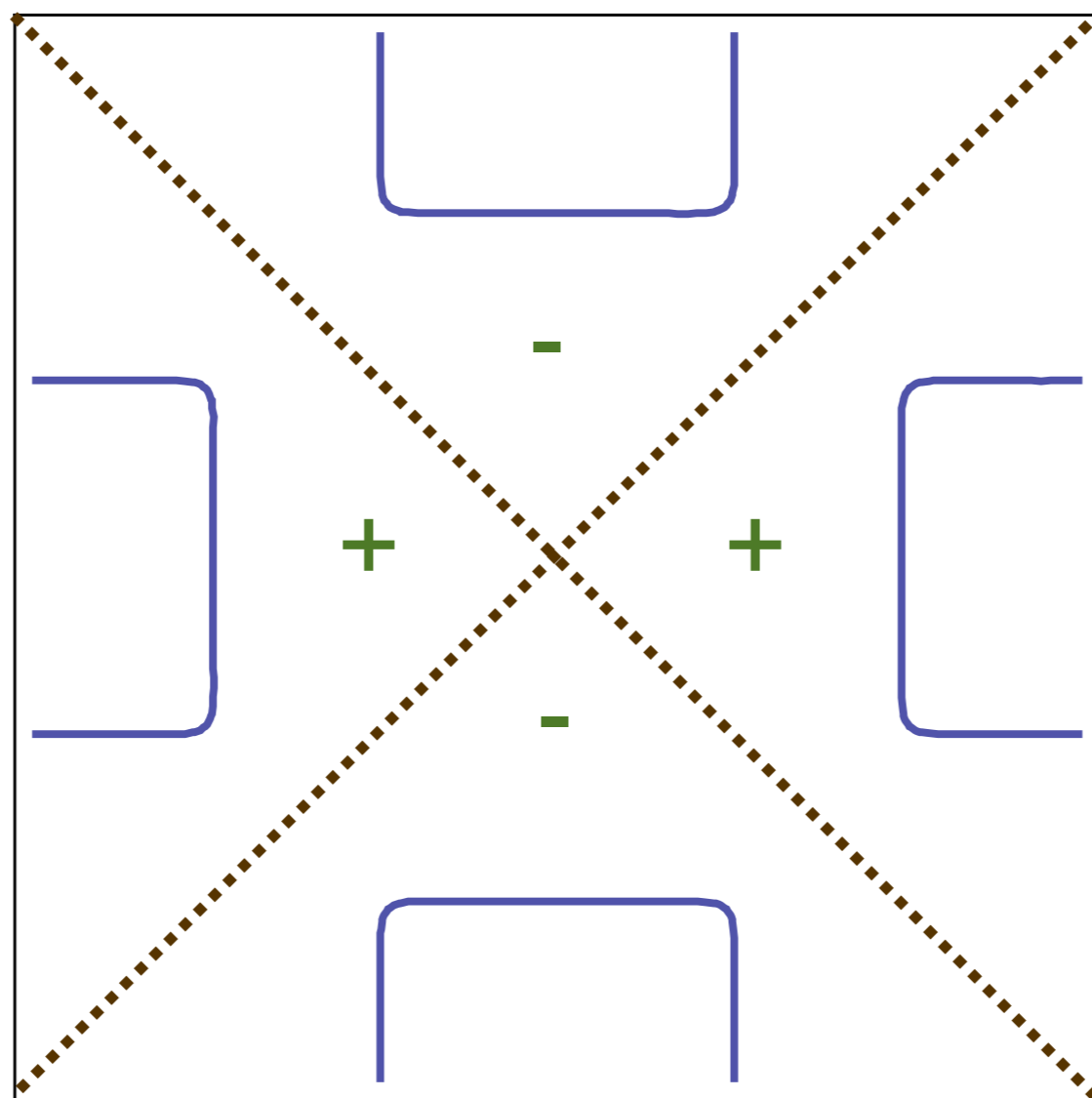
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Fermions fully gapped

(b)

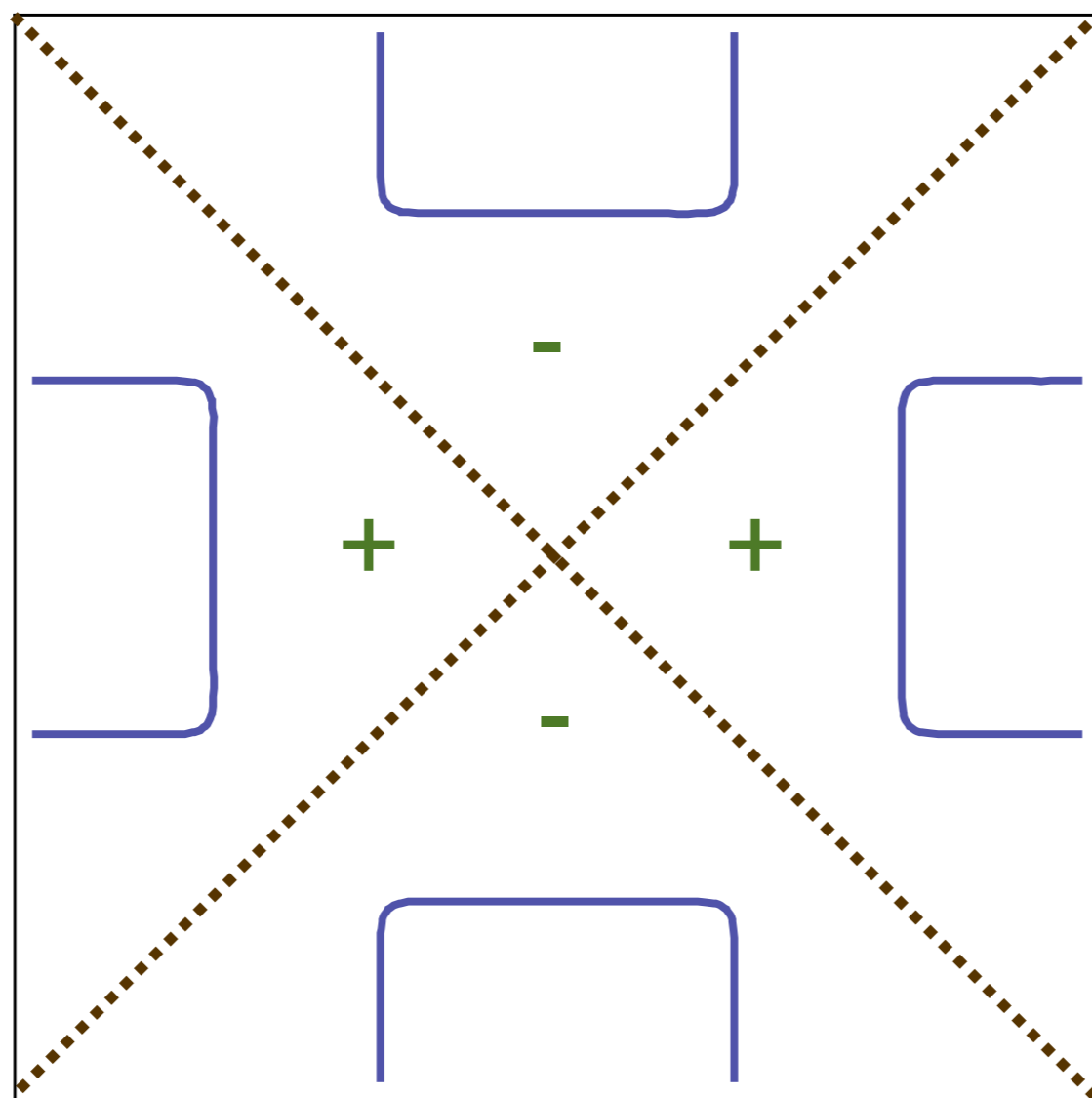
Conventional phases

BCS superconductor/supersolid

$$\langle z_\alpha \rangle = 0, \quad \langle g_+ g_- \rangle \neq 0, \quad \langle f_+ f_- \rangle \neq 0$$

Absence of low energy fermions causes monopoles to proliferate.
(This could also be the fate of the holon superconductor.)

What is the nature of the resulting confining state ?



Fermions fully gapped

t - J model of electrons

AFM metal

g_{\pm} Fermi pockets, $\langle z_{\alpha} \rangle \neq 0$
Monopoles suppressed

O(4) criticality

Doublon metal

g_{\pm} Fermi pockets, $\langle z_{\alpha} \rangle = 0$
Monopoles suppressed

t - J model of bosons

AFM boson superfluid

$\langle g_{\pm} \rangle \neq 0$, $\langle z_{\alpha} \rangle \neq 0$
Monopoles suppressed

O(4) criticality

Paired boson superfluid

$\langle g_{\pm} \rangle \neq 0$, $\langle z_{\alpha} \rangle = 0$
Monopoles suppressed

Comparison with a toy t - J model in which the g_{\pm} and f_{\pm} are bosons

t - J model of electrons

AFM metal

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Monopoles suppressed

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Paired boson superfluid

$\langle g_{\pm} \rangle \neq 0$, $\langle z_{\alpha} \rangle = 0$
Monopoles suppressed

AFM paired boson superfluid

$\langle g_{\pm} \rangle = 0$, $\langle g_+ g_- \rangle \neq 0$, $\langle z_{\alpha} \rangle \neq 0$
Monopoles suppressed

CP¹ criticality

Paired boson supersolid with periods
determined by total boson density

$\langle g_{\pm} \rangle = 0$, $\langle g_+ g_- \rangle \neq 0$, $\langle z_{\alpha} \rangle = 0$
Monopoles proliferate

t - J model of electrons

AFM metal

g_{\pm} Fermi pockets, $\langle z_{\alpha} \rangle \neq 0$
Monopoles suppressed

O(4) criticality

Doublon metal

g_{\pm} Fermi pockets, $\langle z_{\alpha} \rangle = 0$
Monopoles suppressed

AFM superconductor

$\langle g_+ g_- \rangle \neq 0$, $\langle z_{\alpha} \rangle \neq 0$
Monopoles suppressed

CP¹ criticality

Large "Fermi surface" d -wave
superconductor or supersolid

$\langle g_+ g_- \rangle \neq 0$, $\langle z_{\alpha} \rangle = 0$
Monopoles proliferate

t - J model of bosons

AFM boson superfluid

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AFM paired boson superfluid

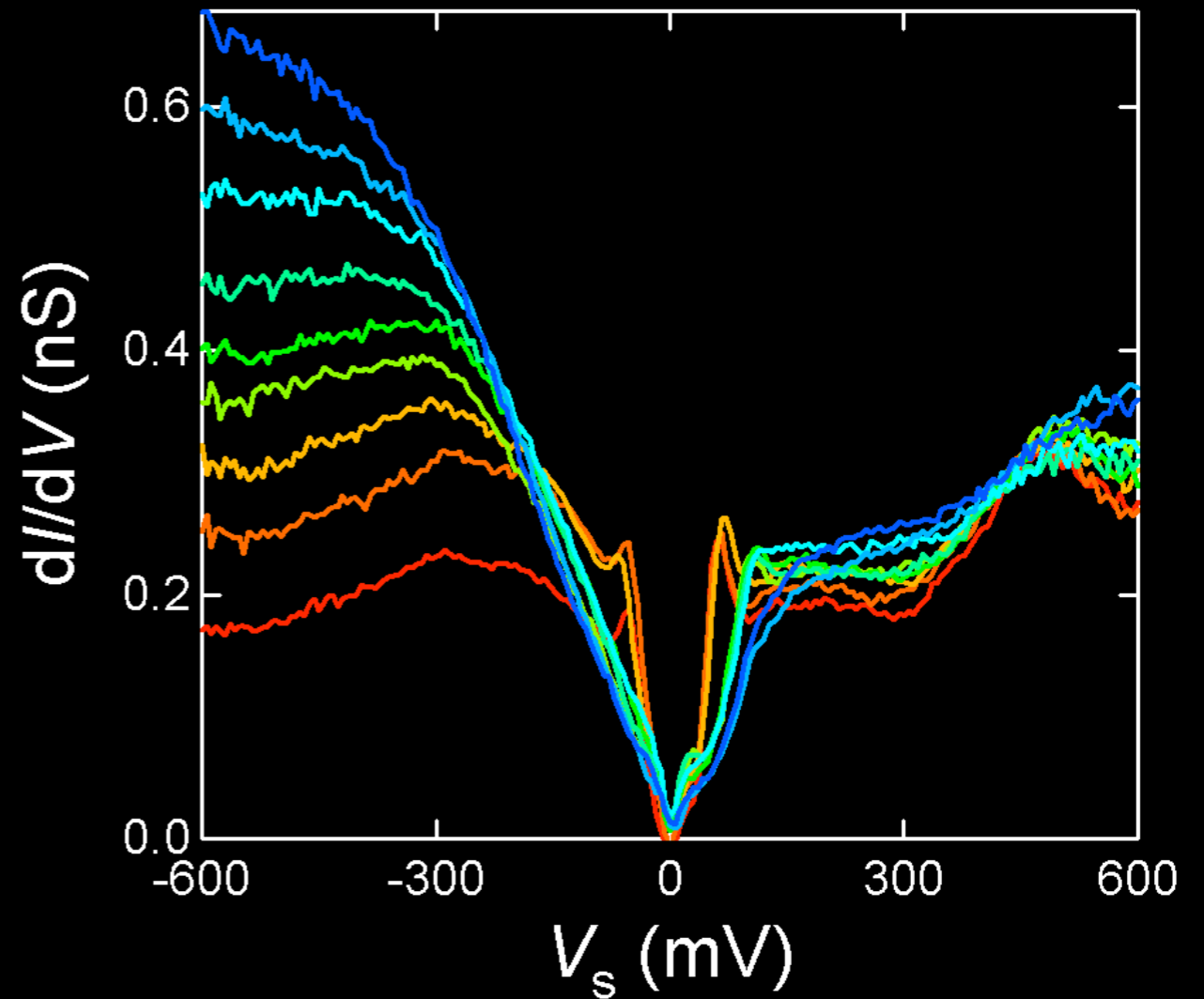
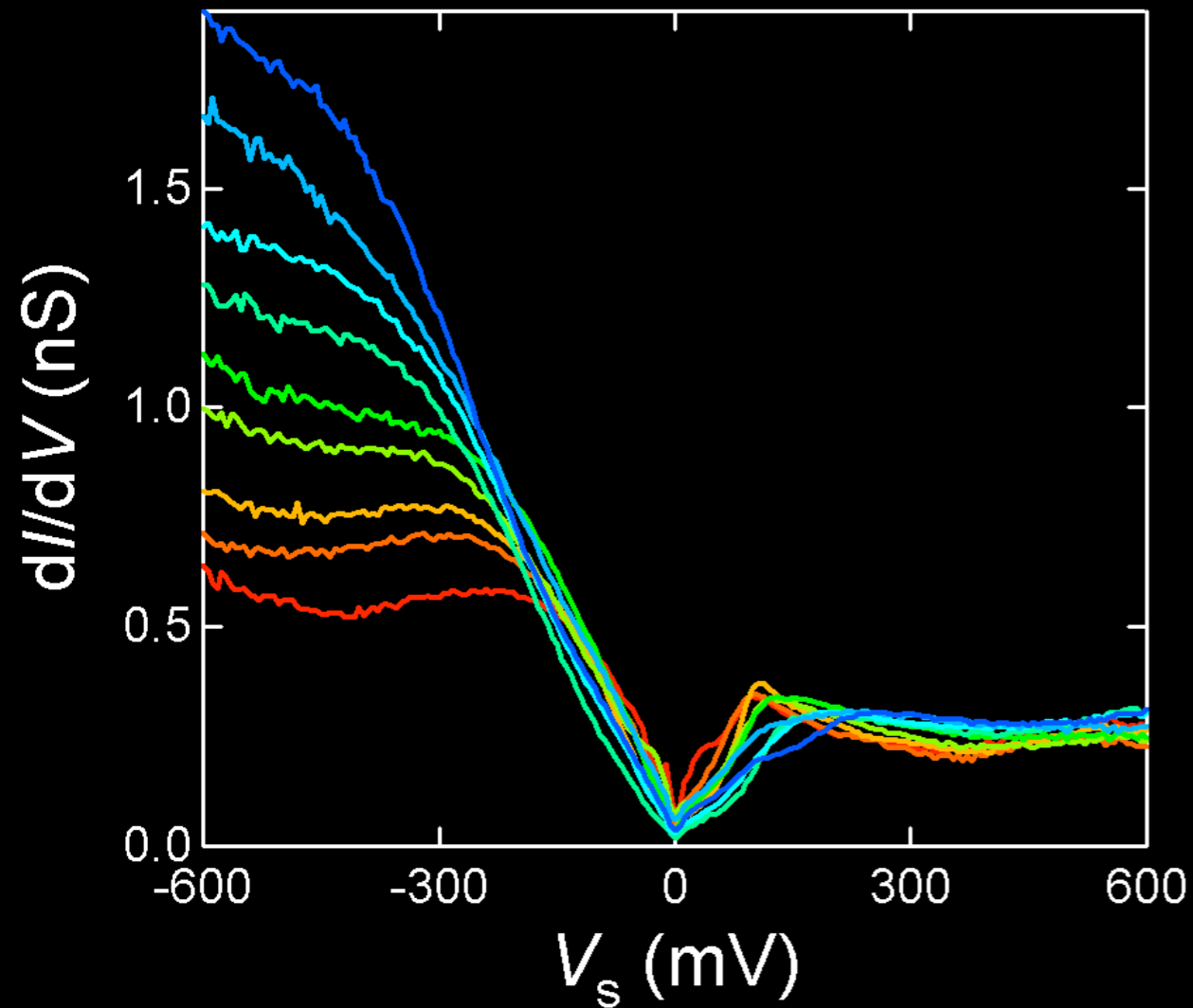
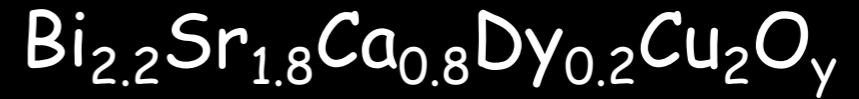
$\langle g_{\pm} \rangle = 0$, $\langle g_+ g_- \rangle \neq 0$, $\langle z_{\alpha} \rangle \neq 0$
Monopoles suppressed

CP¹ criticality

Paired boson supersolid with periods determined by total boson density

$\langle g_{\pm} \rangle = 0$, $\langle g_+ g_- \rangle \neq 0$, $\langle z_{\alpha} \rangle = 0$
Monopoles proliferate

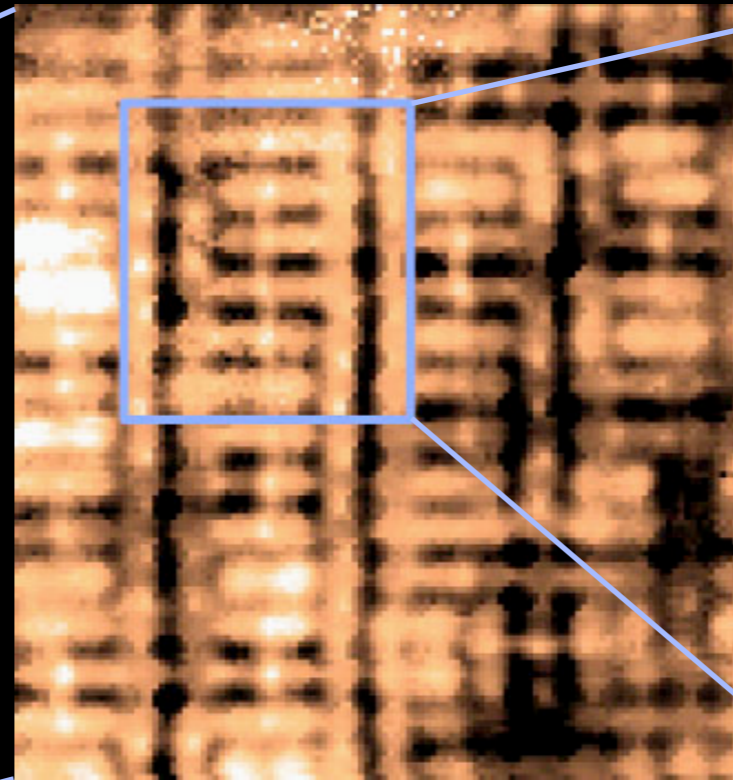
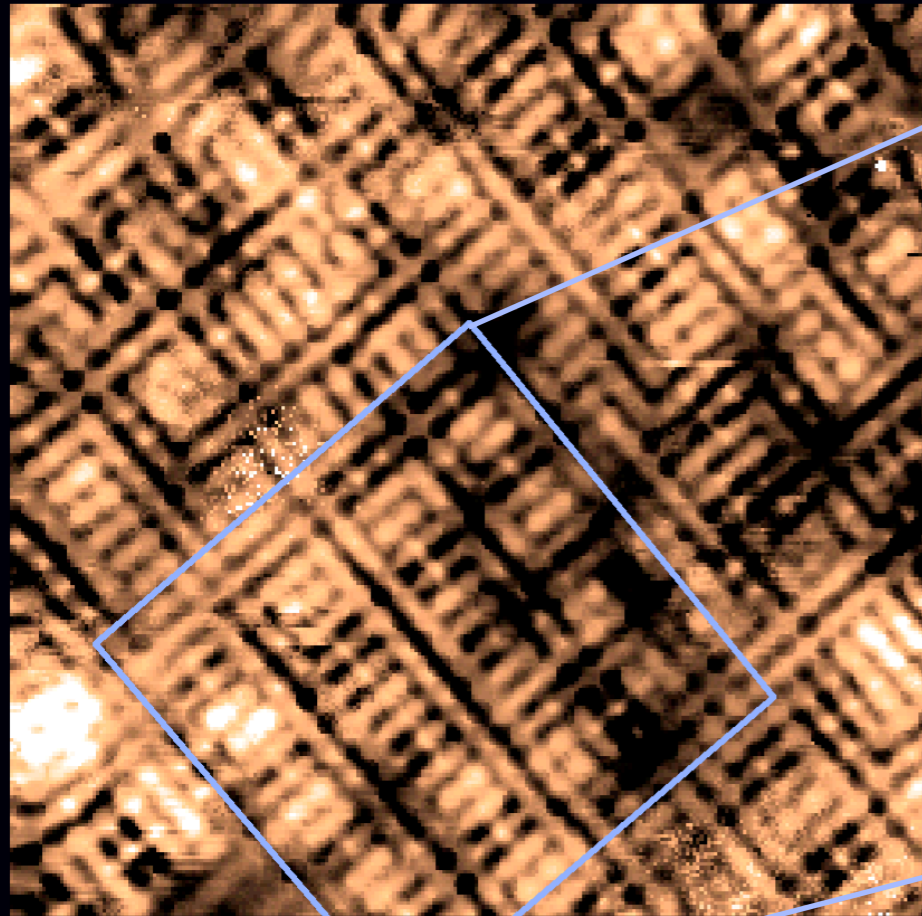
dI/dV Spectra



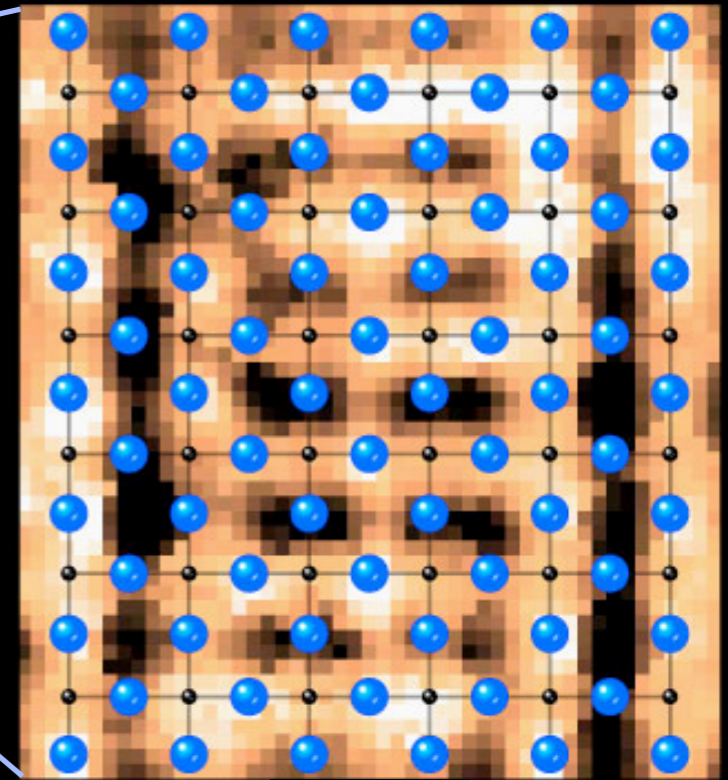
Intense Tunneling-Asymmetry (TA)
variation are highly similar

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

R map (150 mV)



$\text{Ca}_{1.88}\text{Na}_{0.12}\text{CuO}_2\text{Cl}_2$, 4 K

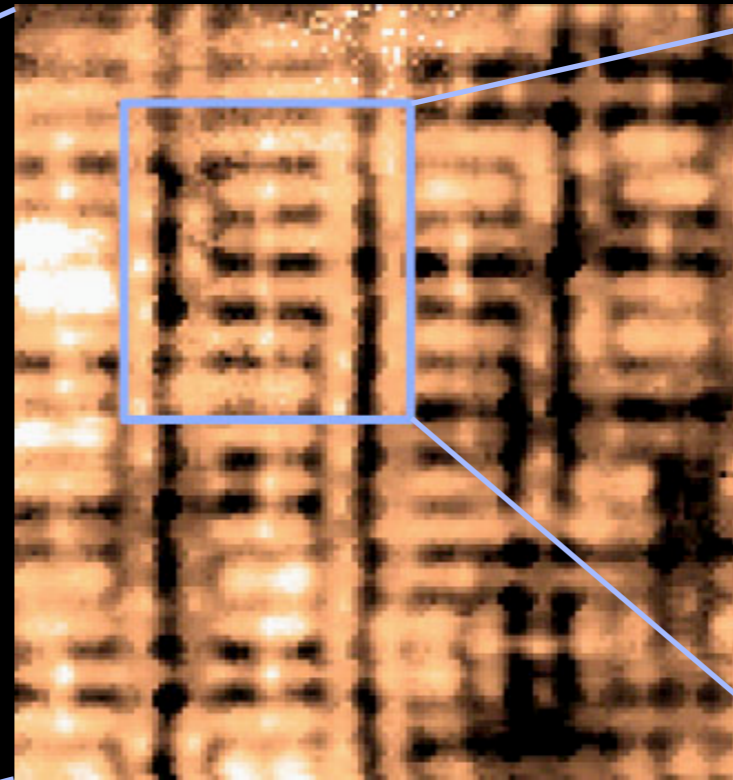
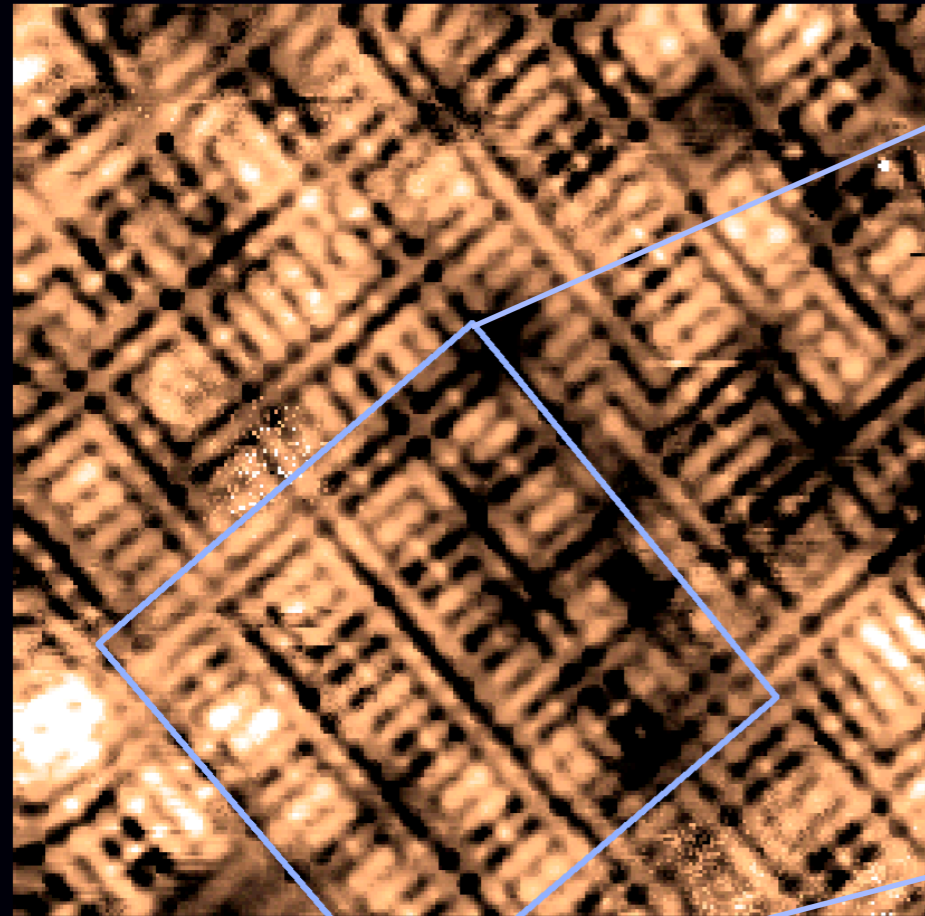


$4a_0$

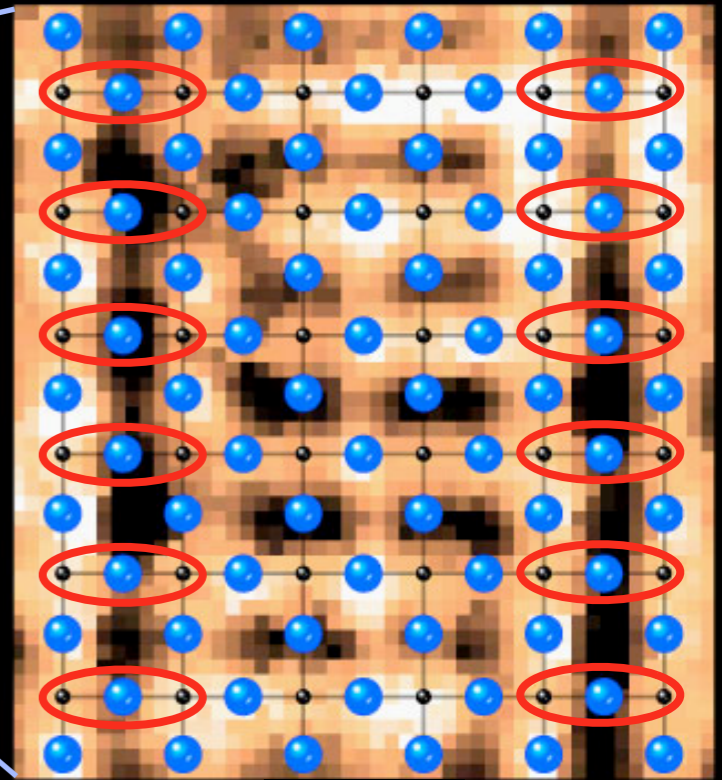
12 nm

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

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$\text{Ca}_{1.88}\text{Na}_{0.12}\text{CuO}_2\text{Cl}_2$, 4 K



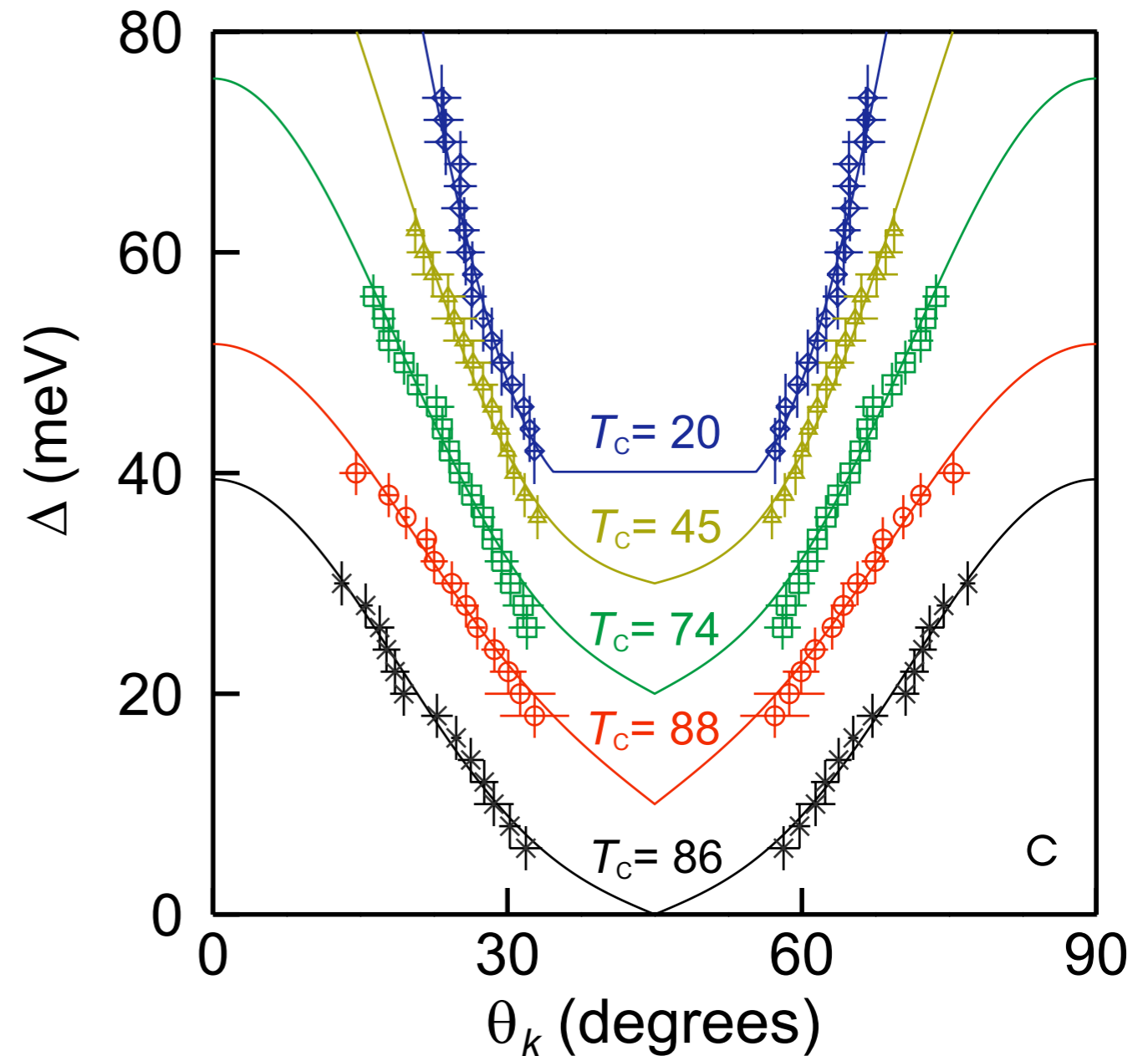
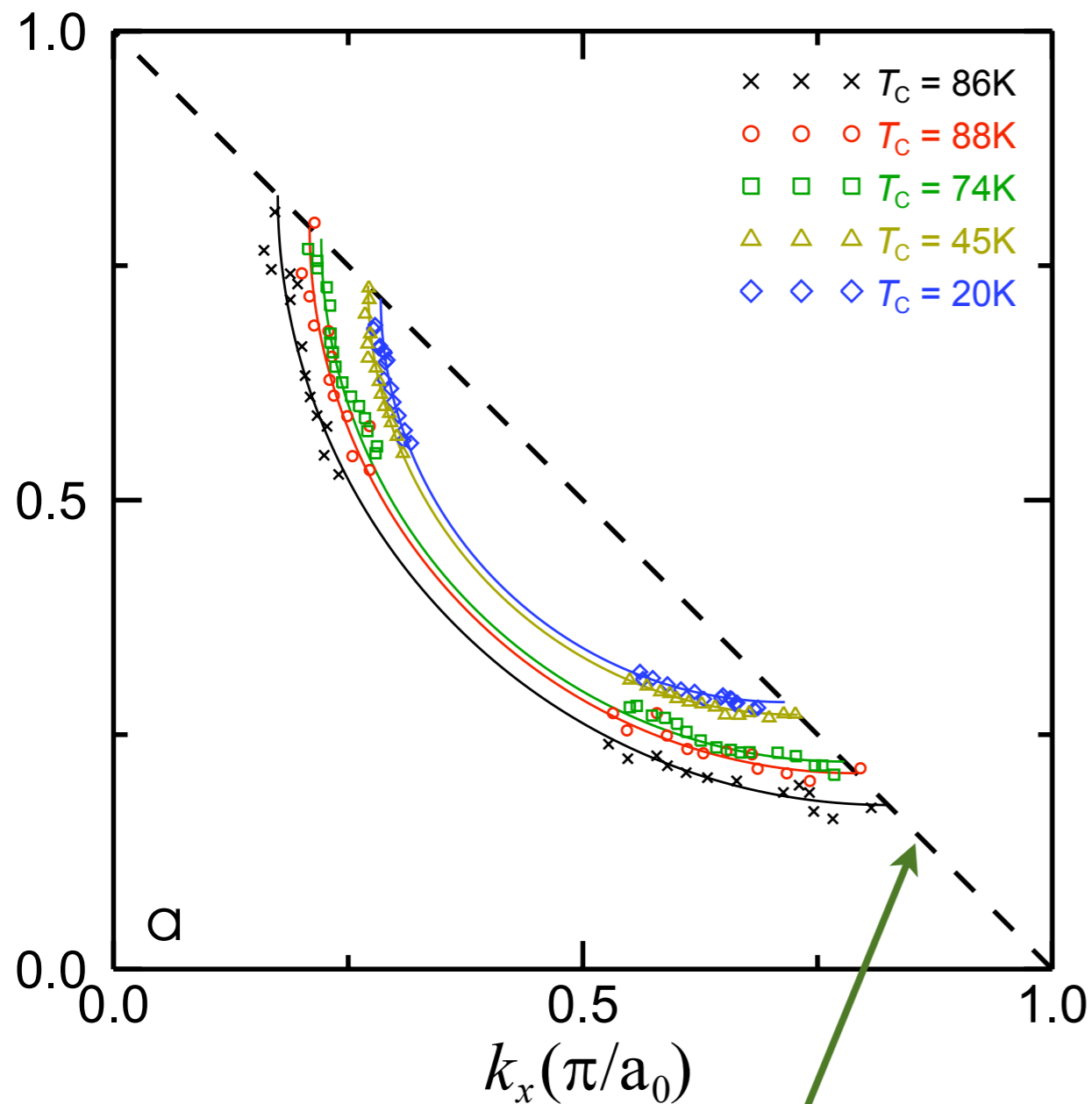
$4a_0$

12 nm

Valence bond supersolid with periods determined by total electron density

S. Sachdev and N. Read, *Int. J. Mod. Phys. B* 5, 219 (1991).
S.A. Kivelson, E. Fradkin, V.J. Emery, *Nature* 393, 550 (1998).
M. Vojta and S. Sachdev, *Phys. Rev. Lett.* 83, 3916 (1999).

Quasiparticle spectra from STM on BSCCO



Note role of AF zone boundary

Y. Kohsaka, C. Taylor, P. Wahl, A. Schmidt, Jinhwan Lee, K. Fujita, J. Alldredge, Jinho Lee, K. McElroy, H. Eisaki, S. Uchida, D.-H. Lee, & J.C. Davis, preprint

(b) Conventional phases

BCS superconductor/supersolid

$$\langle z_\alpha \rangle = 0, \quad \langle g_+ g_- \rangle \neq 0, \quad \langle f_+ f_- \rangle \neq 0$$

Absence of low energy fermions causes monopoles to proliferate.
(This could also be the fate of the holon superconductor.)

What is the nature of the resulting confining state ?

Can the confining state be a large Fermi surface BCS superconductor with no broken translational symmetry ?

(b) Conventional phases

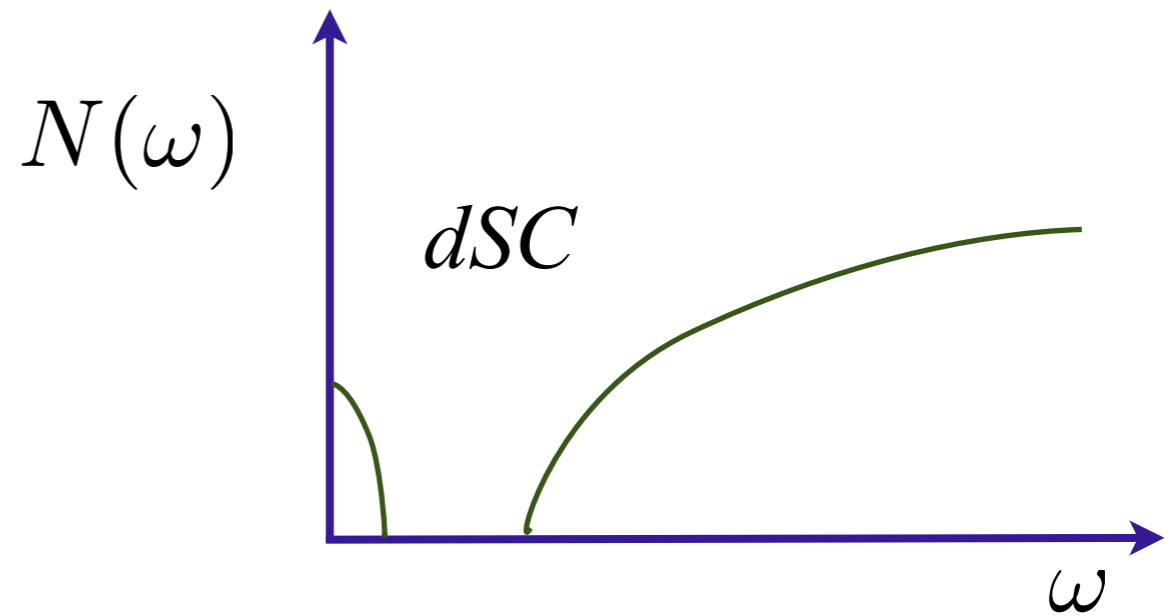
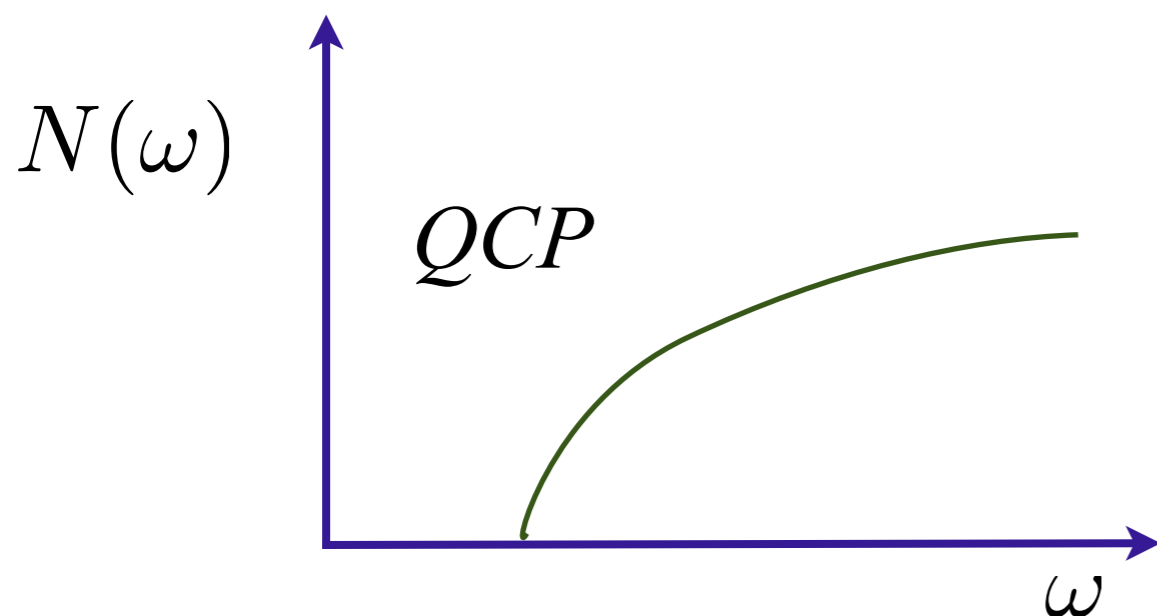
BCS superconductor/supersolid

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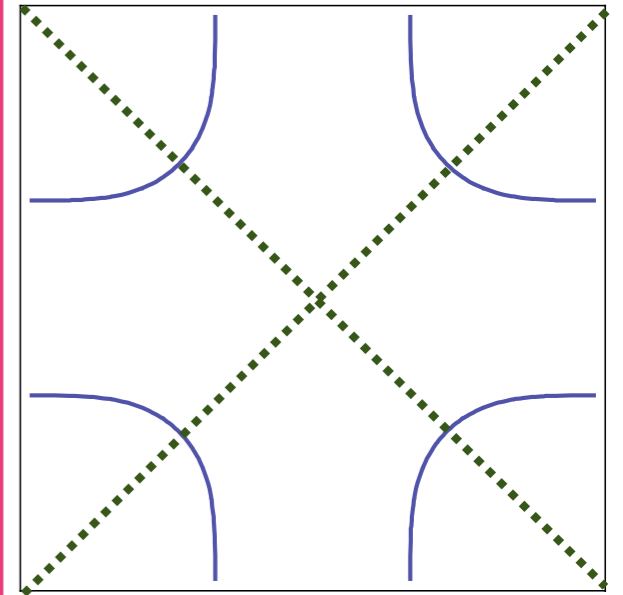
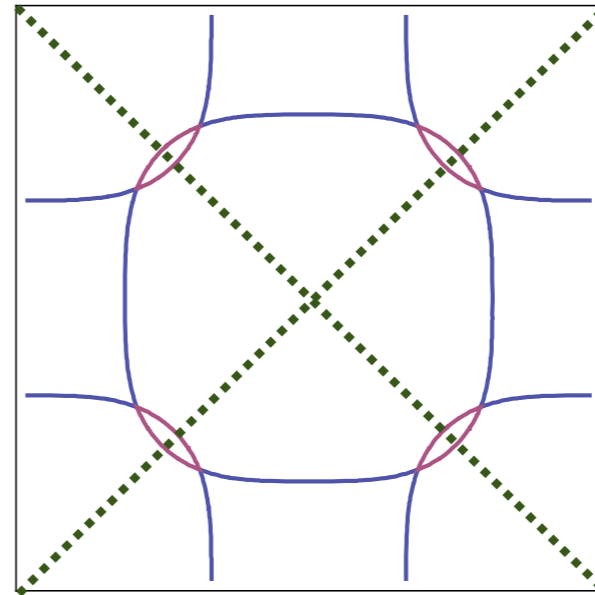
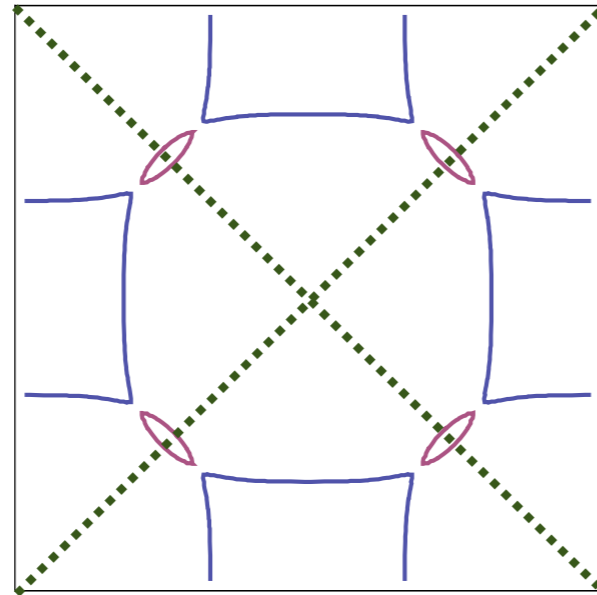
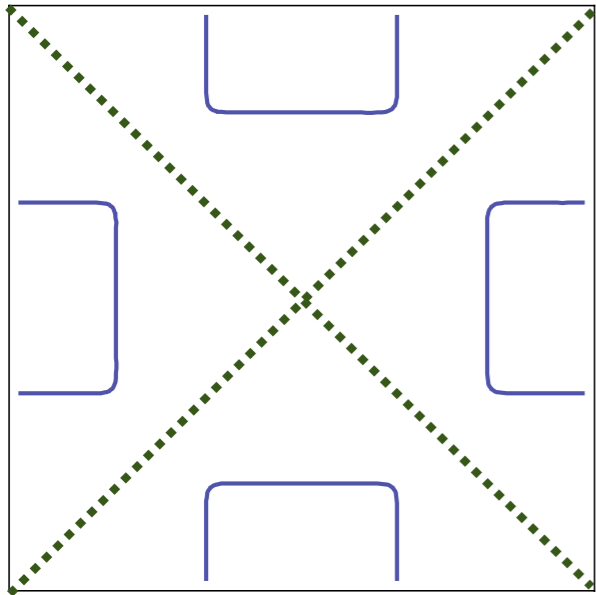
Absence of low energy fermions causes monopoles to proliferate.
(This could also be the fate of the holon superconductor.)

What is the nature of the resulting confining state ?

Can gapless fermions appear in the confining state, even though they are absent at the QCP ?



← Néel order —



dSC - no nodes

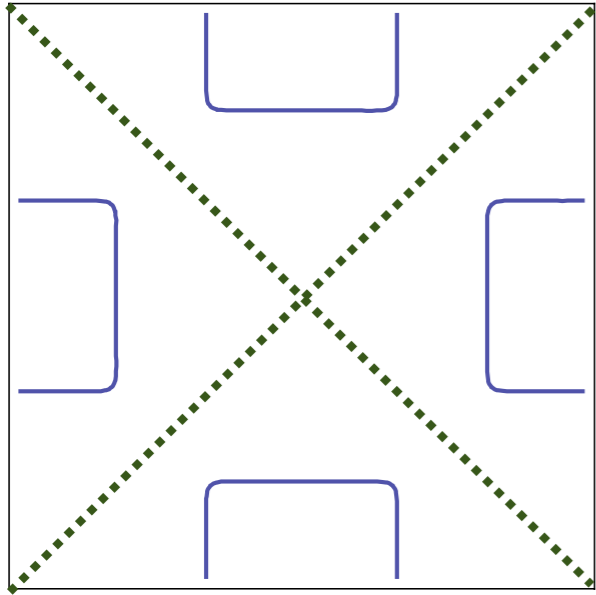
dSC - 8 nodes

dSC - 8 nodes

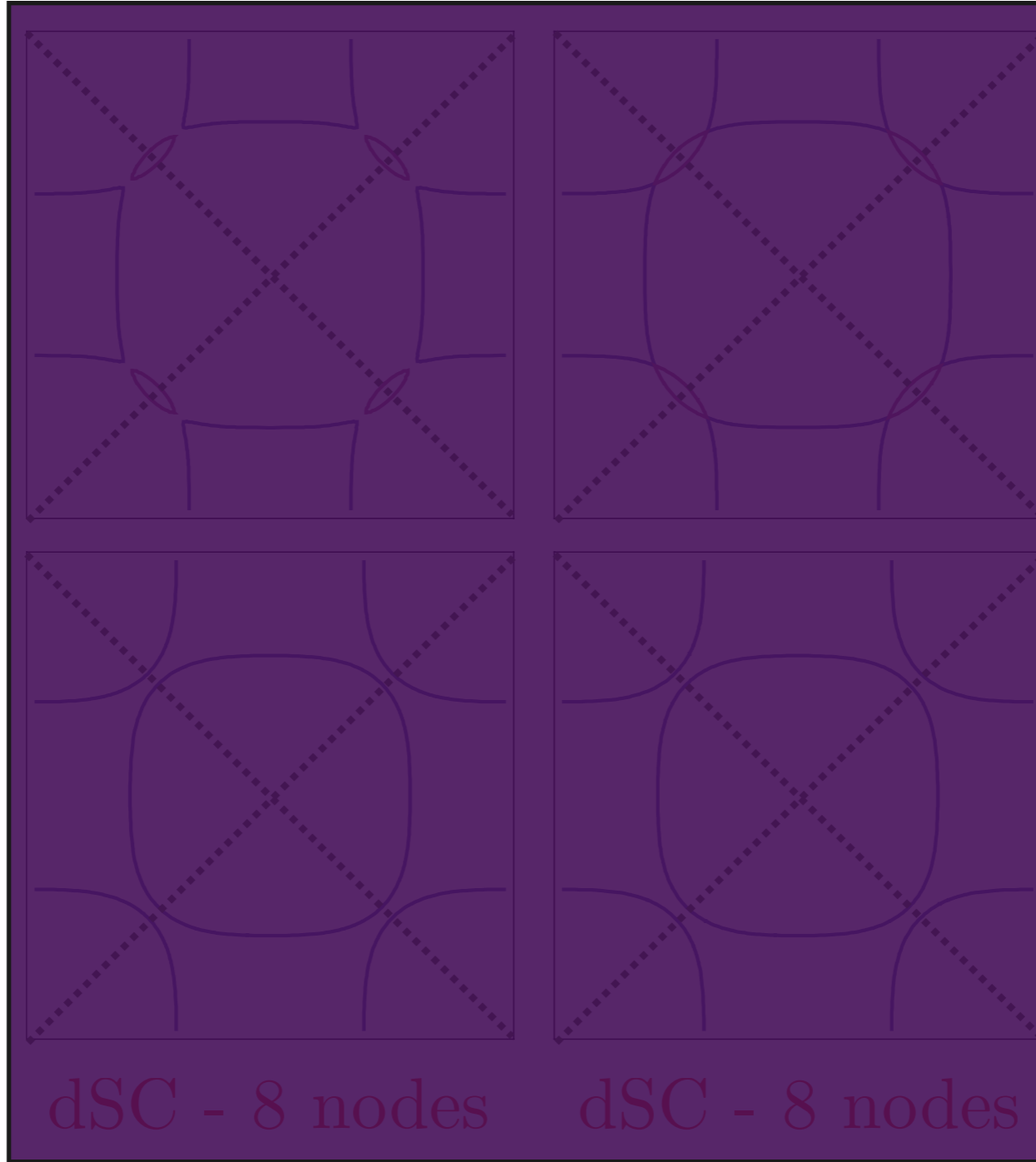
dSC - 4 nodes

→ x

← Néel order —

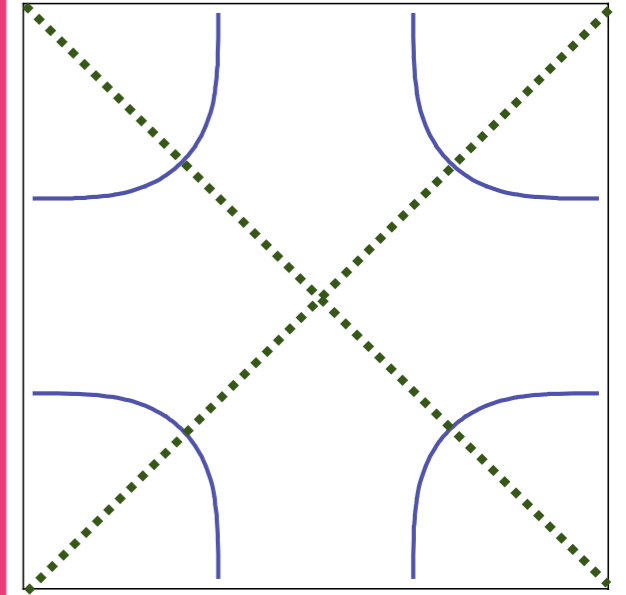


dSC - no nodes



dSC - 8 nodes

dSC - 8 nodes



dSC - 4 nodes

→ x

